ON THE THREE-DIMENSIONAL INSTABILITY OF ELLIPTICAL VORTICES SUBJECTED TO STRETCHING

S. LE DIZÈS
I.R.P.H.E., Universités d'Aix-Marseille I & II, CNRS, 12 Av. Général Leclerc, F-13003 Marseille, France.

M. ROSSI
L.M.M., Université de Paris VI, 4 Place Jussieu, F-75252 Paris cedex 05, France.

AND

H. K. MOFFATT
D.A.M.T.P., University of Cambridge, Silver street, Cambridge CB3 9EW, England.

The presence of organized structures in turbulent flows has been recently emphasized by physical and numerical experiments. In particular, vorticity is mainly concentrated in localized tube-like regions: the so called "worms" or "sinews" (Moffatt *et al.*, 1994). If one agrees that such local structures are important for the global turbulent field, it is certainly worth studying their elementary dynamical behavior. Furthermore, it is known that two-dimensional vortices are subjected to generic three-dimensional instabilities (Waleffe, 1990). This phenomenon, located near the core of vortices, depends on the eccentricity of their streamlines. Since vortices present in turbulent flows are not purely two-dimensional and experience a three-dimensional strain arising from the mean field, it is natural to ask how the elliptical instability carries on when a velocity component is added along the span of the previously two-dimensional vortex.

In this work, we specifically consider the stability of the core of a three-dimensional elliptical vortex subjected to an axial strain. For this case, an exact time-dependent solution can be found. The stability of such a flow is analyzed by linearizing the Navier-Stokes equations around this basic state. The complete system for the perturbations is reduced to a single equation for the perturbed velocity along the vortex span. This equation is unfortunately non-separable in space and time variables: standard Fourier analysis is hence of no use and one should resort to a new method to solve the

S. Gavrilakis et al. (eds.), Advances in Turbulence VI, 355-356.

problem. Fortunately enough, one is guided by the specific case of a pure solid body rotation. It is known that this flow supports neutral waves called inertial waves for which analytical expressions are available. In this work, we construct the equivalent of these solutions for the general case under study. Time-dependent equations for both amplitude and wavenumber are thus obtained. It is shown that the wavenumber modulus and angle with respect to the span increase while this vector rotates more and more rapidly around the same axis. In the limit of weak stretching and weak ellipticity, a perturbation theory can be performed and leads to a WKBJ approximation for the solution. This procedure shows that the linearized Navier-Stokes equations are transformed into a Hill equation with slowly evolving coefficients. According to this slow variation, it is possible to understand the perturbation evolution as follows. When no stretching is applied, the elliptical instability is active: an initial perturbation is unstable when its angle with the span is located inside an interval around $\pi/3$. When stretching is present, the wavector always crosses this unstable region during a finite period of time. In that area, the perturbation amplitude hence grows and an exponential factor can be expected for the global amplification. When the effect of stretching becomes important compared to the destabilizing effect of eccentricity, the wavevector remains in the unstable area during a period of time which is too short to let the perturbation reach a sufficiently large amplitude. As a result, the vortex structure is not affected by the three-dimensional elliptical instability.

The global amplification factor can indeed be computed which demonstrates that a small amount of stretching is capable to prevent the appearance of three-dimensional instabilities for vortices with a low enough eccentricity. Since most vortices are slightly elliptical (Moffatt *et al.*, 1994) in turbulent flows, the above computations are expected to cover a wide range of experimental cases. In particular, it is tentatively argued that this mechanism may explain recent experimental observations (Cadot *et al.*, 1995).

References

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