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THE INTERACTION OF TURBULENCE WITH STRONG WIND SHEAR

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ABSTRACT

The linear inviscid response of an initially weak random velocity perturbation to a uniform shearing motion $U = (\alpha y, 0, 0)$ is analysed, first in terms of the individual Fourier components of the perturbation field, then in terms of the development of its spectrum tensor. This analysis reveals that the dominant contribution, both to the disturbance energy and to the Reynolds stress generated, come ultimately from eddies having a cylindrical structure, the axes of the cylinders being parallel to the shear force. The results are relevant to two aspects of turbulent shear flow: (*a*) the equilibrium structure of a small turbulent 'parcel' of fluid subjected to persistent almost uniform shear, and (b) the structure of the 'large eddies' which derive their energy directly from the shearing of the mean flow.

Certain aspects of the distortion of turbulence by shear that is weakly nonuniform are also considered, and it is found that the gradient in the Reynolds stress generated tends to make the mean velocity profile propagate elastically as a shear wave, suggesting that in certain respects turbulent fluid behaves like a visco-elastic fluid in its response to shear. Finally, the relevance of these results to the phenomenon of the propagation of sharp turbulent-nonturbulent interfaces is discussed.

1. INTRODUCTION

When homogeneous turbulence is subjected to a uniform irrotational mean straining motion, it develops an equilibrium structure which reflects a balance between the effect of the uniform straining field and the nonlinear adaptation of the turbulence. The experimental evidence for the structural equilibrium and the dependence of the parameters that describe it on the parameters describing the strain, have been summarized by Townsend ([20], Chap. 4). The initial stages of the strain (and the complete history if strain is rapid enough) can be described by a linear theory (Batchelor and Proudman [2]; Townsend [19]), the results of which are in reasonable agreement with experiment.

The work on irrotational distortion was originally pursued with the aim of gaining some understanding of the mechanics of shear flow turbulence. In a two-dimensional turbulent flow, such as a turbulent wake, jet or boundary layer, any volume element of turbulent fluid, whose scale is small compared with the lateral scale of the mean flow, is subjected to the persistent, locally uniform, shear of the mean flow. This shear is composed of a rigid body rotation together with an irrotational plane strain with principle axes at 45° to the direction of the mean flow (Fig. 1). Townsend ([20], § 6.3) based an important part of his 'large eddy' theory of the mechanics of shear flow turbulence on the assumption that the structural equilibrium attained by the turbulence

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under pure shear is approximately the same as that produced by the irrotational component alone, and he applied the results of the irrotational strain analysis and experiments to shear flows, although he recognized that this was at best an approximation. It is true that rigid body rotation would have no effect if the principle axes of strain shared this rotation; but they remain fixed in space, and therefore rotate relative to the fluid, so that the process of stretching of vortex lines is very much less efficient when the rigid body rotation is present than when is not. In fact, there seems little *a priori* justification for Townsend's assumption, and it seems worthwhile calculating directly

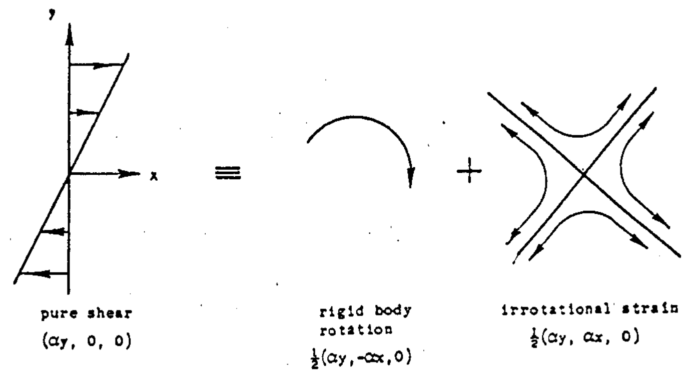

Fig. 1. Decomposition of pure shear into its rotational and irrotational components

the structure imposed by rapid uniform shear, at any rate during the initial stages, when a linear treatment is valid. This is the primary purpose of this paper.

The difference between the effects of pure shear and irrotational strain have been emphasized by Pearson [15], who carried out a linear calculation similar to that of Batchelor and Proudman [2], but including the effect of viscous forces. One result which illustrates the difference in striking manner is that the energy density of the turbulence ultimately decreases to zero under pure shear, whereas it ultimately increases exponentially under irrotational plane strain (in spite of the accelerating influence of viscosity). It seems likely that this marked difference in the 'linear response' of turbulence will also carry over to the nonlinear response when the amplitude of the turbulent fluctuations becomes too large to be described adequately by a linear theory, and when (presumably) the nonlinear self-modulation of the velocity field (rather than direct viscous dissipation) limits the orienting effect of the strain field. Pearson gave no information about the orienting effect induced by pure shear, and this aspect of the problem will be emphasized in this paper.

In §§ 2 and 3, the effect of pure shear on a random velocity perturbation is analysed, first by examining the effect of shear on a single Fourier component of the field, then by examining the development of the spectrum tensor. An important result of the analysis is that both the energy of the disturbance and the Reynolds stress that develops are ultimately dominated by contributions from a family of eddies of almost cylindrical structure, having very little variation in the direction of the mean flow.

These results seem to have some bearing on the problem of the generation of large eddies as described by Townsend ([20], § 6.1). These large eddies are an order of magnitude larger than the energy-containing eddies' of the turbulence, contain a fraction (about 1/5) of the total turbulent energy, and are believed to control both the rate of transfer of energy from mean flow to turbulence, and the rate of spread of aturbulent region into surrounding laminar fluid. The large eddies derive their energy directly from the mean flow, and pass it on to the energy-containing eddies, and the assumption that these two processes are in equilibrium leads to an important relation between the Reynolds stresses and the mean rate of strain. In order to get quantitative results, a structure has to be assumed for the large eddies, the only constraint being that a random superposition of these eddies should lead to correlation functions compatible (at large separations) with the experimentally measured curves. On the limited experimental evidence available to him, Townsend chose a cylindrical structure for the large eddies, the axes of the cylinders being parallel to the mean flow, and the analysis of this paper confirms that this *is* the type of eddy that is «selected» by uniform shear. More extensive measurements, however, by Grant [9] in wakes and boundary layers show that Townsend's cylindrical eddies cannot account for all the experimental correlations, and make it clear that in these flows more complicated mechanisms than a mere distortion by approximately uniform shear are responsible for the production of large eddies. Nevertheless, this simple process (described by Townsend) is an important one, and it is possible that in other shear flows, such as mixing layers and jets, it *is* the dominant means of production of large eddies.

The relevance of the analysis given in §§ 2 and 3 to the large eddy problem is discussed further in § 4.

The most important single quantity involving the turbulent fluctuations is the Reynolds stress, since, if the distribution of this is known, the mean flow can be determined. The Reynolds stress that develops during the linear stage of the shearing is calculated in § 3 and is found to be proportional to the total strain experienced (for small times) suggesting that the turbulent fluid responds in an elastic rather than in a viscous manner. The consequences of this conclusion, when the rate of shear is nonuniform, are investigated in § 5 and a tendency for the mean velocity profile to propagate as a shear wave is revealed. The result has some bearing on the dynamics of the interface separating rotational from irrotational flow near the edge of a turbulent wake or jet. The existence and persistence of sharp wavy interfaces of this kind, causing intermittency in signals received at a fixed point, has been recognized for many years, and a first attempt to understand the local dynamics was published by Corrsin and Kistler [4]. However, there are still aspects of the problem that have defied explanation (see, for example, Liepmann [13]), and it is hoped that the approach adopted in § 6 may represent a step in the right direction.

Some of the results obtained in § 3 of this paper have been obtained previously by Deissler [5], and further developments have been reported by Fox [7] and Deissler [6].

The problem has also been treated by Khazen [10, 11, 12]. It is felt, however, that the approach adopted in § 2, in which the history of a single Fourier component is followed, is physically illuminating and may help towards a better understanding of the computed solutions of the spectral equations described by Deissler.

In atmospheric turbulence, eddy structure is controlled both by wind shear and by thermal stratification. However, even under conditions of neutral stratification, a distinct eddy structure can be inferred (see, for example, Lumley and Panofsky [14], Chap. 5), the eddies being primarily of 'corksscrew' type, with axes parallel to the wind. Evidence of sharp turbulent interfaces in the atmosphere is less well established, although it seems at least a strong possibility that some of the 'layer' radar echoes described at this meeting arise from refractivity discontinuities at turbulent-laminar interfaces similar to those at a turbulent wake boundary. The analysis of this paper has therefore a particular relevance for the topics discussed at this meeting.

2. THE DISTORTION OF A PLANE WAVE BY UNIFORM SHEAR

Consider the action of a uniform shear flow

$$U = (\alpha y, 0, 0), \tag{2.1}$$

on a weak sinusoidal disturbance $u(x, t)$ given at some initial instant $t = 0$ by

$$u(x, 0) = A_0 e^{ik_0 x}. \tag{2.2}$$

The linearized equations for the development of the disturbance (assuming incompressibility) are

$$\begin{gathered}\frac{\partial u}{\partial t} + u\cdot\nabla U + U\cdot\nabla u = -\frac{1}{\rho}\nabla p + \nu\nabla^2 u,\\ \nabla\cdot u = 0,\end{gathered} \tag{2.3}$$

where $p(x, t)$ is the associated pressure perturbation, and ρ, and ν are the density and kinematic viscosity of the fluid. The omitted nonlinear term $u\cdot\nabla u$ is, in fact, identically zero for a plane wave, but this is no longer true when superpositions of waves of different wave vectors are considered (as in § 3). We shall be particularly interested in large-scale disturbances for which the viscous term in (2.3) may be ignored, and in §§ 2 and 3 we shall, for simplicity, follow an inviscid analysis with $\nu = 0$*. Both viscous and nonlinear effects become important after a sufficient time has elapsed, and in most cases of interest it is the nonlinear forces which become important first (see § 4).

The equations (2.3) (with $\nu = 0$) admit the solution

$$u = A(t)\, e^{ik(t)\cdot x}, \qquad p/\rho = \pi(t)\, e^{ik(t)\cdot x}, \tag{2.4}$$

provided

$$\dot{A} + i(\dot{k}\cdot x)A + \alpha A_2(1, 0, 0) + i\alpha y k_1 A = ik\pi, \tag{2.5}$$

and

$$k\cdot A = 0. \tag{2.6}$$

The coefficients of x, y and z in equation (2.5) must vanish; hence

$$\dot{k}_1 = 0, \quad \dot{k}_2 = -\alpha k_1, \quad \dot{k}_3 = 0, \tag{2.7}$$

so that

$$k_1 = k_{01}, \; k_2 = k_{02} - \alpha t k_{01}, \; k_3 = k_{03}. \tag{2.8}$$

In these equations, and in what follows, the suffix 0 refers to conditions at $t = 0$, and the suffices 1, 2 and 3 refer to components in the x, y and z directions. The constancy of k_1 and k_3 allows us to omit the suffix 0 for these components unless special emphasis of initial conditions is required. If $k_1 = 0$, then the wave vector $(0, k_2, k_3)$ remains constant. If $k_1 \neq 0$, then the effect of the shear is asymptotically to align the wave vector in the (0, 1,0) direction and to increase its magnitude linearly with time. The effect of the shear on wave fronts in these two cases is represented graphically in Fig. 2*a* and *b*. Fig. 2*c* and *d* show respectively the orienting effect on a single wave vector and the orienting effect on those initial wave vectors on the sphere

$$k_{01}^2 + k_{02}^2 + k_{03}^2 = k_c^2. \tag{2.9}$$

The notation in these figures will be referred to in what follows.

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* The only modification required if viscous forces are retained is the inclusion of a factor

$$\exp\left[-\nu \int k^2\, dt\right] = \exp\left\{-\nu\left[k_0^2 t - k_1 k_{02}\alpha t^2 + \frac{1}{3}k_1^2\alpha^2 t^3\right]\right\}$$

in each component in equation (2.11) below. This causes a viscous damping of all wave amplitudes and of the disturbance energy after a sufficient time has elapsed.

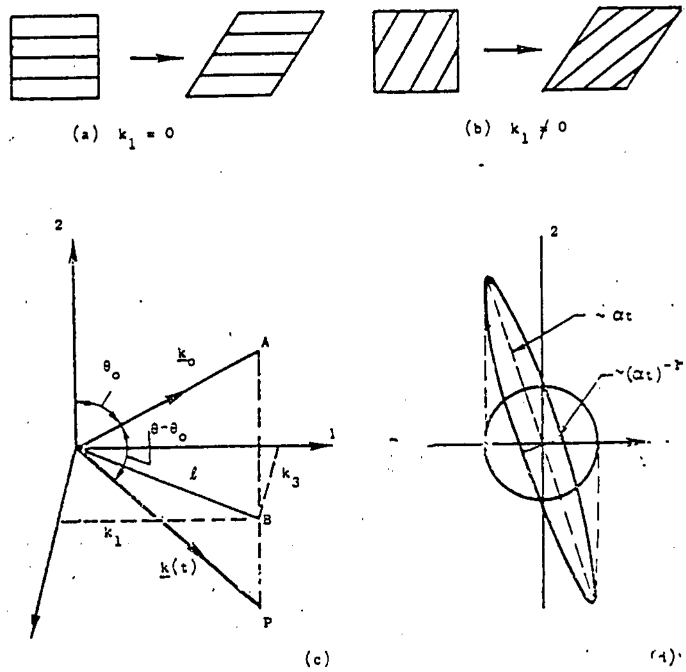


Fig. 2. Effect of shear on the wave fronts and wave vectors of a superposed disturbances

Now from equation (2.6), $\dot{\boldsymbol{k}} \cdot \boldsymbol{A} + \boldsymbol{k} \cdot \dot{\boldsymbol{A}} = 0$, so that from equation (2.5), writing $k^2 = \boldsymbol{k} \cdot \boldsymbol{k}$,

$$-ik^2 \pi = -\dot{\boldsymbol{k}} \boldsymbol{A} + \alpha A_2 k_1 = 2 \alpha A_2 k_1$$

from equation (2.7). The part of equation (2.5) not involving x, y and z is then satisfied provided

$$\dot{\boldsymbol{A}} + \alpha A_2 (1, 0, 0) = -i\boldsymbol{k}\pi = 2 \alpha A_2 k_1 \boldsymbol{k}/k^2. \tag{2.10}$$

Integration of the second component of this equation, then of the first and third components, is straightforward and leads to the following dependence of the amplitude components on time:

$$\left.\begin{aligned} A_1(t) &= A_{01} - A_{02} \left\{ \frac{k_0^2 k_3^2}{k_1 l^3} [\theta] + \frac{k_1 k_0^2}{l^2} \left[\frac{k_2}{k^2} \right] \right\}, \\ A_2(t) &= A_{02} k_0^2/k^2, \\ A_3(t) &= A_{03} + A_{02} \frac{k_3 k_0^2}{l^3} \left\{ [\theta] + l \left[\frac{k_2}{k^2} \right] \right\}; \end{aligned}\right\} \tag{2.11}$$

here, $l^2 = k_1^2 + k_3^2$, $\tan \theta = l/k_2$ (see Fig. 2*c*), and the notation $[\psi]$ is used to indicate $\psi(t) - \psi(0)$. In the particular case $k_1 = 0$, these expressions reduce to the very simple form (of course, derivable directly from equation (2.10))

$$\left.\begin{aligned} A_1(t) &= A_{01} - \alpha t A_{02}, \\ A_2(t) &= A_{02}, \\ A_3(t) &= A_{03}. \end{aligned}\right\} \tag{2.12}$$

We shall be particularly interested in the behaviour of $A_i(t)$ for small and for large times. For $\alpha t k_1 \ll k_{02}$,

$$k^2 \sim k_0^2 - 2 \alpha t k_1 k_{02}, \quad [\theta] \sim -\alpha t k_1 l/k_0^2,$$

and

$$[k_2/k^2] \sim \alpha t (2 k_0^{-4} k_1 k_{02}^2 - k_0^{-2} k_1),$$

so that

$$\left.\begin{aligned} A_1 &= A_{01} - \alpha t A_{02}\left(1 - \frac{2 k_1^2 k_{02}^2}{l^2 k_0^2}\right) + 0(\alpha t)^2, \\ A_2 &= A_{02} + 2\alpha t A_{02} k^{-2} k_1 k_{02} + 0(\alpha t)^2, \\ A_3 &= A_{03} + 2\alpha t A_{02} k^{-2} l^{-2} k_1 k_{02}^2 k_3 + 0(\alpha t)^2. \end{aligned}\right\} \tag{2.13}$$

If $k_1 \ll k_{02}$, this linear behaviour continues for a long time (until $\alpha t \approx \approx k_{02}/k_1$) and $|A_1|$ can become very large during this period.

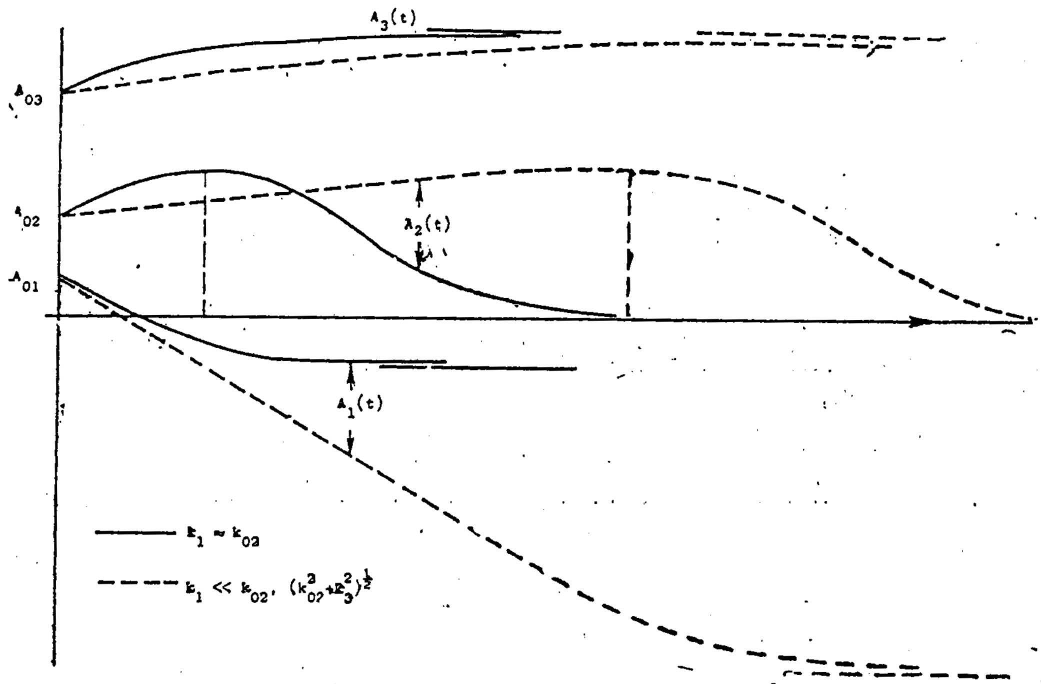


Fig. 3 Typical behaviour of amplitude components as given by equation (2.11)

For $\alpha t k_1 \gg m = (k_{02}^2 + k_3^2)^{1/2}$, we have $k^2 \sim (\alpha t)^2 k_1^2$, $\theta \to \pi$, and $k_2/k^2 \to 0$, so that

$$\left.\begin{aligned} A_1 &\sim A_{01} - A_{02}\{k_0^2 l^{-3} k_1^{-1} k_3^2 (\pi - \theta_0) + l^{-2} k_1 k_{02}\}, \\ A_2 &\sim A_{02} k_0^2 k_1^{-2} (\alpha t)^{-2}, \\ A_3 &\sim A_{03} + A_{02}\{k_0^2 l^{-3} k_3 (\pi - \theta_0) - l^{-2} k_{02} k_3\}. \end{aligned}\right\} \tag{2.14}$$

Thus the energy density of the disturbance approaches the constant level $\frac{1}{2}[|A_1(\infty)|^2 + |A_3(\infty)|^2]$. If k_1 is small (so that $l \approx k_3$) then in general (provided $A_{02} \neq 0$, and $\theta_0 \neq \pi$) this energy density is approximately

$$\frac{1}{2}(\pi - \theta_0)^2 |A_{02}|^2 k_0^4 k_1^{-2} k_3^{-2},$$

and this is large when k_1 is small; this is because wave vectors of this kind take a long time to become aligned, and $|A_1|$ increases linearly throughout most of the alignment time. Typical behaviour of the three amplitude components is sketched in Fig. 3.

3. DISTORTION OF A WEAK RANDOM PERTURBATION BY PURE SHEAR

If the initial disturbance is such that it can be represented as a superposition of plane waves in the form*

$$u_i(\boldsymbol{x}, o) = \sum_{k_0} \text{ or } \int d^3 \boldsymbol{k}_0 \, A_{0i}(\boldsymbol{k}_0)\, e^{i\boldsymbol{k}_0 \boldsymbol{x}*}, \tag{3.1}$$

then, during the linear development of the field, the perturbation at time t is given from equation (2.11) by

$$u_i(\boldsymbol{x}, t) = \sum_{k} \text{ or } \int d^3 \boldsymbol{k} \, A_i(\boldsymbol{k}, t)\, e^{i\boldsymbol{k}\boldsymbol{x}}, \tag{3.2}$$

where

$$A_i(\boldsymbol{k}, t) = L_{ij}(\boldsymbol{k}, t)\, A_{0j}(\boldsymbol{k}_0), \tag{3.3}$$

with

$$L_{ij} = \begin{pmatrix} 1 & \frac{k_0^2}{l^3 k_1}\left\{k_3^2 [\theta] + l k_1^2 \left[\frac{k_2}{k^2}\right]\right\} & 0 \\ 0 & k_0^2/k^2 & 0 \\ 0 & \frac{k_0^2 k_3}{l^3}\left\{[\theta] + l\left[\frac{k_2}{k^2}\right]\right\} & 1 \end{pmatrix} \tag{3.4}$$

$\boldsymbol{k}_0$ being related to $\boldsymbol{k}$ through equation (2.8). The volume element $d^3\boldsymbol{k}$ in wave vector space is invariant, since from equation (2.9)

$$d^3 \boldsymbol{k} = \frac{\partial(k_1, k_2, k_3)}{\partial(k_{01}, k_{02}, k_{03})}\, d^3 \boldsymbol{k}_0 = d^3 \boldsymbol{k}_0 \,. \tag{3.5}$$

The spectrum tensor of the disturbance at time t is

$$\Phi_{ij}(\boldsymbol{k}, t) = \lim_{d^3 \boldsymbol{k} \to 0} \overline{A_i(\boldsymbol{k}, t)\, A_j^*(\boldsymbol{k}, t)\, d^3 \boldsymbol{k}}, \tag{3.6}$$

the star indicating a complex conjugate, and the overbar an ensemble average. Hence, using equations (3.4) and (3.5),

$$\Phi_{ij}(\boldsymbol{k}, t) = L_{il} L_{jm} \Phi^0_{lm}(\boldsymbol{k}_0), \tag{3.7}$$

where

$$\Phi^0_{lm}(\boldsymbol{k}_0) = \lim_{d^3 \boldsymbol{k}_0 \to 0} \overline{A_{0l}(\boldsymbol{k}_0)\, A^*_{0m}(\boldsymbol{k}_0)\, d^3\boldsymbol{k}_0}$$

is the spectrum tensor of the initial disturbance, which may be considered known. If $\boldsymbol{k}_0$ is replaced by $\boldsymbol{k} + \alpha t k_1 (0, 1, 0)$, equation (3.7) gives $\Phi_{ij}(\boldsymbol{k}, t)$ explicitly in terms of its initial value. In order to go further it is expedient to make some simple assumption about the form of the initial spectrum tensor, and the argument in what follows will be confined to two extreme possibilities.

Case A:

Suppose that the initial disturbance is two-dimensional with *no variation in the x-direction*; it may be visualised as a superposition of cylindrical eddies with axes parallel to the mean shearing flow. (The velocity component u_1 need not, however, be zero.) The Fourier components of such a field all have $k_1 = 0$, and we have seen in § 2 that these are the only Fourier components of a completely random field which can receive unlimited energy through inviscid shearing. The spectrum tensor of such a disturbance is of the form

$$\Phi^0_{lm}(\boldsymbol{k}) = \Psi^0_{lm}(k_2, k_3)\, \delta(k_1). \tag{3.8}$$

* For a stationary random function, the Fourier-Stieltjes representation $u(\boldsymbol{x}, o) = \int e^{i\boldsymbol{k}_0 \cdot \boldsymbol{x}}\, dZ_0(\boldsymbol{k}_0)$ would be appropriate. The final result (3.6) is unaffected by this refinement.

The tensor L_{ij} degenerates in this case to the simple form (from (2.12))

$$L_{il} = \begin{pmatrix} 1 & -\alpha t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.9}$$

The quantities containing the most interesting physical information are Φ_{ii} and Φ_{12} which integrate to give the energy density and the Reynolds stres respectively:

$$T(t) \equiv \frac{1}{2}\overline{u_i u_i} = \frac{1}{2}\int \Phi_{ii} d^3 k, \quad \tau(t) \equiv -\overline{u_1 u_2} = -\int \Phi_{12}\, d^3 k. \tag{3.10}$$

From equations (3.7) and (3.9), it is evident that, for $\alpha t \gg 1$, Φ_{ii} is dominated (for almost all values of k) by those terms on the right for which $i = l$, $l = m = 2$, and in fact, using equation (3.8),

$$\Phi_{ii}(k, t) \sim (\alpha t)^2\ \psi^0_{22}(k_2, k_3)\, \delta(k_1), \tag{3.11}$$

and so

$$T \sim \frac{1}{2}(\alpha t)^2\, \overline{u^2_{02}} \quad \text{as} \quad \alpha t \to \infty. \tag{3.12}$$

This energy is concentrated almost entirely in the component u_1. If the turbulence is initially isotropic in the y-z plane so that $\overline{u^2_{02}} = \overline{u^2_{03}} = T_0$, say, then for $\alpha t \gg 1$

$$T \sim \frac{1}{2}(\alpha t)^2 T_0. \tag{3.13}$$

Similarly, from equations (3.7) and (3.9),

$$\Phi_{12}(k, t) = -\alpha t\, \Phi^0_{22}(k), \tag{3.14}$$

so that

$$\tau(t) = \alpha t\, T_0. \tag{3.15}$$

Hence the stress established is proportional to the *total strain* αt experienced, suggesting an elastic rather than a viscous type of response. This idea will be pursued further in § 5.

Case B:

Now suppose that the initial disturbance is isotropic, so that (Batchelor [1])

$$\Phi^0_{ij}(k_0) = \frac{E_0(k_0)}{4\pi k_0^4}\,(k_0^2 \delta_{ij} - k_{0i} k_{0j}); \tag{3.16}$$

$E_0(k_0)$ is the energy spectrum function satisfying $^1/_2 \overline{u_0^2} = \int\limits_0^\infty E_0(k_0) dk_0$. The easiest way to describe the qualitative development of the spectrum function is by reference to Fig. 2, *d*. Suppose first that

$$E_0(k_0) = T_0\, \delta(k_0 - k_c), \tag{3.17}$$

where, in this case, T_0 represents the initial energy density, so that initially the wave vectors of all the Fourier components end on the sphere $k_0 = k_c$. Fig. 2,*d* shows how these wave vectors change with time; at time t, they end on the surface of the spheroid

$$k_1^2 + (k_2 + \alpha t k_1)^2 + k_3^2 = k_c^2, \tag{3.18}$$

of which a section in a plane k_3 = const is shown in the Figure. The distance of this surface from the origin varies continuously between a maximum and a minimum value, which for large αt behave like and it is evident that the

$$k_{\max} \sim \alpha t k_c, \quad k_{\min} \sim (\alpha t)^{-1} k_c, \tag{3.19}$$

initial δ-function is spread out continuously over the finite range of the wave number magnitude axis between k_{max} and k_{min} (Fig. 4). At the same time, the total energy of those Fourier components whose wave number magnitudes remain of order k_c increases as $(\alpha t)^2$; but the area of the initial sphere corresponding to such wave vectors decreases as $(\alpha t)^{-1}$, so that the total energy increases as αt. The energy of any finite band of Fourier components that are swept into the range $k \gg k_c$ tends to a constant value. The energy

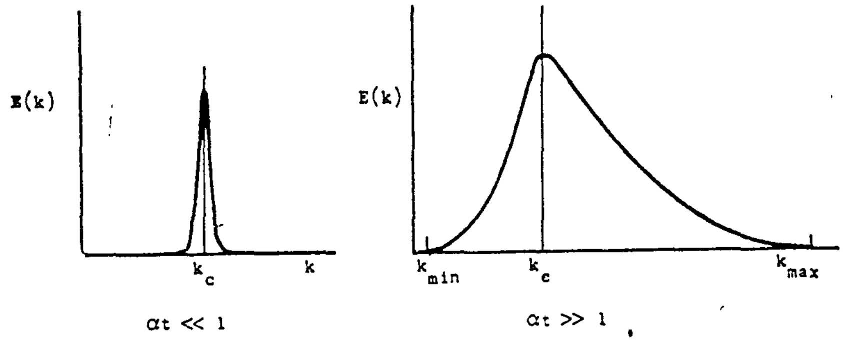


Fig. 4. Development of $E(k, t)$ under shear when $E(k, 0) = T_0\,\delta(k - k_c)$

spectrum becomes severely anisotropic, being dominated asymptotically by those Fourier components for which $k_1/k_c = O(\alpha t)^{-1}$, and the energy increase again being concentrated in the u_1 component of velocity.

More rigorously, for general $E_0(k_0)$, we have

$$T = \frac{1}{2}\int L_{il}\, L_{im} \frac{E_0(k_0)}{4\pi k_0^4}\,(k_0^2\,\delta_{lm} - k_{0l}\,k_{0m})\, d^3 k; \tag{3.20}$$

the largest contribution to the integral (for $\alpha t \gg 1$) comes from the term for which $l = m = 2$ (due to the presence of the factor k_1^{-1} in L_{12}). Hence, replacing d^3k by d^3k_0 and putting $k_{01} = \xi/\alpha t$,

$$T \sim \frac{\alpha t}{8\pi} \iiint_{-\infty}^{\infty} E_0\,[(l^2 + k_{02}^2)^{1/2}] \left[\tan^{-1}\left(\frac{l}{k_{02} - \xi}\right) - \tan^{-1}\frac{l}{k_{02}}\right]^2 \frac{\partial\xi}{\xi^2}\, d\,k_{02}\, dk_{03} \tag{3.21}$$

with $l = [(\xi/\alpha t)^2 + k_{03}^2]^{1/2}$. As $\alpha t \to \infty$, $l \to k_{03}$; this limiting process can legitimately be carried out under the integral sign in equation (3.21), since the integrand is uniformly convergent for all ξ as $\alpha t \to \infty$. (The presence of the factor ξ^{-2} ensures uniform convergence in the range of large ξ that would otherwise be troublesome.) Hence, letting $\alpha t \to \infty$,

$$T \sim \frac{\alpha t}{8\pi} \iiint_{-\infty}^{\infty} E_0\,[(k_{02}^2 + k_{03}^2)^{1/2}] \left[\tan^{-1}\frac{k_{03}}{k_{02} - \xi} - \tan^{-1}\frac{k_{03}}{k_{02}}\right]^2 \frac{d\xi}{\xi^2}\, d\,k_{02}\, dk_{03}. \tag{3.22}$$

This integral (a linear functional of $E_0(k)$) is certainly convergent for any reasonable $E_0(k)$, and independent of t, so the result $T \sim \alpha t$ is true for a general initial isotropic spectrum.

Similarly, the expression for the Reynolds stress is

$$\tau = -\int L_{1l}\, L_{2m}\, \overline{\Phi}_{lm}^{0}(k_0)\, d^3 k. \tag{3.23}$$

Here we shall require the behaviour for small as well as for large times. For $\alpha t \ll 1$, the forms (2.13) lead simply to

$$\tau = \frac{2}{3}\,\alpha\, t\, T_0 + o\,(\alpha\, t)^2. \tag{3.24}$$

For $\alpha t \gg 1$, the dominant terms in equation (3.23) are again those for which $l = m = 2$, and if we keep only the first part of L_{12} which gives the dominant contribution to the integral, we are led to the form

$$\tau \sim \frac{T_0}{4\pi} \int_0^{\pi} \left\{ \int_0^{2\pi} \sec \varphi_0 \left[\cot^{-1} (\cot \theta_0 - \alpha t \cos \varphi_0) - \theta_0\right] \partial \varphi_0 \right\} \sin \theta_0 \, d\theta \, . \tag{3.25}$$

As $\alpha t \to \infty$, this is dominated by values of φ_0 near $\varphi_0 = \pi/2$ and $3\pi/2$ (corresponding again to initial wave vectors which are slow to align). Putting $\mu = \alpha t (\varphi_0 - \pi/2)$, (and noting that the contributions from the two neighbourhoods are equal), we have for $\alpha t \gg 1$,

$$\tau \sim - \frac{T_0}{2\pi} \int_0^{\pi} \left\{ \int_0^{\pi\alpha t} \left[\cot^{-1} (\cot \theta_0 + \mu) - \theta_0\right] \frac{\partial \mu}{\mu} \right\} \sin \theta_0 \, d\theta \sim$$

$$\sim \frac{T_0}{2\pi} \int_0^{\pi} \theta_0 \log \pi \alpha t \, \sin \theta_0 \, d\theta_0 \sim \frac{1}{2} T_0 \log \alpha t. \tag{3.26}$$

The increase of T and τ with αt, are again due to the linear increase in amplitude of Fourier components with $k_1 \approx 0$. However, T and τ increase less rapidly in case B than in case A, because in case B the Fourier components which experience the linear increase correspond to an initial wave-vector band which decreases in width as $(\alpha t)^{-1}$, whereas in case A, the field consisted only of those components which experience the linear increase indefinitely.

4. EFFECTS OF NONLINEAR FORCES. THE LARGE EDDY PROBLEM

The existence of a distinct family of eddies in turbulent shear flows, containing an appreciable fraction of the total turbulent energy and a scale comparable with the scale of the mean flow, is generally accepted (see the discussion in § 1), but there is, as yet, little theoretical indication as to what the statistical structure of these eddies may be (and the experimental evidence is by no means conclusive), and there is no theoretical estimate of the large eddy intensity. Townsend's 'large eddy theory' is based on an assumption that these eddies are critically stable (as far as their energy budget is concerned), but their actual intensity seems to play no part in the theory. Can we obtain any information on these questions of structure and intensity from the analysis of the preceding sections?

Firstly, on the question of structure, it is clear from §§ 2 and 3 that in the absence of any initial anistropy, a mean shear preferentially amplifies cylindrical eddies of the type proposed by Townsend [20], (see Fig. 5). However, the energy supply to such eddies from the mean shear does not cease when the plane of circulation is perpendicular to Ox as suggested by Townsend ([20], § 6.1); rather, its energy increases as t^2 (from § 3, case A) indefinitely for a cylindrical eddy, and for a time of order $(\varepsilon\alpha)^{-1}$ for an eddy of initial length: width ratio ε.

This energy increase will ultimately be checked when nonlinear forces become important. This happens when the magnitude of the eddy vorticity $|\omega|$ becomes comparable with α, since then the rotational part of $u \cdot \nabla u$, i. e., $- u \wedge \omega$, becomes comparable with $u \cdot \nabla U$. For the plane wave considered in § 2,

$$\omega = i\, k \times A\, e^{ik.x}, \tag{4.1}$$

so that, from equations (2.8) and (2.14), in general,

$$\omega \sim i\,(-\alpha t\, k_1 A_{\infty 3},\ k_3 A_{\infty 1},\ \alpha t\, k_1 A_{\infty 1})\, e^{ik(\varepsilon)x}, \tag{4.2}$$

where A_∞ is the ultimate steady value for A. [As $k_1 \to 0$, the product $k_1 A_{\infty 1}$ tends to a finite nonzero value.] Thus, asymptotically the disturbance vortex lines tend to lie in the x-z plane. Note that for an initially isotropic field, the vorticity is *not* dominated in the limit by the nearly cylindrical eddies which contribute most to the energy, but receives significant contributions from all Fourier components. From equation (4.2), the disturbance vorticity becomes comparable with α after a time of order $(k_1/A_\infty)^{-1}$; for a plane wave with $k_1 \to 0$, from equation (2.14), this time is

$$t_n = 0\,(l^3 k_1 / A_{02} k_0^2 k_3^2). \tag{4.3}$$

When a continuous spectrum of waves is sheared, this means that nonlinear forces become important after a time inversely proportional to the initial disturbance amplitude.

At this stage, two possible effects can be distinguished, in addition to the direct interact on of the initially independent waves. First, when $|\omega_1|$ becomes

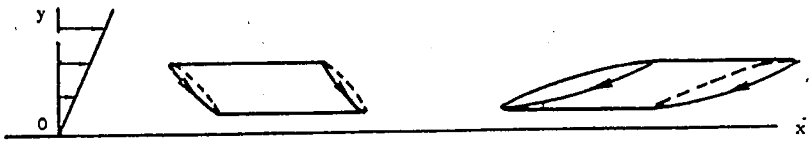


Fig. 5. Shearing of a Townsend Eddy

comparable with α, the vortex lines of the total flow undulate considerably about the z-direction. We have seen that when the basic shearing motion is uniform, continuous energy transfer is possible only to those eddies having little or no variation in the x-direction. When the shear is considerably perturbed, however, the direction of the principal axes of strain, and the local rotation axis, will likewise vary through angles of order $\pi/4$, and a larger range of wave vectors will become available for the receipt of energy from the *perturbed* mean flow. In this respect, the large eddies play the role of an *essential intermediary* in the process of energy transfer from the mean flow to the turbulence; without them, continuous energy transfer to components of turbulence with wave vectors in random directions is not possible.

Secondly, as $|\omega_3|$ increases, points of inflection appear in the profile of the x-component of the perturbed velocity field, and it seems likely that, when the vorticity gradient becomes large enough, the perturbed profile will become unstable to smaller scale disturbances in the manner described by Gill [8]. For this mechanism to operate, the initial amplitude must be large enough for nonlinear effects to become important before viscous forces damp out the disturbance.

The prediction that the large eddy vorticity must be of the same order as the mean flow vorticity receives some support from Townsend's experimental estimates in the case of the turbulent wake. The large eddy energy density is approximately 1/8 that of the mean velocity defect; their scale is somewhat less than half the scale of the mean velocity. Hence, the vorticity ($\approx$ (energy)$^{1/2}$/scale) of the large eddies and of the mean flow are of the same order of magnitude. This evidence, however, must be viewed with caution, since, as already mentioned in § 1, the observations of Grant [9], show that other processes in addition to distortion by shear, are responsible for large eddy generation; in particular, Grant suggests that instabilities of the von Karman vortex street originating near the cylinder, and persistent instability of the wake associated with the continual build-up of Reynolds stresses, may give rise to eddy structures not inconsistent with correlation measurements.

5. THE REACTION OF THE TURBULENCE ON THE MEAN FLOW

If homogeneous turbulence is distorted by uniform shear, then a uniform Reynolds stress is established and this has no effect on the mean flow. If the shear is nonuniform, however, i. e., if $\alpha = \alpha(y)$, or if the turbulence is nonhomogeneous initially, then a nonuniform stress field is established and this tends to change the mean flow. For a strictly parallel flow $U = (U(y, t), 0, 0)$, the mean flow equation is

$$\frac{\partial U}{\partial t} = \frac{\partial \tau}{\partial y}. \tag{5.1}$$

Hence U and so $\alpha = dU/dy$ become functions of t, and the total strain αt that appeared throughout §§ 2 and 3, must be replaced by the quantity

$$\beta = \int_0^t \alpha dt. \tag{5.2}$$

The derivation of the stress τ in § 3 is otherwise unchanged, and gives as before

$$\text{in case } A, \quad \tau = T_0\beta \tag{5.3}$$

$$\left.\begin{aligned} \text{in case } B, \quad \tau &= \tfrac{2}{3}T_0\beta + O(\beta^2) \qquad (\beta \ll 1), \\ &\sim \tfrac{1}{2}T_0 \log\beta \qquad (\beta \gg 1). \end{aligned}\right\} \tag{5.4}$$

The relation $\tau = c^2\beta$ with $c^2 = T_0$ or $(2/3)\,T_0$ (assumed uniform) in case A or B respectively, leads to

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial}{\partial y}\frac{\partial \tau}{\partial t} = c^2\frac{\partial}{\partial y}\frac{\partial \beta}{\partial t} = c^2\frac{\partial \alpha}{\partial y},$$

or

$$\frac{\partial^2 U}{\partial t^2} = c^2\frac{\partial^2 U}{\partial y^2}. \tag{5.5}$$

We have already remarked that a linear relation between stress and strain implies an elastic response, so that the appearance of the wave equation as a consequence is not surprising. The derivation requires implicitly that the scale of the mean strain inhomogeneity should be large compared with the scale of the strained turbulence, so that the assumption of uniform local strain may be reasonable. However, if this is not the case (as, for example, for the large eddies in wake turbulence), one might still expect the equation (5.5) to have some qualitative, if not quantitative, significance.

Two further qualifications must be made. First, the stress-strain relations (5.3) and (5.4) are valid only during the linear response period $t \ll t_n$. In a sense, the time $t \approx t_n$ marks the 'yield point' of the turbulent medium. For $t \gtrsim O(t_n)$ the stress presumably approaches an asymptotic level determined by the *rate* of strain. The time t_n may also be interpreted as the 'relaxation time' of the turbulence when the mean rate of strain changes. If t_s is a time characteristic of this change, then if $t_n \ll t_s$ the response (for $t_s \ll t_n$) will be purely elastic, while if $t_n \ll t_s$ the response will be purely viscous. The analogy between turbulent flow and the flow of a non-Newtonian fluid was first pointed out by Rivlin [17], and this point of view has been further recommended by Liepmann [13]; the approach adopted here suggests that turbulent fluid is visco-elastic in its response to a changing rate of shear.

Secondly, if there is a point of inflection in the mean velocity profile, i. e., a point where $\alpha'(y) = 0$, then this will be a natural source of instabilities which grow exponentially rather than linearly with time. However, in the limit of uniform shear (i. e., $\alpha'(y) = 0$ everywhere), the growth rate of such disturbances is zero, and it is reasonable to expect that for sufficiently small curvature in the mean profile, the dominant contribution to the Reynolds stress is still given by equation (5.3) or (5.4).

To be more specific, consider the following initial value problem: at time $t = 0$, suppose that the mean profile is of simple jet-type,

$$U(y, 0) = U_0 \operatorname{sech}^2 y/b, \tag{5.6}$$

and suppose that the superposed weak turbulence is of the two-dimensional type considered in case A of § 3. The development of the mean flow is then described by equation (5.5) with $c^2 = T_0$, and the solution is

$$U(y, t) = \frac{1}{2} U_0 \left[\operatorname{sech}^2 \frac{y - ct}{b} + \operatorname{sech}^2 \frac{y + ct}{b}\right], \tag{5.7}$$

representing two shear waves travelling with velocities $\pm c$. If the eddies are of lateral scale b (and this is a maximum estimate), their vorticity at time t is of order $(T_0^{1/2}/b)\,\beta \approx (c/b)\overline{\alpha}t$ where $\overline{\alpha} \approx U_0/b$ is a mean value of the shear in the jet; this vorticity becomes comparable with $\overline{\alpha}$ after a time $t_l \approx b/c$, and this is just the time that the initial profile takes to split into its two components. Hence, the linear development described by equation (5.7) can be valid only near the outer edges of the jet where the total strain experienced is still sufficiently small (i. e., $\beta \ll U_0/c$).

If the superposed turbulence is initially isotropic, then the same description holds for $\beta \ll 1$, but for $U_0/c \gg \beta \gg 1$, the asymptotic law $\tau \sim (1/2)T_0 \log \beta$ is appropriate. Writing $c^2 = (1/2)\, T_0$, the mean flow equation is then most compactly expressed in terms of β in the form

$$\frac{\partial^2 \beta}{\partial t^2} = c^2 \frac{\partial^2}{\partial y^2} \log \beta. \tag{5.8}$$

This equation is still hyperbolic, and presumably exhibits solutions of propagating type, though the nonlinearity suggests that the development of the profile will depend strongly on its amplitude. For $\beta \gtrsim U_0/c$, the linear theory for the Reynolds stress development breaks down.

6. THE STEEPENING AND PROPAGATION OF TURBULENT INTERFACES

Turbulent wakes, jets, mixing layers, and boundary layers are characterized, in general, by the presence of sharp undulating interfaces which separate the rotational turbulent region from an irrotational region in which turbulent fluctuations decay rapidly with distance from the interface. (The fluctuation energy decreases asymptotically as the inverse fourth power of this distance, according to Phillips [16].) In the irrotational region, the assumption that the field of fluctuations is homogeneous in directions parallel to the interface, leads to the result that the Reynolds stress component $\tau = -\overline{u_1 u_2}$ is zero (Corrsin and Kistler [4]). Weak inhomogeneity in wakes and jets in the x-direction does produce a nonzero τ, whose only effect, however, is to modify the mean pressure distribution (Stewart [18]). The mean flow in the irrotational region is the uniform velocity at infinity, together with a component associated with the finite amplitude fluctuations of the interface, which, as demonstrated by Stewart, can contribute to the mean velocity defect (for a wake or boundary layer), or increment (for a jet). However, any genuine rotational shear that is present is necessarily confined to the rotational region where the Reynolds stress is nonzero (and rotational). Fluid is continually entrained across the interface from the irrotational to the rotational side; equivalently one may say that the interface propagates towards the irrotational region.

Two outstanding questions regarding these interfaces are (*a*) why do they remain sharp, and (*в*) what determines their velocity of propagation? Corrsin and Kistler have proposed the following answers to these questions:

(*a*) The rate of production of mean-square vorticity $\overline{\omega^2}$ ($= \omega'$ say) in homogeneous turbulence is proportional to the mean-square vorticity already present. In any local gradient of ω' (in a nonhomogeneous turbulence) the higher rate of production in the region of higher ω'-density will tend to steepen the gradient. In particular, near the edge of a free turbulent flow where ω' decreases from its value well inside the turbulent region to zero outside, the gradient of ω' will increase until direct viscous dissipation intervenes, when the thickness of the transition layer is of order $(\nu/\omega')^{1/2}$, (the local inner Kolmogorov scale). This argument predicts steepening of the gradient of ω' whether a mean shear is present or not. It is open to the criticism that the rate of *destruction* of ω' in homogeneous turbulence by viscous dissipation is also proportional to $\overline{\omega^2}$ and that the rates of production and destruction are approximately in balance, at any rate so long as the turbulence is not too far from statistical equilibrium.

(*b*) The propagation of the interface is essentially the transmission of vorticity by viscous stresses through the interface. On dimensional grounds, the propagation velocity V^* must be of order the inner Kolmogorov velocity scale:

$$V^* = O\,(\nu\omega')^{1/2}; \tag{6.1}$$

the presence of mean shear, and therefore of an additional (kinematic) Reynolds stress, τ_L (just inside the layer) would tend to augment this velocity, giving

$$V^* = O\left(\tau_L^2 + (\nu\omega')^2\right)^{1/4}. \tag{6.2}$$

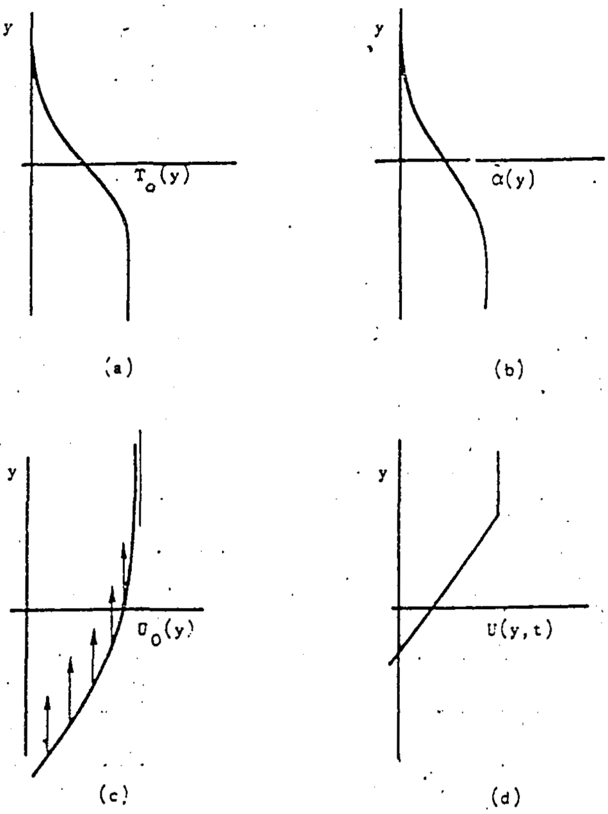
An objection to the form (6.1) raised by Coles [3] is that the *mean* propagation velocity $\overline{V^*}$ is known to be independent of viscosity (at any rate in regions o flow where Reynolds number similarity has any meaning). The limiting form of equation (6.2) at high Reynolds number, $V^* = O\,(\tau_L)^{1/2}$, seems reasonable, but it suffers from the weakness (common to all results of dimensional analysis), that the precise propagation mechanism remains obscure.

An approach which avoids the above criticisms (though doubtless it invites others) can be based on the shear wave equation derived on the basis of equations (5.1) and (5.4*a*). Let us again formulate an initial value problem, intended to represent the situation near the instantaneous boundary of a turbulent wake (Fig. 6): at $t = 0$, let $\alpha\ (y) = \partial U/\partial y$ be a monotonic function decreasing from a positive value α_0 at $y = -\infty$ to zero at $y = +\infty$, and let $T_0\ (y)$, the (unstrained) kinetic energy of superposed weak fluctuations, be a similar monotonic decreasing function. The equation for $U\ (y,\ t)$ is then

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial}{\partial y} c^2 \frac{\partial U}{\partial y}, \tag{6.3}$$

where $c^2 \propto T_0$, the constant of proportionality being of order unity (and depending on the precise degree of anisotropy of the turbulence). For a unique solution, we should also specify $\partial U/\partial t$ at $t = 0$; this is equivalent to prescribing the degree to which the turbulence has been strained prior to the initial instant. Let us simply stipulate that $\partial U/\partial t$ has the form that gives rise to a single wave propagating in the positive y-direction. If T_0 were uniform, this wave would propagate without change of shape (the necessary qualifications about the 'linear phase' of Reynolds stress development discussed in §5 being understood).

Fig. 6. Development of a discontinuity in mean shear



But when T_0 is a function of y, the wave moves faster where T_0 is greater and steepens, as indicated in Fig. 6*c*. If $T_0(y)$ decreases to zero as $y \to +\infty$, either as some power of y, or exponentially, then equation (6.3) certainly predicts the development of a discontinuity in the mean shear as time proceeds. If some other effect does not intervene first, this development will be checked only when viscous forces act directly to smooth out the discontinuity; (inclusion of viscous forces in equation (6.3) gives an additional term $\nu\partial^3 U/\partial t\partial y^2$ on the right-hand side). Actually, in deriving equation (6.3) we required that the scale of inhomogeneity of the mean flow should be large compared with the scale of the strained turbulence, and clearly this cannot remain true as the discontinuity develops. However, although this will certainly change the *details* of development, the *tendency* towards a discontinuity implied by equation (6.3) is highly suggestive.

What of the propagation speed, once the discontinuity is formed? On the model suggested above, it would appear that this speed ($\alpha T_0^{1/2}$) decreases to zero as the interface moves out to increasing values of y. However, here we must admit that a turbulent interface does not remain plane, but rather undulates on the scale of the local large turbulent eddies. These undulations cause irrotational fluctuations outside the turbulent region, which are «pushed ahead» of the interface as it propagates. In other words, on the length scale of the interface thickness, the local turbulent kinetic energy on the irrotational side does not decrease to zero, but always equals the mean turbulent kinetic energy at the undulating interface, T_e say; this is of the same order as the kinetic energy of the large eddies discussed in § 4. Treating c^2 as a constant in equation (6.3), the discontinuity in $\partial U/\partial y$ then continues to propagate as a discontinuity (just as would a similar discontinuity on a stretched string) with velocity $c\alpha T_e^{1/2}$ (the constant of proportionality being of order unity). Viscous forces are important only in the layer separating the two regions, and in this layer, they are resposible for generating vorticity in the previously irrotational fluid. They need only generate a small amount of vorticity, which is subsequently amplified by ordinary turbulent stretching of vortex lines.

It will be noticed that the above model for discontinuity development and propagation explicitly requires the presence of mean shear in the turbulent region as well as initial inhomogeneity of the turbulence. This is in contrast to the description of the process by Corrsin and Kistler which applies equally whether mean shear is present or not. There seems to be no direct experimental evidence for the formation of discontinuities when there is no mean shear. Townsend ([20], § 3.9) has suggested that the scale of inhomogeneity in this case will *increase*, (like the integral scale of homogeneous turbulence during decay) rather than decrease, in the direction of a discontinuity. An experiment distinguishing between these possibilities would be illuminating.

ACKNOWLEDGMENTS

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Discussion

E. A. Novikov—Will the elastic response of the turbulent flow remain if we take into account the nonlinear effects?

H. K. Moffatt — The response of turbulent flow to sudden disturbances of the mean velocity profile has an elastic character but it has a certain relaxation time and only at the initial stage does it have elastic features. The nonlinear effects certainly modify the simple elastic response and probably one should regard the fluid as having a finite memory.

W. L. Jones — You indicate that variations in the mean flow can be propagated «elastically» by large scale turbulence; one can also think of this as a propagation of the fluid vorticity. Does this represent the true transmission of vorticity from one fluid element to another, as is the case in waves, or perhaps more simply the transport of fluid possessing certain vorticity?

H. K. Moffatt — The fluid is not transported by this mechanism, but the mean velocity profile tends to propagate relative to the fluid.