

PERSPECTIVES

APPLIED PHYSICS

Helicity—invariant even in a viscous fluid

Observing and probing writhe and twist in vortex dynamics

By **H. Keith Moffatt**

The vortex ring is a fundamental phenomenon of fluid dynamics, recognized since the seminal investigations of Helmholtz (1) and Kelvin (2). Its familiar manifestation as a “smoke ring” in air derives from the fact that both smoke and vorticity (local fluid spin) are transported with the flow, which is “induced” by the vortex itself; so the smoke

provides a natural visualization of the vorticity (see the photo). Vortex rings can also be generated in water and visualized either by dye or by small air bubbles that migrate to the low-pressure region at the core of the vortex. On page 487 of this issue, Scheeler *et al.* (3) explore a particular property of a vortex ring whose core is helical rather than circular in form. This property, helicity, is an integral over the fluid domain that expresses the correlation between velocity and vorticity, and an invariant of the classical Euler equations of ideal (inviscid) fluid flow. The question addressed by Scheeler *et al.* is the extent to which the helicity remains

invariant when fluid viscosity, unavoidable in reality, is taken into consideration.

Helicity is of special interest because it admits topological interpretation: It is precisely because vortex lines are frozen in the fluid, thus conserving their topology, that helicity is conserved too. As Kelvin recognized, if two vortex tubes are linked, then that linkage survives in an ideal fluid for all time; if a vortex tube is knotted, then that knot survives in the same way for all time. Helicity is the integral manifestation of this invariance: For two linked tubes, it is proportional to the product of the two circulations (each conserved by Kelvin’s circulation

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This “steam ring” was emitted by an eruption of Mount Etna in 2000. The steam marks a turbulent vortex ring, which is transported by its own self-induced velocity. It is only approximately circular, implying that there must be a continuous interchange between twist and writhe helicity as it propagates. It is also subject to slow dissipative decay through the action of viscosity associated with the turbulence.

dye periodically arranged around the edge. The vortex is shed from this trailing edge, carrying the dye with it. This then marks the progress of the vortex as it propagates downstream. The movement of the blobs along the core of the vortex allows direct measurement of the component of velocity instantaneously parallel to the vorticity, leading immediately to determination of the local helicity density.

The authors then generate a helical vortex and a following circular vortex, so that a familiar “leapfrog” regime is established (see the figure). The helical vortex is periodically expanded and contracted, through its interaction with the circular vortex, and the total helicity as well as the writhe and twist are tracked as functions of time. In this situation, the authors demonstrate that, despite the influence of viscosity, helicity is indeed conserved at least during the first complete leapfrog cycle, and precisely by the mechanism of conversion of writhe to twist (and vice versa), as described above.

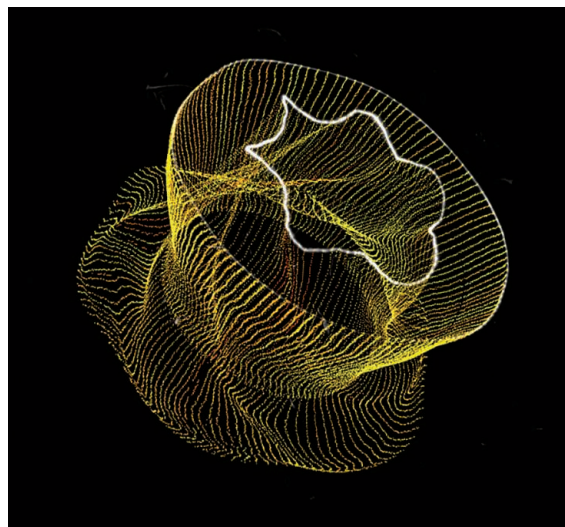
By following a single propagating helical vortex for a longer time, the authors similarly demonstrate that twist is preferentially dissipated by viscosity, the helicity being ultimately entirely associated with writhe, and nearly constant from that point on.

In earlier work, Kleckner and Irvine (6) provided an equally remarkable experimental demonstration of the production of a vortex in the form of a trefoil knot, just as envisaged by Kelvin, who, hoping that such knotted vortices would be stable, based his vortex atom theory on this expectation. Kelvin’s search for stable knotted vortices failed, and the theory was eventually abandoned. The knotted vortex in the experiment of Kleckner and Irvine did indeed turn out to be quite violently unstable, leading to rapid viscous reconnection of vortex lines, with change of topology to two unlinked but deformed vortex rings. Whether helicity is conserved through such reconnection events is still unclear, but there is some evidence that the writhe of the trefoil

is converted at least in part to writhe plus twist within each of the rings that emerge from the reconnection process. Computational investigation of this process, a complex, fully three-dimensional phenomenon, is extremely challenging.

Helicity is of key importance in turbulent thermal convection in a strongly rotating fluid, in which gravity and rotation conspire to generate it. In such contexts, it is vital to understand how helicity inhibits the energy cascade to small scales at which turbulent energy is dissipated as heat; hence, the value of understanding prototypical processes in which attention is focused on the role of helicity.

In the wider context of planetary physics and astrophysics, helicity gains exceptional importance, because it is known that its presence in any large expanse of



Volumetric image of a helical vortex leapfrogging through a vortex ring in water, with dye-blob tracks overlaid in warm colors.

conducting fluid is strongly conducive to dynamo action—that is, to the generation of a large-scale magnetic field. The geomagnetic field is the result of helical convection combined with differential rotation in Earth’s liquid-metal outer core. The magnetic fields of the Sun and stars are similarly generated by helical convection and differential rotation deep within their subsurface convection zones—a research field of central importance in current theoretical astrophysics. ■

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theorem), whereas for a single knotted tube, or a deformed unknotted tube, it is proportional to its “writhe plus twist,” as encountered in differential geometry—a property of knotted ribbons that is invariant under continuous deformation (4, 5).

Thus, for example, if an untwisted ribbon that goes twice round a circle before closing on itself is unfolded and untwisted back to circular form, then its writhe will decrease continuously from 1 to 0, and its twist will increase continuously from 0 to 1, by way of compensation. Such conversion of writhe to twist is familiar to anyone seeking to straighten out a coiled garden hose.

Scheeler *et al.* approach the measurement of helicity in a highly original manner. They generate a helical vortex ring by jerking into motion a specially designed toroidal airfoil with a sharp trailing edge in the form of the desired closed helix. This sharp trailing edge is colored with dye, uniformly except for tiny concentrated blobs of

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