LIQUID METAL MHD AND THE GEODYNAMO

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ABSTRACT. The magnetic field of the Earth is generated by dynamo action associated with the upwelling of buoyant material in the liquid outer core. It is argued that this upwelling occurs in the form of mushroom-shaped blobs of material released from the mushy zone at the inner core boundary (ICB), and having a very small density defect $\delta\rho/\rho$. The rise of buoyant material with velocity w is compensated by the slow rate of growth of the solid inner core. The resulting mass balance, combined with approximate geostrophic force balance in the core leads to estimates

$$\delta\rho/\rho \sim 3 \times 10^{-9} \quad , \quad w \sim 2 \times 10^{-4}\,\mathrm{m/s}.$$

Each rising blob drives a Taylor column, and the helicity and α-effect associated with this flow is estimated. A mean-field dynamo driven by this α-effect in conjunction with differential rotation generates a magnetic field whose strength is determined in order of magnitude by the plausible assumption of magnetostrophic equilibrium.

1. Introduction

The most plausible scenario for the dynamics of the geodynamo is that first proposed by Braginskii [1], in which gravitational energy is released through the process of solidification at the inner core boundary (ICB) of the iron alloy of which the liquid outer core is composed. Pure iron solidifies, leaving a layer of molten alloy slightly richer in the lighter ingredient (which we shall assume to be sulphur, although this is not the only possibility). Dendrites grow and collapse under their own weight, forming a 'mushy zone' [2] of depth of the order of a few kilometers, this layer providing a continuous source of the lighter fluid material.

This layer of lighter fluid is gravitationally unstable by the Rayleigh-Taylor mechanism. The character and growth-rate of the instability are controlled partly by conditions within the mushy zone, which may be most simply treated as a porous medium with a Darcy law of resistance, and partly by the dynamics of the overlying liquid zone in which both Coriolis forces and Lorentz forces play an important, and probably dominant, role.

Whatever the nature of the instability, the resulting rise of lighter material is compensated by the descent of heavier material and by the gravitational condensation through solidification onto the solid inner core. In this process, gravitational

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energy is released at a rate of order 10^{12}W[3], and is converted partly to kinetic energy of the rising blobs, and partly to heat. If there were no dynamo action and therefore no magnetic field, all of this power would have to be dissipated by viscosity; the flow would be turbulent with energy dissipation rate

$$\epsilon \sim u^3/\ell \sim 10^{-8}\mathrm{W/m^3}$$

where u and ℓ are velocity and length-scales of the turbulence. Thus, for example, if $\ell \sim 10$ km, then $u \sim 5$ cm/s., a hundred times larger than the value that is inferred from studies of the secular variation of the Earth's magnetic field extrapolated down to the core-mantle boundary (CMB) [4].

Dynamo action *does* however occur, because strong Coriolis forces associated with the Earth's rotation ensure that the buoyancy-driven convection has a helical character, with helicity distribution [5] that is antisymmetric about the equatorial plane. Provided the typical scale ℓ of the convection is small compared with the core radius (and this is the essence of the process proposed in this paper), a mean-field approach [5-8] is relevant, and dynamo action occurs through the α-effect that is an inevitable concomitant of the non-zero helicity. Thus the kinetic energy is immediately converted to magnetic energy which is in turn converted to heat by Joule dissipation. The rate of Joule dissipation associated with a predominantly toroidal field of order 20G is of order 4×10^8W. The actual Joule dissipation will be greater than this, because of the dissipation associated with the small-scale field; and it may be supposed that the system adjusts itself so that the net rate of dissipation of energy into heat is just equal to the gravitational power supplied.

This description of the dynamo process is set out schematically in figure 1, in which some of the important subsidiary mechanisms are also indicated. Broadly speaking, these involve interactions between small-scale and large-scale fields, and between their poloidal and toroidal ingredients. A number of feedback processes are identified, which can all play a part in the delicate balance that is established. Ideally, the dynamo theoretician would like to explain not only the existence and present behaviour of the Earth's magnetic field, but also its long-term behaviour, including the phenomenon of reversals, over geological time. This target is still some way off, but it is evident that the complexity of the nonlinear effects represented in figure 1 is quite sufficient to incorporate such phenomena as reversals, without seeking more exotic physical mechanisms.

2. Bubble Convection

Following the above discussion, we postulate that Rayleigh-Taylor instability causes intermittent release of buoyant blobs (or 'bubbles') from the dendritic layer where the density defect is continuously created (figure 2). The molecular diffusivity D of sulphur in iron just above the melting point is estimated [9] to be

$$D \sim 10^{-8}\mathrm{m^2/s}$$

and is very small compared with the product $w\ell$ of the expected scale ℓ and rise velocity w (see below). In a first approximation, we may neglect this molecular

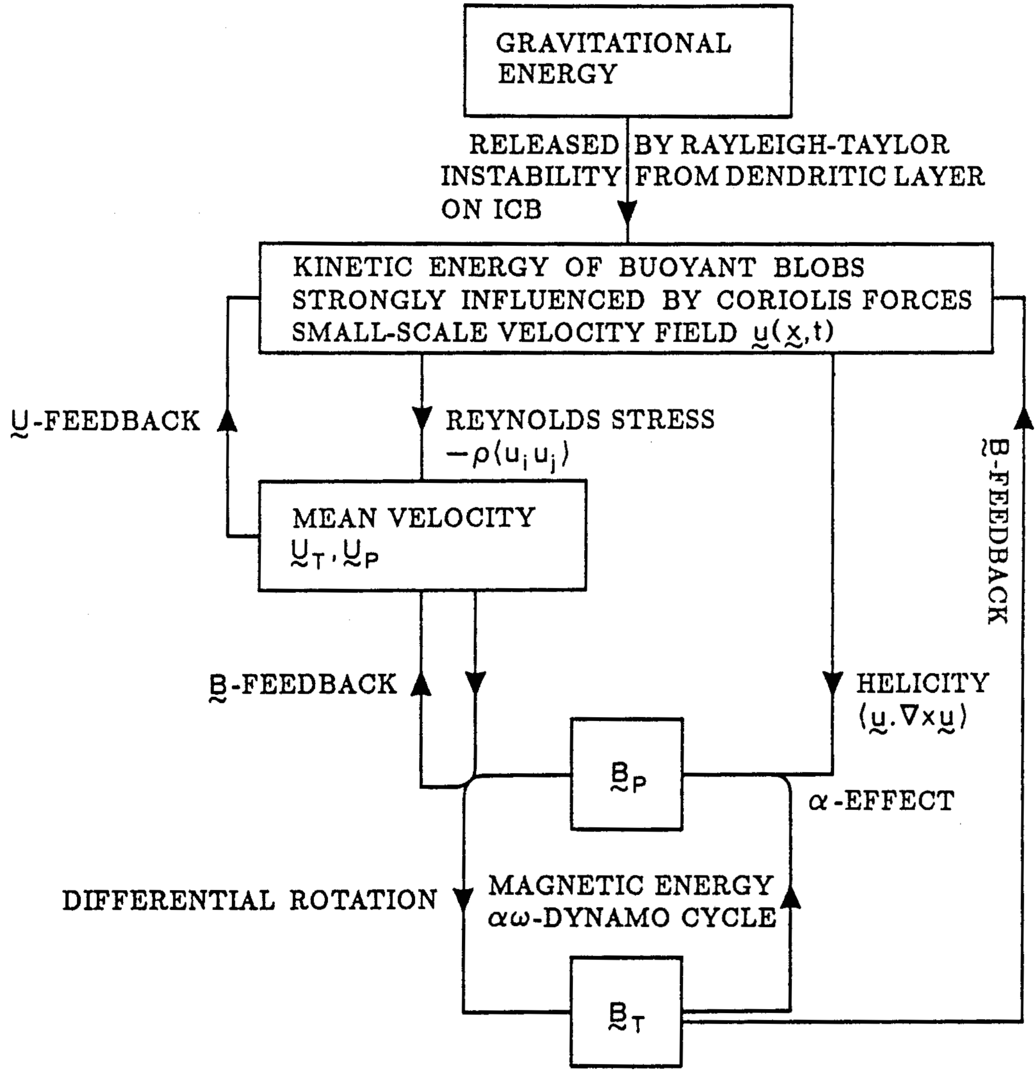

Figure 1. Energetics and feedback cycles of the Geodynamo

diffusion altogether. The bubbles, once formed, then rise through the outer core, and spread out horizontally as they approach the CMB. [This picture of bubble convection has some affinity with the bubble convection proposed by Howard [10] as a model for thermal convection at very high Rayleigh number.]

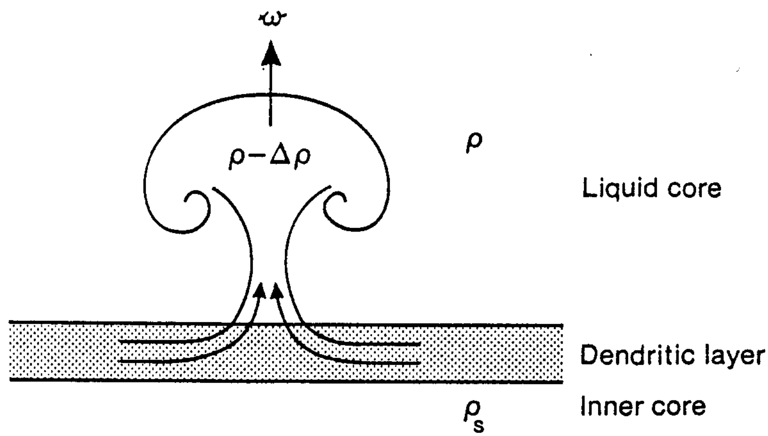

Figure 2. Release of buoyant blob from dendritic layer on ICB

Consider first the mass balance associated with this process (figure 3). Let w represent the mean upward velocity of blobs near the ICB, carrying density defect $\Delta\rho$; this is compensated by a downward mass flux

$$\frac{dM}{dt} = 4\pi R_I^2 \dot{R}_I(\rho_s - \rho)$$

required to provide the growth of the inner core; here R_I is the inner core radius, ρ_s the density of the solid inner core, and ρ the density of the liquid core, both taken near the ICB, and we assume $\rho_s - \rho \approx \rho/20$. This mass balance then gives

$$w\ \Delta\rho/\rho \approx \dot{R}_I/20. \tag{2.1}$$

Secondly, consider the force balance which determines the rate of rise of the bubble. In a non-rotating environment, this rate of rise would be given (on dimensional grounds) by

$$w \sim (ga\ \Delta\rho/\rho)^{1/2}, \tag{2.2}$$

where a is the radius of the bubble (or the radius of curvature at its upper surface); this merely reflects a balance between the buoyancy force $\Delta\rho\ \underline{g}$ and the convective acceleration term $\rho\underline{u}.\nabla\underline{u}$ of the Navier Stokes equation. In a rotating medium with

$$a\Omega/w \gg 1, \tag{2.3}$$

the Coriolis acceleration $\rho\underline{\Omega} \wedge \underline{u}$ replaces the convective acceleration in the force balance, so that (2.2) is replaced by

$$w \sim (g/\Omega)\Delta\rho/\rho, \tag{2.4}$$

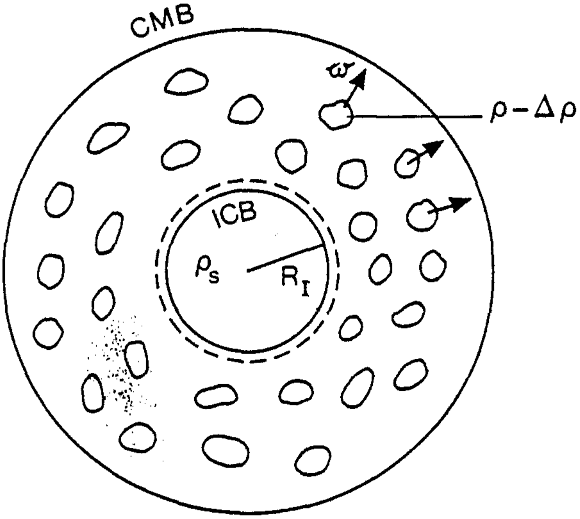

Figure 3. The rise of blobs with density defect $\Delta\rho$ implies a downward mass flux just sufficient to account for the slow growth of the inner core.

independent of the size of the bubble.

Combining (2.1) and (2.3), we find

$$\frac{\Delta\rho}{\rho} \sim \left(\frac{\Omega \dot{R}_I}{20\, g}\right)^{1/2} \quad , \quad w \sim \left(\frac{g\, \dot{R}_I}{20\, \Omega}\right)^{1/2} . \tag{2.5}$$

With $\Omega \sim 7 \times 10^{-5} \mathrm{s}^{-1}$, $g \sim 3\ m/s^2$ (near the ICB) and $\dot{R}_I \sim 10^{-11}\mathrm{m/s}$ (on the assumption that the inner core has been growing steadily over the whole lifetime of the Earth), (2.5) gives

$$\frac{\Delta\rho}{\rho} \sim 3 \times 10^{-9} \quad , \quad w \sim 2 \times 10^{-4} \mathrm{m/s}. \tag{2.6}$$

This estimate of w is entirely plausible, being of the same order of magnitude as velocities at the CMB inferred from the secular variation studies referred to previously. Note that $w/\Omega \sim 3\mathrm{m}$ so that any blob of scale a much greater than a few meters will easily satisfy the condition (2.3).

3. Helicity and Dynamo Action Induced by Bubble Rise

Under the same condition (2.3), the rising bubble may be expected to drive flow in a Taylor column [11] consisting of the cylinder of fluid circumscribing the bubble and parallel to the rotation vector $\underset{\sim}{\Omega}$. The effect is difficult to analyse in the spherical annulus geometry, but may be understood at least qualitatively with reference to the plane geometry indicated in figure 4. As the bubble rises, it pushes the column of fluid ahead of it; this column must spread out in a layer near the upper boundary where convective acceleration associated with the change of direction of the flow is not negligible; and the fluid must then descend and flow in to feed the Taylor column at the lower boundary.

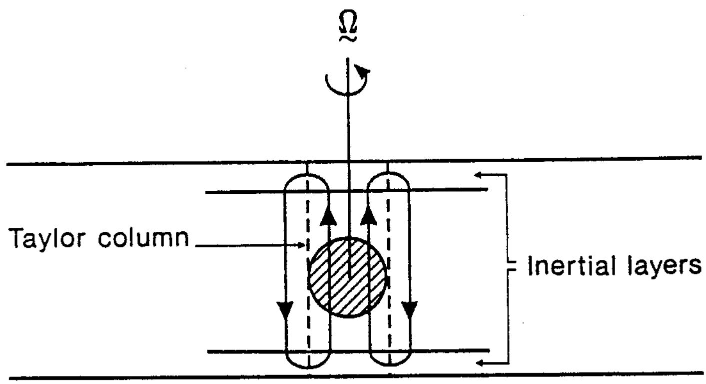

Figure 4. Taylor column associated with a rising bubble in a plane geometry; inertial layers are needed at top and bottom to satisfy the condition of zero normal velocity on the boundaries.

The upward flux in the column is $Q = \pi a^2 w$, and this is compensated by an equal and opposite downward flux outside the column. The downward moving fluid rotates more slowly than the upward moving fluid on account of the tendency to conserve angular momentum in the inertial layers at the top and bottom, this reduction in angular velocity being proportional to Ω. Hence the total helicity of the flow, defined by

$$\mathcal{H} = \int \underset{\sim}{u} \cdot \underset{\sim}{\omega} \, dV \tag{3.1}$$

is given in order of magnitude by

$$\mathcal{H} \sim V_c w \Omega, \tag{3.2}$$

where V_c is the volume of the Taylor column. More generally, we may expect that if there are a number of such columns randomly distributed, then the local mean helicity density will be

$$\bar{\mathcal{H}} \sim c\, w\Omega \tag{3.3}$$

where c is the proportion of the volume occupied by columns. [A similar mechanism for the production of helicity has been found by Hunt and Hussain [12] in the opposite limit of a weakly rotating fluid.]

The situation is more complicated in the spherical annulus geometry, as can be seen from figure 5. Outside the cylinder circumscribing the inner core and parallel to $\underset{\sim}{\Omega}$, the helicity induced by blobs originating from northern and southern hemispheres at the ICB will tend to cancel; no such cancellation occurs however *inside* this circumscribing cylinder, and the mean helicity in the fluid part of this region will be given by $\bar{\mathcal{H}} \sim \pm cw\Omega$ in the northern and southern hemispheres respectively.

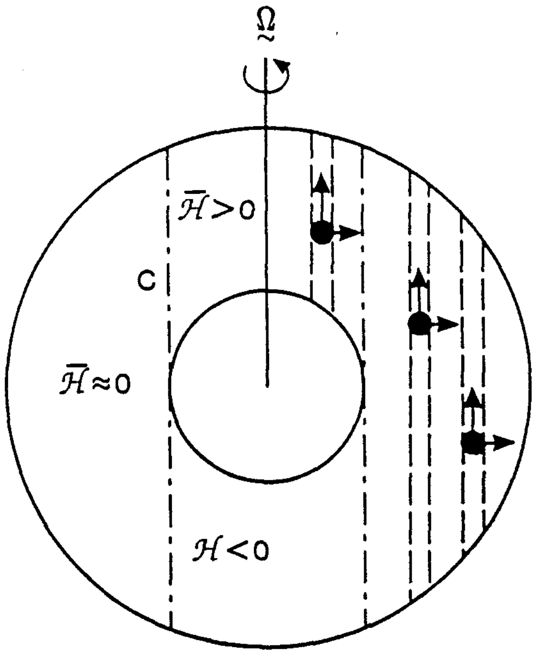

Figure 5. Taylor columns associated with rising blobs in spherical geometry. Helicities associated with blobs outside the cylinder C tend to cancel.

We may now estimate the α-effect [6,7] associated with this helicity. To be specific, let us suppose that the scale of the bubbles is of order $a \sim 10$km, which seems quite plausible in relation to the scale of the dendritic layer from which the bubbles originate. The magnetic diffusivity of the liquid core, η , is of order $3\text{m}^2/\text{s}$, so that the magnetic Reynolds number associated with the bubble rise is

$$R_m \cong \frac{wa}{\eta} \sim 1 \,. \tag{3.4}$$

Under this condition, α may be determined by first-order smoothing theory [7], and is given in order of magnitude by

$$\alpha \sim -\bar{\mathcal{H}} a^2/\eta \sim c(\Omega a) R_m \,. \tag{3.5}$$

The magnetic Reynolds number based on α and the core radius R_c is then

$$R_\alpha = \left|\frac{\alpha R_c}{\eta}\right| \sim c\left(\frac{\Omega R_c}{w}\right) R_m^2 \tag{3.6}$$

so that, for example, with $c \sim 0.01$, we find

$$R_\alpha \sim 3 \times 10^3, \tag{3.7}$$

a value that is amply sufficient to drive a dynamo of either α^2 or $\alpha\omega$-type [5,8].

The above discussion of course neglects the back-reaction of the magnetic field on the flow, via the Lorentz force. It is to be expected that the magnetic field will saturate at a level at which the three forces of buoyancy, Coriolis and Lorentz are comparable:

$$\underline{g}\Delta\rho \sim 2\rho\underline{\Omega} \wedge \underline{u} \sim \underline{j} \wedge \underline{B}. \tag{3.8}$$

With $j \sim \sigma w B$, this leads to the estimate

$$B \sim (2\rho\Omega\mu_o\eta)^{1/2} \sim 20G. \tag{3.9}$$

Since the poloidal field in the core is of order 1G, this suggests that the dynamo is of $\alpha\omega$-type with a dominant toroidal field of order 20G.

4. Discussion

We have adopted the view [1] that the source of energy for dynamo action in the Earth's core is the gravitational potential energy that is released when iron alloy solidifies at the ICB. We have argued that this process leads to the release of bubbles of material with density defect $\Delta\rho/\rho \sim 3 \times 10^{-9}$, rising with velocity $w \sim 2 \times 10^{-4}$m/s; that these rising blobs generate helicity and so an α-effect in the rotating environment; and that this, in conjunction with differential rotation, drives an $\alpha\omega$-dynamo.

The description appears self-consistent as far as orders of magnitude are concerned. A number of problems arise, however, which need closer investigation: (i) the problem of Rayleigh-Taylor instability in a dendritic layer, taking due account of Coriolis forces and Lorentz forces in the overlying liquid; (ii) the problem of the rise of a blob of slightly buoyant material in a spherical annulus, the dynamics being dominated by Coriolis and Lorentz forces; (iii) the horizontal spread of such a blob when it reaches the CMB, and the associated transient coupling between core and mantle that this may induce.

There may also be a longer period cyclical behaviour associated with the gradual growth of the layer of more buoyant material near the CMB. The rate of growth of this layer is such that it will fill the spherical annulus in a time of approximately 4000 years. At this stage the density would be $\rho - \Delta\rho$ throughout the liquid core, and the process would in effect start again. Over the whole lifetime ($\sim 4 \times 10^9$

years) of the Earth, the mean density of the liquid core has decreased by an amount $\Delta\rho$ about 10^6 times if this picture is correct, the total decrease being of order $.002\rho$, just sufficient to compensate the growth of the denser inner core. It is tempting to speculate that the oscillations of the Earth's dipole moment on time-scales in the range $10^3 - 10^4$ years may be caused by this cyclical process in the core, and even that field reversals may be triggered by this mechanism.

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