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THE EARTH'S MAGNETISM PAST ACHIEVEMENTS AND FUTURE CHALLENGES

H.K. Moffatt

Department of Applied Mathematics and Theoretical Physics
University Cambridge
Silver Street, Cambridge CB3 9EW, UK

1. Historical Perspectives

My subject is the magnetic field of the Earth and the manner of its variation in space and time, a subject that has challenged the imagination of mankind since the dawn of civilisation. My particular concern is the root cause of terrestrial magnetism, the mechanism that I shall describe as the 'dynamo process' by which the magnetic field of the Earth has been generated and maintained since the earliest geological times. The history of the magnetic field is revealed by the study of ancient rocks which were susceptible to and magnetised by the Earth's field as they solidified and cooled through the Curie point – anything from 100 K to 700 K depending on the mineral. From palaeomagnetic records, we know that the magnetic field has existed at roughly its present strength for at least 3.5×10^9 years, and that reversals of its polarity have occurred at random intervals, typically between one and 5 times 10^5 years, throughout these aeons of time. But only in the last 50 years have we come anywhere near to an understanding of this extraordinary global phenomenon. I want to describe some of the developments of these last 50 years, to indicate where we stand now, and to consider the nature of the critical problems that still confront us. But first, allow me to place this problem in proper historical perspective.

Every child who has played with a magnetic compass knows that the compass needle points North; but he learns as he grows older that magnetic North is not quite the same as 'true' North defined by the Pole star; or to put it differently, that the magnetic dipole axis is slightly inclined to the rotation axis of the Earth. This worrying mismatch was already known to the Chinese of the Sung dynasty. In his great work 'Science and Civilisation in

China', Joseph Needham quotes the Mêng Chhi Pi Than of Shen Kua (c1088) which he translates as follows: "Magicians rub the point of a needle with the lodestone; then it is able to point to the south. *But it always inclines slightly to the east, and does not point directly at the south*"; so the 'declination' of the field was known, at least to the Chinese, more than 900 years ago†. It was rediscovered and charted out by the early navigators of the 15th and 16th centuries and in particular by Christopher Columbus (1492) whose great voyage of discovery will be celebrated next year. We recognize this declination now as a manifestation of a crucial departure from axisymmetry which is essential for the Earth's internal dynamo to operate.

The distinction between local magnetic north and 'true' north is often indicated on large-scale maps. The small print usually warns that the angle between the two directions changes irregularly by up to 1° in 6 years. This is the 'secular variation' of the magnetic field which was known to navigators of the 17th century, and was no doubt a considerable nuisance to them. Edmund Halley considered the possible causes of this secular variation in 1693, and concluded that "the external parts of the globe may well be reckoned as the shell, and the internal as a nucleus or inner globe included within ours, with a fluid medium between ... only this outer Sphere having its turbinating motion some small matter either swifter or slower than the inner Ball." – This was a prophetic vision as far as the inner structure of the earth is concerned, but also remarkable in its perception of the need for *differential rotation*, now recognized as a further key element in the dynamo process.

A modern understanding of this dynamo process is based on Michael Faraday's law of induction. The Bank of England has this year marked the bicentenary of Faraday's birth by printing a new £20 note bearing his image. By painstaking experiments, Faraday discovered in 1832 that if a conductor moves across a magnetic field, and if a path is available for the completion of a current circuit, then, in general, current will flow in that circuit. For this achievement, Faraday was awarded the Copley Medal of the Royal Society of London. The Royal Society citation records that "... he gives indisputable evidence of

† Footnote: Although quoted also in Chapman & Bartel (1940, Vol II, p.902) the quotation is there given a less positive interpretation.

electric action due to terrestrial magnetism alone. An important addition is thus made to the facts which have long been accumulating for the solution of that most interesting problem, the magnetism of the earth." It was in fact more than an important addition; it was the key ingredient of the dynamo process, although this was not recognised till much later.

Faraday is not alone this year among eminent magneticians to be commemorated on a banknote. Carl Friedrich Gauss (1777–1855) also enjoys this distinction on the new 10 mark note issued this year by the Deutsche Bundesbank. In two great papers, Gauss (1832, 1838) established the spherical harmonic decomposition of the Earth's magnetic field and the technique by which secular variation of the field could be quantified. The traditional unit of field intensity in Geomagnetism is of course the gauss (G), and it is regrettable that the Système International of units now favours the tesla (T) ($IT = 10^4 G$). Gauss's spherical harmonic decomposition allows us to extrapolate the field (assumed potential) down to the core-mantle boundary (CMB), to map the contours of constant radial field at the CMB, and to do so at different epochs using all available data; Bloxham, Gubbins & Jackson (1989) present these results in brilliant colour contours. In these maps, the dipole ingredient of the field is still quite evident at the CMB, but there is also a strong presence of quadrupole, octupole and higher order ingredients, as is to be expected from the nature of downward extrapolation towards the region where the 'source' currents are confined. The slow evolution of the pattern (i.e. its secular variation) is also evident.

The high point and climax of electromagnetic theory in the 19th century was the publication in 1873 of Maxwell's Treatise on Electricity and Magnetism. Maxwell built on Faraday's discoveries, and completed the system of equations that bear his name. It is interesting to note however that in the chapter of the treatise devoted to "Terrestrial Magnetism", Maxwell comes nowhere near to any explanation of the real nature of the phenomenon. He confines himself to a description of Gauss's techniques for the determination of the Earth's field and its time variation, and for demonstrating that the dominant sources for the field are of internal rather than external origin; but as to the root cause of the phenomenon, he writes: "The field of investigation into which we are introduced by the study of terrestrial magnetism is as profound as it is extensive. ... What cause, whether exterior to the earth or in its inner depths, produces such enormous changes in the earth's magnetism, that its magnetic poles move slowly from one part of the globe to another? ... These immense changes in so large a body force us to conclude that we are not yet acquainted with one of the most powerful agents in nature, the scene of whose activity lies in those inner depths of the earth, to the knowledge of which we have so few means of access". It was the science of seismology that was to provide the vital means of access, establishing the existence first of a liquid outer core (Jeffreys (1926): "the central core is probably fluid, but its viscosity is uncertain"), and secondly of an inner core (Lehmann 1936)[†] that is solid (Bullen 1946), both now believed to be essential for the operation of the geodynamo.

One of the earliest discussions of possible causes of terrestrial magnetism was given by Arthur Schuster in his Presidential address to the Physical Society of London in 1912. Schuster discussed the arguments for and against a current system in the earth's interior, and concluded that "the difficulties which stand in the way of basing terrestrial magnetism on electric currents inside the earth are insurmountable" – strong words, which have since been invalidated with the passage of time, and the birth and advance of magnetohydrodynamics. Nevertheless, even as late as 1940, Chapman & Bartel in their great treatise on Geomagnetism, came to the same defeatist conclusion as Schuster. They discussed Larmor's (1921) suggestion concerning the possibility of self-exciting dynamo action but stated that "Cowling, however, has shown that such self-excitation is not possible. Consequently, Schuster's view still holds, that 'the difficulties ... are insuperable [sic]! ". Cowling (1934) had not in fact shown that such self-excitation is not possible: he had merely shown that it was not possible for axisymmetric systems, and the tilt of the magnetic dipole which had been known for centuries shows that we are dealing with an emphatically non-axisymmetric system. Nevertheless the fact that Chapman & Bartel could be so easily persuaded that

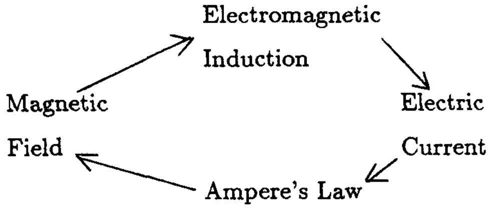
† An illuminating discussion of the developments leading to these discoveries, and the reasons for attributing them to Jeffreys and Bullen respectively, is given in the paper "Discovery of the Earth's Core" by Brush (1980).

Cowling's anti-dynamo theorem (see, for example, Moffatt 1978) closed the matter is an indication of the powerful influence that this theorem then had – this no doubt because it was one of the few exact results of the subject. The year 1940 marks a high point in the collection and systematisation of geomagnetic data, but also marks the nadir as regards real understanding of the origins of terrestrial magnetism.

The post-war years have seen a profound transformation in the situation, to the point at which a dynamo theory of the origin of the earth's magnetic field is now universally accepted among geophysicists. The progress in dynamo theory has been dramatic, and the theory applies with equal force to planets other than the Earth. Statements in textbooks of the 1980's are as vigorously positive as Schuster's (1912) statement was negative. Thus, for example, Cook (1980) writes "There is no theory other than a dynamo theory that shows any signs of accounting for the magnetic fields of the planets"; and Jacobs (1984) writes "There has been much speculation on the origin of the Earth's magnetic field ... The only possible means seems to be some form of electromagnetic induction, electric currents flowing in the Earth's core". It is a dynamo theory based on the principles of magnetohydrodynamics, and ultimately on a suitable exploitation of Faraday's law of induction, that has led to this remarkable revolution in our understanding of Nature. We now turn to the essential ingredients of this dynamo theory.

2. Essential Ingredients of the Dynamo Process

The most plausible scenario for the geodynamo, now widely accepted, is that first advanced by Braginskii (1964a). In this scenario, the ultimate source of energy for the dynamo is gravitational energy released by the slow cooling of the Earth, and consequential slow growth of the solid inner core. As this inner core solidifies, a mushy zone is created at the inner core boundary (ICB), within which buoyancy is generated due to the slow release of the lighter ingredients (sulphur, oxygen, silicon ...) in the liquid iron alloy of which the outer core is composed, this buoyancy force being compensated partly by the Coriolis force due to the Earth's rotation, and partly by the Lorentz force due to the magnetic field, whose presence we seek to explain. The Coriolis force induces the vital property of *helicity* in the convective motion, while an associated tendency to conserve angular momentum leads to a state of *differential* rotation in the core about the axis of rotation (defined with respect to the mantle). These properties are now known to be highly conducive to self-exciting dynamo action, represented by the 'bootstrap' cycle



The magnetic field grows as an instability associated with this cycle, and equilibrates when the Lorentz force is strong enough to exert a braking and controlling action on the convection. Within this scenario, reversals of polarity can appear simply as a flipping between two unstable states of a complex nonlinear dynamical system.

These essential ingredients can be modelled by a combination of two idealised systems (figure 1a,b); the first is the self-exciting disc dynamo, in which a disc rotates in the presence of a magnetic field associated (via Ampere's law) with the current I induced by Faraday's law. The simplest equation governing this system is of the form

$$L\frac{dI}{dt} + RI = M\Omega I \tag{1}$$

where L and R are the self-inductance and resistance of the total current circuit, M is the mutual inductance between the wire loop and the rim of the disc, and Ω is the angular velocity of the disc. The right-hand side of eqn. (1) represents the Faraday induction effect. Actually, eqn. (1) oversimplifies the situation; for it implies that when Ω is constant and greater than R/M, the current grows like e^{pt} where

$$p = L^{-1}(M\Omega - R) \quad , \tag{2}$$

a dynamo instability. As $R \to 0$, this gives $p \sim M\Omega/L$, a finite growth rate, the hall-mark of what is now described as a 'fast' dynamo. In this respect, the model equation (1) is misleading, but the deficiency is not lethal and may be corrected (Moffatt 1979) in a way that shows that the disc dynamo is not in fact fast, but slow ($p \to 0$ as $R \to 0$).

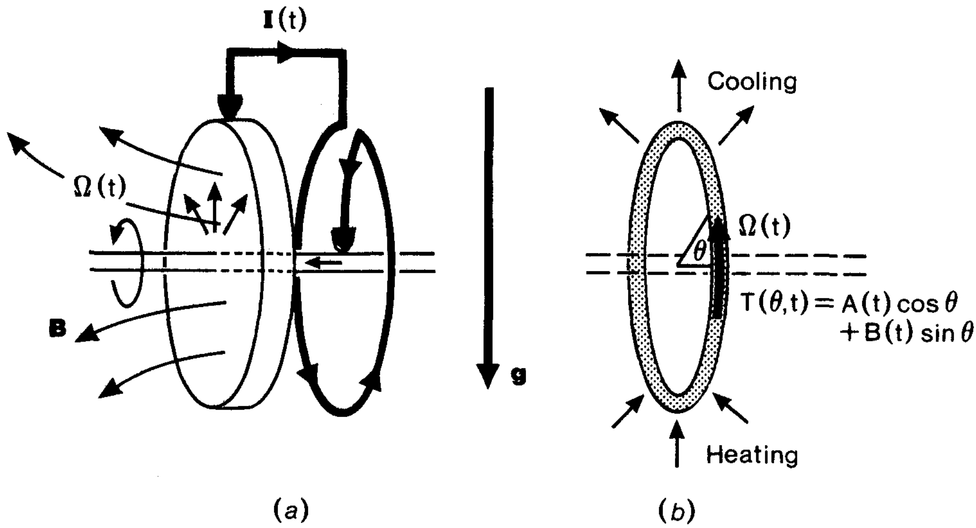


Figure 1. Marriage of (a) the Bullard homopolar disc dynamo and (b) the Welander fluid loop, coupled through gravity, to provide a model thermally driven dynamo.

The second model ingredient (figure 1b) is a fluid loop of radius a in a vertical plane, heated from below and cooled from above, a potentially unstable situation (Welander 1967). If the fluid flows in the θ-direction with angular velocity $\Omega(t)$ then the temperature perturbation $T(\theta, t)$ satisfies an equation of the form

$$\frac{\partial T}{\partial t} + \Omega \frac{\partial T}{\partial \theta} = D \frac{\partial^2 T}{\partial \theta^2} + S(\theta) \tag{3}$$

where D is a measure of thermal diffusivity in the θ-direction and $S(\theta)$ represents the differential heating, which for simplicity we may take to be

$$S(\theta) = -\sigma \sin\theta \tag{4}$$

where σ is a positive constant. The solution of (3) then has the form

$$T(\theta, t) = A(t)\cos\theta + B(t)\sin\theta \tag{5}$$

where

$$\frac{dA}{dt} + \Omega B = -DA \quad , \tag{6}$$

$$\frac{dB}{dt} - \Omega A = -DB - \sigma \quad . \tag{7}$$

The density perturbation is $\Delta\rho = -\alpha T$ where α is the coefficient of thermal expansion, and the gravitational torque acting on the fluid is then

$$G = \int_0^{2\pi} \alpha g T a \cos\theta (V/2\pi) d\theta = \tfrac{1}{2}\alpha g a V A(t) \tag{8}$$

where V is the volume of fluid in the tube.

The equation of motion of the fluid is

$$C\frac{d\Omega}{dt} = G - k\Omega \tag{9}$$

where C is the moment of inertia of the fluid about its axis of rotation, and k is a frictional resistance parameter, associated with the viscosity of the fluid. The state $A = 0, B = -\sigma/D, \Omega = 0$ is unstable if

$$\gamma \equiv \tfrac{1}{2}\alpha g V a > D^2 k/\sigma \quad , \tag{10}$$

and then the preferred (stable) states are given by

$$\Omega = \pm\left(\frac{\gamma\sigma}{k} - D^2\right)^{\frac{1}{2}} \quad , \quad A = \frac{\sigma\Omega}{D^2 + \Omega^2} \quad , \quad B = -\frac{D\sigma}{D^2 + \Omega^2} \quad , \tag{11}$$

so that the fluid rotates (either clockwise or anticlockwise) with steady angular velocity.

Suppose now that we combine these two ingredients, i.e. place the fluid loop around the rim of the disc in such a way that the torque G is communicated to the disc so that this rotates with the same angular velocity as the fluid. We reinterpret C as the total moment of inertia and k as the total frictional resistance coefficient. As well as the torque G, the disc now experiences an electromagnetic torque $-MI^2$ (due to the Lorentz force distribution within the disc) so that (9) is replaced by

$$C\frac{d\Omega}{dt} = G - k\Omega - MI^2 \quad . \tag{12}$$

The composite system is now governed by (1), (6), (7), (8) and (12), a 4th order nonlinear dynamical system.

Consider now what happens if the thermal forcing parameter σ is increased very slowly from zero. A first bifurcation occurs at $\sigma = D^2 k/\gamma$, and for $\sigma > D^2 k/\gamma$ one of the steady convective solutions (11) is established. As σ increase further, the state with $\Omega > 0$ becomes unstable to magnetic perturbations when $\Omega = R/M$, i.e. when

$$\sigma = \frac{k}{\gamma}\left(D^2 + (R/M)^2\right) \quad , \tag{13}$$

and there is a second bifurcation to states given by

$$\Omega = R/M \quad , \quad \frac{G}{\gamma} = A = -\frac{\sigma B}{D} = \frac{\sigma \Omega}{D^2 + \Omega^2} \quad , \quad I = \pm[(G - k\Omega)/M]^{\frac{1}{2}} \quad . \tag{14}$$

The state with $\Omega < 0$ is not subject to this type of dynamo instability. The difference between the states $\Omega > 0$ and $\Omega < 0$ arises because the twist of the wire in figure 1a breaks the mirror symmetry of the system.

This system is of course highly idealised, but it does contain the essential ingredients of dynamo action: first some kind of basic convective instability driven by buoyancy forces; second, differential rotation which is here concentrated at the sliding contacts between the rotating disc (whether fluid or solid) and the fixed wire; and third, lack of mirror symmetry, here deliberately and necessarily introduced through the twist of the wire. This is the most subtle aspect of the dynamo process, which, in a homogeneous conductor, is generally associated with the helicity of the flow.

One noteworthy property of the solution (14) is that as σ increases, the kinetic energy $\frac{1}{2}C\Omega^2$ of the system remains constant, whereas the magnetic energy $\frac{1}{2}LI^2$ increases linearly with σ and (for large σ) is much greater than the kinetic energy. Likewise the Joule dissipation RI^2 is much greater than the 'viscous' dissipation $k\Omega^2$. The geodynamo process is believed to operate in a similar regime.

3. Helicity and the α-Effect

The year 1966 marked the publication of a paper which was to give greater impetus to the subject of dynamo theory in both geophysical and astrophysical contexts than any other of the post-war era. This was the paper of Steenbeck, Krause & Rädler of the Institute for Astrophysics in Potsdam, in which they showed that turbulence whose statistical properties lack mirror symmetry can have the effect of twisting the mean current in a conducting fluid towards alignment with the mean magnetic field, the 'mean' being defined as a local mean over scales large compared with the scale of the turbulence. This effect, which they described as the 'α-effect', had been found also in earlier work of Parker (1955) and Braginskii (1964b), but the subtlety and complexity of these earlier theories had obscured the basic simplicity of the effect, and the achievement of Steenbeck et al (1966) was to recognize this basic simplicity and thus to lay the foundations for many subsequent developments.

The effect can be very easily understood through consideration of a simple example. Suppose that a velocity field

$$\mathbf{u} = u_0(\sin kz, \cos kz, 0) \tag{15}$$

acts upon a uniform magnetic field $\mathbf{B}_0 = (0, 0, B_0)$. Note that the field (15) has the property that

$$\boldsymbol{\omega} = \nabla \wedge \mathbf{u} = u_0 k(\sin kz, \cos kz, 0) = k\mathbf{u} \tag{16}$$

so that the vorticity $\boldsymbol{\omega}$ is parallel to $\mathbf{u}$, i.e. $\mathbf{u}$ is a Beltrami field. The *helicity* of the flow is $\mathcal{H} = \langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle = k u_0^2$. The induction equation of magnetohydrodynamics is

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl}\,(\mathbf{u} \wedge \mathbf{B}) + \eta \nabla^2 \mathbf{B} \tag{18}$$

where η is the magnetic diffusivity of the fluid. Hence, a perturbation field $\mathbf{b}$ is generated, which under steady conditions, is given by

$$\eta \nabla^2 \mathbf{b} = -(\mathbf{B}_0 \cdot \nabla)\mathbf{u} = -B_0 u_0 k(\cos kz, -\sin kz, 0) \quad , \tag{19}$$

so that

$$\mathbf{b} = \frac{B_0 u_0}{\eta k}(\cos kz, -\sin kz, 0) \quad . \tag{20}$$

The lines of force of the total field $\mathbf{B}_0 + \mathbf{b}$ are then helices. Moreover, from (15) and (19), there is a mean electromotive force

$$\mathcal{E} \equiv \langle \mathbf{u} \wedge \mathbf{b} \rangle = \alpha \mathbf{B}_0 \quad , \quad \alpha = -\mathcal{H}/\eta k^2 \tag{21}$$

where the angular brackets represent averaging over z. (Actually this averaging is not needed in this simple example, but it is needed for more general choices of $\mathbf{u}$.) Equation (20) displays the α-effect in its simplest form: non-zero helicity implies an electromotive force parallel to the ambient magnetic field $\mathbf{B}_0$ and consequently has the effect of driving current parallel to $\mathbf{B}_0$. Note that both $\mathcal{H}$ and α are pseudo-scalar, and are non-zero only if (as here) the flow lacks reflexional (i.e. mirror) symmetry. For more general random velocity fields with isotropic statistics and helicity spectrum function $H(\mathbf{k})$, the expression for α is (Moffatt 1970)

$$\alpha = -\frac{1}{3\eta} \int H(\mathbf{k}) k^{-2} d^3\mathbf{k} \tag{22}$$

the factor $\frac{1}{3}$ appearing through averaging over all directions.

Allowing now for slow spatial variation of $\mathbf{B}$ and taking the local average of (18), the mean field evolves according to the equation

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl } (\alpha \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad . \tag{23}$$

If α is constant, then the most unstable structures associated with this equation are indeed those for which $\mathbf{J} = \text{curl } \mathbf{B}$ is parallel to $\mathbf{B}$; writing curl $\mathbf{B} = K\mathbf{B}$, such modes grow like e^{pt} where

$$p = \alpha K - \eta K^2 \quad , \tag{24}$$

so that we have dynamo instability provided

$$|\alpha K| > \eta K^2 \tag{25}$$

i.e. provided the scale K^{-1} of the mean field is sufficiently large. The physical mechanism of this instability is simply understood through consideration of two linked flux tubes carrying fields $\mathbf{B}_1$ and $\mathbf{B}_2$. The α-effect generates a current $\mathbf{B}_1$ in tube 1, $\mathbf{J}_2$ in tube 2, which (via Ampere's law) act as the source currents for the fields $\mathbf{B}_2$ and $\mathbf{B}_1$ respectively. Linked flux tubes are associated with nonzero *magnetic helicity* (Moffatt 1989); in the terrestrial context, the linked fields are the toroidal (or zonal) and poloidal (or meridional) ingredients of the mean field.

The beauty of the α-effect was that it allowed a very neat circumvention of Cowling's theorem which implied that the geodynamo, if it existed at all, had to be non-axisymmetric. It is non-axisymmetric motions that give rise to the α-effect in geodynamo theory, but the coefficient α is itself axisymmetric. Many dynamos of both steady and oscillatory type have been found corresponding to various choices of the function $\alpha(r,\theta)$ (in spherical polar coordinates r,θ,φ); the problem is to find the appropriate form for this function that is compatible with the background dynamics of the flow.

4. Macrodynamics of the Geodynamo

The Navier-Stokes equation governing the fluid dynamics of the core is (in the Boussinesq approximation)

$$\rho_0 \frac{D\mathbf{U}}{Dt} + 2\rho_0 \mathbf{\Omega} \wedge \mathbf{U} = -\nabla p + \delta\rho \mathbf{g} + \mathbf{j} \wedge \mathbf{B} + \rho_0 \nu \nabla^2 \mathbf{U} \tag{26}$$

where ρ_0 is the mean fluid density, $\delta\rho$ the perturbation associated with thermal and/or compositional buoyancy, p the pressure field, ν the kinematic viscosity of the fluid, and $\mathbf{j} \wedge \mathbf{B}$ the Lorentz force distribution ($\mathbf{j} = \mu_0^{-1} \mathrm{curl}\, \mathbf{B}$). We here use $\mathbf{U}$ for the large-scale velocity (i.e. neglecting any small-scale turbulence or internal waves). Reasonable estimates of $|\mathbf{U}|$ and of ν invariably indicate that, except possibly in boundary layers on the ICB and CMB, the inertia term $\rho_0 D\mathbf{U}/Dt$ and the viscous term $\rho_0 \nu \nabla^2 \mathbf{U}$ are both completely negligible compared with the Coriolis term $2\rho_0 \mathbf{\Omega} \wedge \mathbf{U}$; the force balance is then 'magnetostrophic':

$$2\rho_0 \mathbf{\Omega} \wedge \mathbf{U} = -\nabla p + \delta\rho \mathbf{g} + \mathbf{j} \wedge \mathbf{B} \quad . \tag{27}$$

There is an immediate constraint on $\mathbf{B}$ that may be derived from this (J.B. Taylor 1963).

With cylindrical polar coordinates (s, φ, z), the φ-component of (27) is

$$(\mathbf{j} \wedge \mathbf{B})_\varphi = \frac{1}{s}\frac{\partial p}{\partial \varphi} + 2\rho_0 \Omega U_s \quad . \tag{28}$$

Let $C(s)$ be a cylinder of radius s within the liquid core and bounded by the CMB (and the ICB if $s < R_I$); then incompressibility implies that U_s integrates to zero over $C(s)$, and since p is single-valued, it follows that

$$\mathcal{J}(s) \equiv \int_{C(s)} (\mathbf{j} \wedge \mathbf{B})_\varphi dS \equiv 0 \quad . \tag{29}$$

We are therefore led to seek fields $\mathbf{B}$ satisfying this *Taylor constraint* as legitimate candidate fields resulting from a dynamo process. Unfortunately the fields $\mathbf{B}$ that are excited by a dynamo instability with prescribed mean velocity $\mathbf{U}(\mathbf{x})$ and α-effect $\alpha(\mathbf{x})$ do not in general satisfy this constraint. This is of course because a purely kinematic prescription of $\mathbf{U}$ and α fails to recognize the dynamics associated with (27) and its consequence (29).

This difficulty can be tackled in two ways. The first was explored by Malkus & Proctor (1975) and Proctor (1977); for a detailed discussion, see Moffatt (1978, §12.2). Briefly, the idea is to consider that the α-effect is *prescribed* and to solve the coupled equations (23) and (26) (with $\delta\rho = 0$), either by perturbation techniques when the dynamo mechanism is just supercritical, or by numerical techniques. Proctor (1977) found a tendency towards satisfaction of the Taylor constraint as the system evolved, but with superposed torsional oscillations which failed to decay when the diffusion parameters η and ν were very small compared with ΩR^2 (as they are in the geo-context).

The second approach (Braginskii 1975, 1978) is based on the idea that a small nonzero value of the Taylor function $\mathcal{J}(s)$ may be accommodated through the effects of weak core-mantle coupling at the CMB, e.g. through the effects of Ekman suction in a viscous boundary layer on the CMB. This is Braginskii's 'model-Z' which yields field structures which have very small s-component in the core. By careful choice of an α-effect concentrated in the equatorial region, Braginskii (1978) was able to demonstrate the possibility of evolution towards such a state. Roberts (1989) has demonstrated that the asymptotic state, whether a Taylor state (with $\mathcal{J}(s) \equiv 0$), or a model-Z state ($|B_s| \ll |B_z|$) is very dependent on the intensity and structure of the α-effect that is assumed. The question of which (if either) of these types the geodynamo actually is remains open.

A further important step towards a self-consistent dynamo model has been taken by Fearn & Proctor (1987) who considered a driven dynamo in a spherical geometry, the idea being to assume a magnetic structure $\mathbf{B}(\mathbf{x})$, calculate the instabilities $\{\mathbf{u}, \mathbf{b}\}$ to which this structure is subject, derive the associated α-effect, and (with luck), show that $\mathbf{B}$ can be maintained by that α-effect. Obviously, an iterative procedure has to be followed to achieve this aim; as yet no converged solutions have been reported.

The most systematic attack that has as yet been launched on the problem of thermal convection in a rotating spherical annulus and the resulting dynamo action is that of Zhang & Busse (1987, 1988, 1989, 1990) who explore first the finite amplitude convective regime (with $\mathbf{B} = 0$), and secondly the magnetoconvective regime when a non-zero (but weak) field $\mathbf{B}$ is maintained by dynamo action. The simple model system described in §2 above actually provides quite a good indication of the important bifurcations that occur in this system also. It remains to be seen whether these computations can be extended to values of the parameters (Rayleigh number, Ekman number, Prandtl number, ...) that are realistic in the geo-context.

5. Microdynamics of the Geodynamo

An alternative approach, based on consideration of the likely nature of compositional convection driven by release of buoyant material from a mushy zone of order 1 km thick, has been proposed by Moffatt (1989). It is supposed the blobs of fluid with density defect $\Delta\rho$ and scale in the range 1–10 km are released from the mushy zone and rise upwards with mean velocity w. There is then a net mass flux downwards which is needed to account for the slow growth of the inner core. This mass budget gives a first relation between w and $\Delta\rho$:

$$w\frac{\Delta\rho}{\rho} \approx \left(\frac{\rho_s - \rho}{\rho}\right)\dot{R}_I \approx \tfrac{1}{20}\dot{R}_I \quad , \tag{30}$$

where ρ_s is the density of the inner core, and $\dot{R}_I$ the rate of increase of its radius R_I.

The dynamics of the upward rising blobs is supposed to be governed by the magnetostrophic equation (27) i.e.

$$2\rho\mathbf{\Omega}\wedge\mathbf{u} = -\nabla p - \Delta\rho\mathbf{g} + \mathbf{j}\wedge\mathbf{B} \quad , \tag{31}$$

in which the Coriolis, buoyancy and Lorentz forces are assumed to be all of the same order of magnitude. The comparability of Coriolis and buoyancy forces gives a second relation

$$\rho\Omega w \sim (\Delta\rho) g \quad , \tag{32}$$

and so, from (30) and (32), we find

$$\frac{\Delta\rho}{\rho} \sim \left(\frac{\Omega\dot{R}_I}{20g}\right)^{\frac{1}{2}} \quad , \quad w \sim \left(\frac{g\dot{R}_I}{20\Omega}\right)^{\frac{1}{2}} \quad . \tag{33}$$

With the values $\Omega = 7\times 10^{-5}s^{-1}, g \sim 3m/s^2$ (near the ICB), and $\dot{R}_I \sim 10^{-11}$ m/s (on the assumption that the inner core has been growing steadily over at least a substantial fraction of the lifetime of the Earth), these estimates provide

$$\frac{\Delta\rho}{\rho} \sim 3\times 10^{-9} \quad , \quad w \sim 2\times 10^{-4} \quad \text{m/s} \quad . \tag{34}$$

If the Lorentz force $\mathbf{j}\wedge\mathbf{B}$ were absent from (31), then the flow associated with the rising blob would be characterised by a Taylor column parallel to $\mathbf{\Omega}$, and possibly by Ekman layer effects where this column intersects the CMB. However a Lorentz force of magnitude comparable to the other forces eliminates the Taylor column, and allows for a rise of the blob that is uninfluenced by the CMB until it actually arrives there (Loper & Moffatt 1992). The rising blob drifts westward through (in effect) conservation of angular momentum. A random distribution of rising blobs thus induces a mean differential rotation which may in principle be calculated. Moreover, each blob induces helicity in the surrounding flow as it rises, thus generating an α-effect via a relationship of the form (22). Hence we have the basic ingredients for an $\alpha\omega$-dynamo which, if operative, will generate a field dominated by the toroidal ingredient $\mathbf{B}_T$. The Lorentz force impeding the rise of the blob is of order $\sigma w B_T^2$ where σ is the electrical conductivity of the fluid ($\eta = (\mu_0\sigma)^{-1}$) and equating this in order of magnitude to the Coriolis force gives the estimate

$$B_T \sim (2\rho\Omega\mu_0\eta)^{\frac{1}{2}} \sim 20G \quad . \tag{35}$$

A field of this magnitude is sufficient to interfere significantly with the dynamics of an individual blob; a much larger field ($\sim 200G$) is required to interfere with *global* core dynamics.

As a rising blob approaches the CMB, it will generate a local eruption of magnetic field into the lower mantle. Such eruptions have been identified by Bloxham et al (1989), in the CMB magnetic maps discussed earlier.

Finally, it is tempting to speculate on the possible origin of the polarity reversals in relation to this 'blob' model. Note first that compositional convection is different in nature from thermal convection, in that the lighter buoyant fluid, having risen to the CMB, does not fall again, but spreads out forming a stable layer of gradually increasing thickness. As this layer grows, so one may expect the α-effect to be increasingly confined to the lower region. The rate of growth of the layer is such that it will fill the spherical annulus in a time of approximately 4000 years. At this stage, the density of the liquid core is simply reduced to $\rho - \Delta\rho$, and the whole process starts again (recall that $\Delta\rho/\rho \sim 3 \times 10^{-9}$!). 4000 years is somewhat less than the natural decay time of the dipole component of the Earth's field ($\sim 10^4$ years); but a large fluctuation of the α-effect on this time-scale may be expected to induce a correspondingly large fluctuation in the dynamo process. As has often been pointed out, the MHD equations are invariant under change of sign of $\mathbf{B}$, and so occasional reversals of polarity, induced by large amplitude oscillations of the dynamo process, are to be expected. An absence of such reversals, rather than their presence, is what would require explanation!

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