Book Review - *Singularities: Formation, Structure, and Propagation*. J. Eggers & M. A. Fontelos. Cambridge Texts in Applied Mathematics, Cambridge University Press, 2015. Paperback, 453+xvi pp. ISBN 9781107485495. £39.99.

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Abstract

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Just over 50 years ago, Von Foerster, Mora & Amiot (1960) provided an intriguing discussion of human population growth in terms of the equation $\mathrm{d}N/\mathrm{d}t = kN^{\lambda}$, where $N(t)$ is the population at time t , and k and λ were constants chosen to provide the best fit for the growth of $N(t)$ over the previous two millennia. They found that $\lambda = 1.99$, which, being greater than 1, leads inevitably to a singularity ($N = \infty$) within a finite time; their (tongue-in-cheek) prediction of the date of this 'doomsday' crisis was Friday 13th November 2026!

Eggers and Fontelos start their discussion of singularities with this beautifully simple example, which immediately brings into focus the dramatic character of a 'finite-time singularity' (in contrast to merely exponential growth), and the need to resolve such singularities, given that no measurable physical variable can in reality become infinite within a finite time. This sort of example can serve as a warning to those who seek to demonstrate a finite-time singularity for the Euler equations of inviscid incompressible flow: any computational evidence of approach to a singularity of vorticity is likely to fail as the alleged singularity is approached; computation must always be supplemented by rigorous asymptotic theory revealing the spatial structure of the singularity and related characteristics.

To provide such a theory is the main purpose and achievement of this remarkable book which starts with an eye-catching introductory chapter 'What are singularities all about?', describing singularities, wherever they may occur, as the 'fingerprint' of the governing nonlinear partial differential equation. At the outset, the authors introduce scaling and the related dimensional analysis, with particular reference to (i) the problem of drop pinch-off at time t^* (when the minimum connecting thread radius at time t scales like $(t^* - t)^{2/3}$), and (ii) G. I. Taylor's (1950) treatment of a blast wave from an intense explosion (which gives the radius of the blast increasing like $t^{2/5}$). Although such results follow from dimensional analysis, a certain degree of physical intuition, indeed inspiration, is needed to first identify the dominant physical processes involved (surface tension and inertia in the former case, explosion energy and inertia in the latter). Such considerations play an important part in the very wide range of phenomena treated in subsequent chapters.

Singularities usually involve some kind of self-similar behaviour associated with the scaling. The above examples, which involve the exponents $2/3$ and $2/5$ respectively, both exhibit what Barenblatt (1996) called 'self-similarity of the first kind', for which the exponents are determined by dimensional analysis. Barenblatt distinguished this from 'self-similarity of the second kind', for which a scaling exponent can only be determined as an eigenvalue of a problem involving both a partial differential equation and its boundary conditions: two-dimensional flow in a corner driven by remote stirring (treated in chapter 5) provides a good example. (But I note that, in § 3.1, Eggers & Fontalo introduce the term 'self-similarity of the second kind' in a different sense to describe self-similarity for which there are two or more intrinsic length scales; it is not clear to what extent the two uses of the term are equivalent.)

The authors then discuss the generic character of cusp singularities, as formed when, for example, a vertical jet of viscous fluid impinges on a bath of the same fluid. In this situation, a locally two-dimensional cusp (located on a horizontal circle) is generated by viscosity and resolved by surface tension γ which yields a finite radius of curvature, exponentially small as $\gamma \to 0$; this type of cusp is treated later in detail in chapter 14, where the very singular character of the cusping phenomenon is revealed. In a purely mathematical sense, surface tension does indeed resolve the singularity, but from a physically realistic point of view, the 'cusp' remains singular, its radius of curvature being so small as to be at the sub-molecular level, and, as Eggers himself has demonstrated, further resolution that takes account of air pressure within the cusp region is required.

A picture of a cusp (from the experiment of du Pont & Eggers 2006) is shown on the first page of chapter 1: this however is the axisymmetric cusp that can form on the free surface of a viscous fluid that is drained through a hole in the bottom of the container. The problem, of resolving this type of cusp is very difficult, and not amenable to the sort of analysis that works for the corresponding two-dimensional problem; as far as I know, this challenging problem is still unsolved.

Each of the fifteen chapters is supplemented by a set of exercises some more testing than others. Thus for example, even as early as page 13, Exercise 1.6 challenges the reader to show that a drop of dye placed in a stationary fluid will diffuse like $t^{1/2}$, but like $(\epsilon t^3)^{1/2}$ (Richardson's Law) if the fluid is in turbulent motion with energy input ϵ per unit time and mass. The power of dimensional analysis is vividly revealed by such examples.

The chapters progress through a wide range of problems and the techniques needed to solve them. Among these are: Stokes waves, the Taylor cone in electrohydrodynamics, corner flows in general, drop break-up, singularities in Hele-Shaw flow, mean curvature flow, bubbles and cavities, shock waves, chaotic dynamics, point vortex dynamics, vortex filaments and the local induction approximation, instability of vortex sheets leading to the Moore singularity, Ginzburg–Landau vortices, cusps and caustics, the moving-contact-line problem, and finally including even an account of the singularity associated with crack propagation. This is an incredibly rich and varied diet, and the authors have done well to bring a degree of unity to such a diverse range of topics.

The book will serve as an excellent introduction to the field of singularities in continuum mechanics, and a valuable resource for researchers already immersed in any of the above sub-fields. In short, a wonderful achievement!

References

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