Dynamo Theory

Dynamo theory is that branch of MAGNETOHYDRODYNAMICS which is concerned with the self-excitation of magnetic fields in any large rotating mass of conducting fluid in motion (usually turbulent). Self-exciting dynamo action is believed to account for the very existence of magnetic fields in astrophysical systems, whether at the planetary, stellar, or galactic scale (see GEODYNAMO, DYNAMOS: SOLAR AND STELLAR).

The process of self-excitation occurs when fluid motion across a magnetic field generates the current distribution which is itself the source of the 'given' magnetic field. This principle is most simply illustrated by the example of the 'homopolar' disk dynamo sketched in figure 1: a conducting disk rotates about its axis with angular velocity Ω, and a current path between its rim and its axle is provided by the wire twisted as shown in a loop around the axle. Suppose that a current $I(t)$ flows in the loop, the current circuit being closed through the axle of the disk and the disk itself. This current generates a magnetic flux Φ across the disk, and, provided that the conductivity of the disk is not too high, this flux is given by $\Phi = MI$ where M is the mutual inductance between the loop and the rim of the disk. Rotation of the disk induces an electromotive force $E = \Omega\Phi/2\pi$ which drives the current I; the equation for $I(t)$ is then

$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = \mathcal{E} = \frac{M\Omega I}{2\pi} \qquad (1)$$

where L and R are the self-inductance and resistance of the complete current circuit. The device is evidently unstable to the growth of I (and so of Φ) from an infinitesimal level if

$$\Omega > 2\pi R/M. \qquad (2)$$

In this circumstance, the current grows exponentially, as does the retarding torque associated with the Lorentz force distribution in the disk. Ultimately, the disk angular velocity slows down to the critical level $\Omega_0 = 2\pi R/M$ at which the driving torque G just balances this retarding torque, and the current can remain steady. In this equilibrium, the power supplied $\Omega_0 G$ is equal to the rate of Joule dissipation RI^2 (assuming that frictional torques are negligible).

This simple system exhibits the dual property of any self-exciting dynamo: the current in the system generates the magnetic field (without the assistance of any 'external' source), and motion of a conductor across the field generates the current that maintains it. Note the following properties of the system which re-emerge in the more general fluid context, and which appear to be more or less universal concomitants of dynamo action: (1) the system exhibits 'differential rotation' in that the disk rotates, while the wire is at rest (the gradient of angular velocity is here concentrated at the two sliding contacts); (2) in order for the system to act as a dynamo, the wire must be twisted in the same sense as the sense of angular

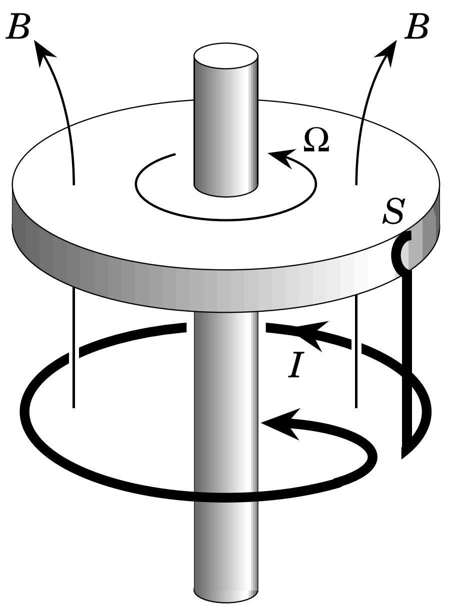

Figure 1. The self-exciting homopolar disk dynamo. Note that the wire is twisted in the same sense as that of the rotation Ω.

velocity of the disk, i.e. there is a definite 'handedness' in the system, a property which, in the fluid context, is generally encountered as a non-zero 'helicity' of the fluid flow (see MAGNETIC HELICITY).

There is a further intriguing property of this simple system, however, which indicates the dangers of too simplistic an argument when dealing with a medium of very high conductivity. If the conductivity of the disk were infinite, then we know that the flux of magnetic field across its rim must remain constant by Alfvén's theorem (magnetohydrodynamics) a result clearly incompatible with exponential growth of flux. What happens in this situation is that an additional current is induced round the rim of the disk which is just such as to maintain the constancy of the trapped flux. This perhaps indicates that, when we move to the much more difficult fluid context, great care will be needed in analysing the high-conductivity limit, which is of particular relevance in stellar and galactic contexts.

The stretch–twist–fold dynamo

Consider an initially circular magnetic flux tube of small cross-section embedded in a highly conducting incompressible fluid. We may easily imagine a doubling of the intensity of the magnetic field by a movement analogous to the doubling of an elastic band: first stretch the tube to double the radius, then twist and fold it in the manner indicated in figure 2. This process evidently doubles the net magnetic flux around the (now composite) tube; if the process is repeated over and over again, then a successive doubling, analogous to exponential growth, occurs. This is what is conventionally known as the stretch–twist–fold dynamo. It relies on the idea that, in

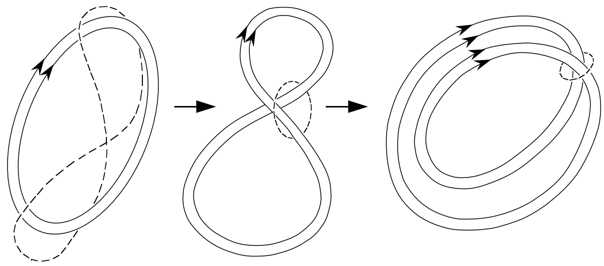

Figure 2. The stretch–twist–fold action applied to a magnetic flux tube; the reciprocal action on a linked material circuit is indicated by the dashed curve.

a perfectly conducting fluid, 'magnetic lines are frozen in the fluid', so that, in a highly conducting fluid, a similar behavior is to be expected. Although the idea is physically plausible, it is difficult to establish the existence of this type of dynamo by rigorous mathematical argument. One difficulty is that the stretch–twist–fold action necessarily introduces a twist within the tube itself (as can be easily seen by applying the twist–fold action to a ribbon in the form of a circle).

If we trace backwards in time the effect of the above distortion on a Lagrangian material circuit round the doubled flux tube (the dashed curve in figure 2), then it is easy to see that this curve must have started as a twisted curve embracing the circular tube twice. Hence the stretch–twist–fold action has the reverse effect on this Lagrangian circuit. An alternative magnetic flux tube, starting in the position of the dashed curve, would thus be diminished by the same distortion. Things are evidently not as simple as they might appear at first sight.

Formal definition of dynamo action

We start from the magnetic induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \wedge (\boldsymbol{u} \wedge \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} \qquad (3)$$

where $\boldsymbol{B}$ is the magnetic field, $\boldsymbol{u}$ the velocity field and η the magnetic diffusivity of the fluid. $\boldsymbol{B}$ also satisfies the supplementary condition $\mathrm{div}\boldsymbol{B} = 0$. If the velocity field $\boldsymbol{u}$ is steady (i.e. $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x})$), then we may look for solutions of equation (3) in the form of 'eigenmodes',

$$\boldsymbol{B}(\boldsymbol{x}, t) = \mathrm{Re}\, \hat{\boldsymbol{B}}(\boldsymbol{x}) e^{pt}. \qquad (4)$$

The amplitude function $\hat{\boldsymbol{B}}(\boldsymbol{x})$ then satisfies the equation

$$p\hat{\boldsymbol{B}} = \boldsymbol{\nabla} \wedge (\boldsymbol{u} \wedge \hat{\boldsymbol{B}}) + \eta\, \nabla^2 \hat{\boldsymbol{B}} \qquad (5)$$

which, when coupled with appropriate boundary conditions, defines an eigenvalue problem for the exponent p, the field $\hat{\boldsymbol{B}}$ being then the corresponding eigenfunction. p may in principle be real or complex: $p = p_r + \mathrm{i} p_i$. If $p_r > 0$ then the corresponding solution of equation (3) exhibits dynamo action: there is evidently a systematic increase in intensity of the magnetic field with growth rate p_r. If $p_i = 0$, the dynamo action is a simple exponential growth, while if p_i is non-zero, then the field at any point oscillates with exponentially growing amplitude.

Solution of the above eigenvalue problem evidently depends on the parameter η which appears in equation (5). Let us suppose that the velocity field is characterized by a length scale l and a velocity scale u_0, so that the magnetic Reynolds number $R_m = u_0 l/\eta$ may be defined. In astrophysical contexts, we are usually concerned with the situation in which R_m is very large, and particular attention must therefore focus on the behavior of p and $\hat{\boldsymbol{B}}$ as R_m tends to infinity. This consideration has led to a formal distinction between 'slow' and 'fast' dynamos. Roughly speaking, the dynamo is fast if p_r is proportional to u_0/l (i.e. independent of η) in the limit $R_m \to \infty$. If, on the other hand, $p_r l/u_0 \to 0$ as $R_m \to \infty$, then the dynamo is slow.

The stretch–twist–fold dynamo considered above is perhaps the best candidate for characterization as a fast dynamo; in a single period of the stretch–twist–fold cycle, it exhibits a doubling of flux, but at the cost of introducing a certain degree of 'fine structure' in the field. Actually, this appears to be a property of any fast dynamo. Equation (5) can be easily analyzed in the formal limit $\eta = 0$. It has been shown that a fast dynamo can exist only if the corresponding field $\hat{\boldsymbol{B}}$ exhibits fine structure on scales of order $R_m^{-1/2}$. In the limit $\eta = 0$, such fields become non-differentiable.

Attempts have been made to circumvent this difficulty by widening the class of solutions of equation (3) to which the term 'fast dynamo' may be ascribed. The reader is referred to the book by Childress and Gilbert (1995) for an account of investigations, based largely on Lagrangian and associated mapping techniques, in this area.

We may note that the homopolar disk dynamo is, for the reasons indicated in the introduction, definitely a slow dynamo, i.e. its growth rate tends to zero as the magnetic Reynolds number, based on disk angular velocity and conductivity, tends to infinity.

Cowling's theorem

In planetary and stellar contexts, it was natural that early investigations should focus on axisymmetric systems. In 1934, COWLING established that axisymmetric dynamo action is impossible, a result which, for many years, falsely led people to assume that all dynamo action was impossible (see Cowling 1957). It is important, nevertheless, to understand the basic nature of Cowling's argument, if only to see how the associated difficulties may be circumvented. Consider an axisymmetric magnetic field generated by currents contained entirely within a sphere (figure 3). The 'toroidal' currents within the sphere (i.e. those in the ϕ direction in a system of cylindrical polar coordinates r, ϕ, z) act as the source of the 'poloidal' part of the magnetic field $\boldsymbol{B}_P$ (i.e. that part with components in the r and z direction). This poloidal field, being associated

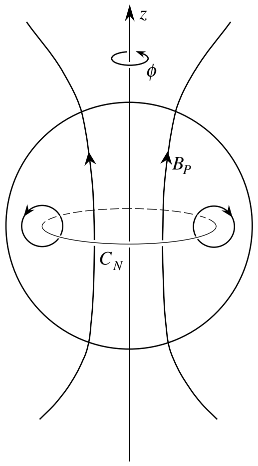

Figure 3. Cowling's theorem: induction cannot maintain current in the neighborhood of the circle C_N if the flow and field are axisymmetric.

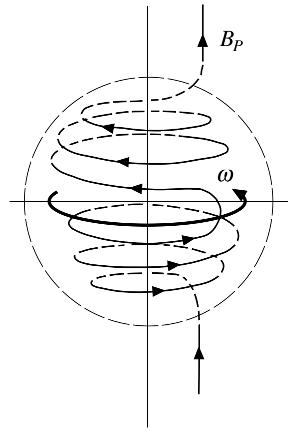

Figure 4. Generation of toroidal field by differential rotation with a variation of angular velocity along each field line.

with a compact source, has at most a dipole structure at a great distance from the sphere. The Earth's magnetic field of course provides an excellent example of this phenomenon. The lines of force of this dipole field must have at least one 'O-type' neutral point, and this neutral point must lie inside the sphere of conducting fluid. There is an associated circle C_N of neutral points around the axis of symmetry. The toroidal current distribution can be driven only by fluid motion across the poloidal magnetic field and this effect is absent on C_N where $\boldsymbol{B}_P = 0$. Consequently any toroidal current on C_N must necessarily decay through normal Ohmic diffusion associated with finite conductivity ($\eta > 0$). Thus, steady maintenance of an axisymmetric magnetic field in a finite volume of conducting fluid is impossible.

This argument of course needs to be formalized, but this can be done, and indeed many extensions of Cowling's theorem have been proved. Essentially, it is well established that, if a steady magnetic field is to be maintained in a finite region containing fluid of finite conductivity, then the field, and the motions that support it, must be non-axisymmetric. This theorem accounts for much of the mathematical difficulty of the subject and for the long period that elapsed following the proof of the theorem before explicit examples of dynamo action in a homogeneous fluid were discovered.

The generation of toroidal field by differential rotation

One mechanism, which is essentially axisymmetric in character, is nevertheless of great importance in the dynamo context. Let us suppose for the moment that a poloidal magnetic field is maintained by some (as yet unspecified) mechanism. Consider the action of a purely toroidal velocity field on this magnetic field. If the angular velocity is constant along every field line, then these field lines are simply rotated around the axis of symmetry with the corresponding angular velocity (the law of 'isorotation'). If, on the other hand, there is any gradient of angular velocity along a field line, then that field line will tend to be drawn out in the toroidal direction, an effect that will be limited only by diffusion effects (figure 4). If the magnetic Reynolds number is large, then the field line may be 'cranked' round the axis many times before this diffusion effect intervenes. Detailed analysis based on equation (3) indicates that the toroidal field that is generated increases until it is $O(R_m)$ times the initial poloidal field. At this stage, in general, Ohmic diffusion establishes a steady state.

This is evidently a powerful mechanism whereby toroidal fields may be generated from a pre-existing poloidal field. The great difficulty is to explain how it is that the poloidal field can be generated in the first place. For this purpose, it is absolutely necessary to consider non-axisymmetric effects. The greatest impetus to this subject has undoubtedly come from the assumption at the outset that the fluid motion is fully turbulent, an assumption that is of course utterly reasonable in stellar and galactic contexts. Even in the case of planetary interiors, although the fluid motions are extremely weak (of the order of fractions of a millimeter per second) it seems highly likely that these motions, while not necessarily fully turbulent, will nevertheless have a significant non-axisymmetric ingredient.

Mean field electrodynamics

The idea behind mean field electrodynamics is that we are generally concerned in dynamo theory with the development of a 'large-scale' magnetic field, i.e. with a field that is coherent over the global scale of the planet or star or galaxy under consideration. The turbulence, or other random motion, which plays a part in the dynamo process in general has a very much smaller characteristic length scale. There is therefore a significant scale separation which can be exploited in the construction of a theory for the large-scale field.

Let us first suppose that, in some region of fluid which contains many constituent eddies of the turbulence, the large-scale ingredient of the field may be treated as nearly uniform. The velocity $\boldsymbol{u}$ distorts the mean field $\boldsymbol{B}_0$, generating a fluctuating component $\boldsymbol{b}$. If we average equation (3) over the scale l of the turbulent fluctuations, then we obtain an equation for the mean field $\boldsymbol{B}_0$, namely

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \boldsymbol{\nabla} \wedge \boldsymbol{\mathcal{E}} + \eta \nabla^2 \boldsymbol{B}_0. \tag{6}$$

Here, a term appears resulting from the interaction between two fluctuating fields, namely

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u} \wedge \boldsymbol{b} \rangle. \tag{7}$$

This is the 'mean electromotive force' associated with the fluid motion. We have assumed that the average of $\boldsymbol{u}$ itself and similarly the average of $\boldsymbol{b}$ are zero. Nevertheless, any correlation between $\boldsymbol{u}$ and $\boldsymbol{b}$ will result in a non-zero value for $\boldsymbol{\mathcal{E}}$.

In order to proceed further, it is necessary to obtain an equation for the fluctuating field $\boldsymbol{b}$. This is obtained by simply subtracting equation (6) from equation (3) to give

$$\frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla} \wedge (\boldsymbol{u} \wedge \boldsymbol{B}_0) + \boldsymbol{\nabla} \wedge (\boldsymbol{u} \wedge \boldsymbol{b} - \langle \boldsymbol{u} \wedge \boldsymbol{b} \rangle) + \eta \nabla^2 \boldsymbol{b}. \tag{8}$$

This equation evidently establishes a linear relationship between $\boldsymbol{b}$ and $\boldsymbol{B}_0$, and hence, via equation (7), between $\boldsymbol{\mathcal{E}}$ and $\boldsymbol{B}_0$. Since $\boldsymbol{B}_0$ is nearly uniform, this linear relationship may be expressed in the form

$$\mathcal{E}_i = \alpha_{ij} B_{0j} + \beta_{ijk} \frac{\partial B_{0j}}{\partial x_k} + \cdots \tag{9}$$

where the coefficients α_{ij}, β_{ijk}, and so on, do not depend on the field $\boldsymbol{B}_0$ but are determined (in principle) solely by the statistical properties of the turbulence $\boldsymbol{u}$ and the magnetic diffusivity η.

The implications of equation (9) are most easily understood if it is assumed that the background turbulence is isotropic (i.e. there is no 'preferred direction'), so that the tensor coefficients α_{ij}, β_{ijk}, etc. are also isotropic. In this case, we may write

$$\alpha_{ij} = \alpha \delta_{ij} \qquad \beta_{ijk} = \beta \epsilon_{ijk} \tag{10}$$

and the corresponding expression for $\boldsymbol{\mathcal{E}}$ is then given by

$$\mathcal{E} = \alpha \boldsymbol{B}_0 - \beta\, \boldsymbol{\nabla} \wedge \boldsymbol{B}_0 + \ldots. \tag{11}$$

The corresponding form of equation (6) then becomes

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \alpha \nabla \wedge \boldsymbol{B}_0 + (\eta + \beta) \nabla^2 \boldsymbol{B}_0. \tag{12}$$

There is here a very important effect to notice, an effect first described in a seminal paper by Steenbeck, Krause and Rädler in 1966 (see Krause and Rädler 1980). Since $\boldsymbol{\mathcal{E}}$ is a polar vector, whereas the magnetic field $\boldsymbol{B}_0$ is an axial vector, the coefficient α in equation (11) must be a pseudo-scalar, i.e. a quantity which changes sign under change from a right-handed to a left-handed frame of reference. By contrast, the coefficient β is a pure scalar, independent of frame of reference. It is evident from equation (12) that β simply serves to augment the molecular diffusivity of the fluid, i.e. we can describe β as a turbulent diffusivity. By contrast, the coefficient α is associated with a term of novel structure in the induction equation; moreover, for a field of very large length scale, it is this term that dominates the right-hand side. The effect, appropriately described as the 'α effect', is crucial to an understanding of dynamo theory.

Dynamo instability associated with the α effect

To understand this effect in the simplest possible way, it is sufficient to consider a field structure $\boldsymbol{B}_0(x)$ satisfying the Beltrami condition

$$\boldsymbol{\nabla} \wedge \boldsymbol{B}_0 = K \boldsymbol{B}_0 \tag{13}$$

where K is a constant (positive or negative). The ABC field

$$\boldsymbol{B}_0 = \begin{pmatrix} B \cos Ky + C \sin Kz \\ C \cos Kz + A \sin Kx \\ A \cos Kx + B \sin Ky \end{pmatrix} \tag{14}$$

provides an example of such a field, as may be easily verified, but there are many others (see FORCE-FREE MAGNETIC FIELDS). Since, for such a field, $\nabla^2 \boldsymbol{B}_0 = -K^2 \boldsymbol{B}_0$, equation (11) simplifies to the form

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \alpha K \boldsymbol{B}_0 - (\eta + \beta) K^2 \boldsymbol{B}_0. \tag{15}$$

Hence $\boldsymbol{B}_0$ is proportional to e^{pt}, where

$$p = \alpha K - (\eta + \beta) K^2 \tag{16}$$

and dynamo instability occurs if $p > 0$, a condition that is always satisfied if $\alpha K > 0$ and $|K|$ is sufficiently small, i.e. provided that the scale $L = 2\pi/|K|$ is sufficiently large. This condition is certainly consistent with the initial assumption that the scale of the mean field be large compared with the scale of the turbulence. Thus, in the presence of a non-zero α effect, the medium is always unstable to the growth of magnetic field perturbations on a sufficiently large length scale.

This effect is one of the most remarkable discoveries of the last 50 years. It indicates how order, represented by the large-scale magnetic field, may emerge from chaos, represented by the small-scale turbulence in the medium.

Evaluation of α

It remains, however, to develop a convincing theory for the determination of the all-important parameter α in terms of the statistical properties of the background turbulence. We have already noted that α is a pseudo-scalar; it can therefore be non-zero only if the statistical properties of the turbulence have a crucial property of 'lack of reflectional symmetry'. The simplest measure of a lack of reflectional symmetry in a turbulent flow is its mean helicity

$$\mathcal{H} = \langle \boldsymbol{u} \cdot \operatorname{curl} \boldsymbol{u} \rangle \tag{17}$$

the mean of the scalar product of the velocity field and the vorticity field curl $\boldsymbol{u}$. This is analogous to magnetic helicity. The helicity is related to the topological structure of the vorticity field, which is conserved in the classical situation in which vortex lines are frozen in the fluid. It may be expected that, if this mean helicity is non-zero, then α will in general be non-zero also.

It is indeed possible to calculate α explicitly in any circumstances in which the fluctuating ingredient of the magnetic field $\boldsymbol{b}$ is weak compared with the mean field. In such circumstances, it is found that

$$\alpha = -\frac{1}{3}\eta \int\int \frac{k^2 H(k,\omega)}{\omega^2 + \eta^2 k^4}\, \mathrm{d}k\, \mathrm{d}\omega \tag{18}$$

where $H(k, \omega)$ is the helicity spectrum of the turbulence, satisfying the defining relationship

$$\langle \boldsymbol{u} \cdot \operatorname{curl} \boldsymbol{u} \rangle = \int\int H(k,\omega)\, \mathrm{d}k\, \mathrm{d}\omega. \tag{19}$$

Thus, α is linearly related to the helicity spectrum of the turbulence. In these integral expressions, k is a wavenumber magnitude and ω a frequency. Note that, in the limit as $\eta \to 0$, the expression (18) for α depends critically on the behavior of the helicity spectrum for small values of frequency ω. Unfortunately, it is in this double limit ($\eta \to 0$, $\omega \to 0$) that the theory yielding the expression (18) is at its least convincing.

Physical mechanism of the dynamo process

The dynamo process associated with the α effect may be represented schematically as in figure 5. Background turbulence with helicity provides an α effect, the dynamo parameter being related to the helicity through equation (18) (or some higher-order version of this equation). This α effect generates an electromotive force (EMF) $\boldsymbol{\mathcal{E}}$ parallel to $\boldsymbol{B}_0$ (through the equation $\boldsymbol{\mathcal{E}} = \alpha \boldsymbol{B}_0 + \cdots$) and this EMF drives a mean current $\boldsymbol{J}_0$ parallel to $\boldsymbol{B}_0$. The resulting mean field has non-zero magnetic helicity, and so the tubes of force of $\boldsymbol{B}_0$ are generally linked. The prototype linkage is shown in figure 5; the current driven round each tube provides (via Ampère's theorem) the mean magnetic field in the other tube.

Three vital conditions ensure the success of this process:

- the turbulence must have non-zero helicity;
- the fluid must have finite non-zero magnetic diffusivity;
- the fluid domain must be large enough to provide space for the growth of the large-scale magnetic field.

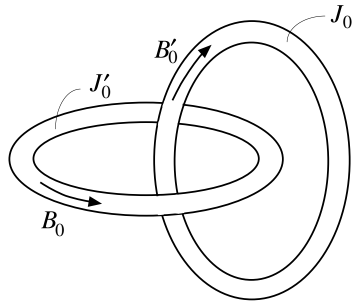

Figure 5. The dynamo cycle associated with the α effect. The current J_0 is associated with the field B_0, and the current J_0' with B_0'

The α–ω dynamo mechanism

As we have seen above, the α effect in isolation can easily give rise to dynamo instability; we have shown this to be true in a medium of infinite extent; however, a similar mechanism certainly occurs in any sufficiently large spherical domain of conducting fluid in turbulent motion.

If the fluid in such a domain is rotating with mean angular velocity Ω, then turbulent convection within the domain in general leads to a state of 'differential rotation' in which the inner regions rotate more rapidly than the outer regions. As we have seen, differential rotation provides a powerful mechanism whereby toroidal field can be generated from poloidal field. The α effect provides the vital complementary mechanism whereby poloidal field can be regenerated from toroidal field. If the turbulence is isotropic, then the same α effect contributes to the regeneration of toroidal field from poloidal field, but the differential rotation effect is generally much more powerful.

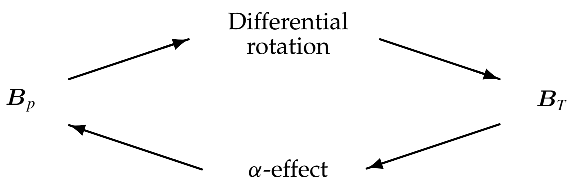

Figure 6. Field regeneration through the α–ω dynamo cycle.

These considerations lead to consideration of dynamos of 'α–ω type' as indicated schematically in figure 6. Here, the dynamo cycle consists simply of two stages: generation of toroidal from poloidal field by differential rotation and regeneration of poloidal from toroidal field by the α effect. Many numerical investigations of the resulting mean field dynamo equations have demonstrated the efficiency of this type of self-exciting dynamo.

Back-reaction of Lorentz forces

The above outline description of the process of dynamo action is purely 'kinematic', in that it assumes that the velocity field, or in the case of turbulent flow the statistics of the velocity field, is prescribed independently of the magnetic field. In circumstances in which dynamo action occurs, the amplitude of the magnetic field grows exponentially in time, and so therefore does the Lorentz force, which is quadratically related to the magnetic field. This Lorentz force reacts back upon the fluid motion so that the velocity field itself depends on the magnetic field that it generates. The dynamo process then enters a nonlinear regime, whose complexities can generally be unravelled only through numerical simulation of the full magnetohydrodynamic equations.

It is also necessary to specify the nature of the forces that sustain the motion, and the associated dynamo, against dissipative effects which now include viscosity as well as Joule dissipation. This is equivalent to specifying the source of energy of the fluid motion; this source may be of thermal origin (as in the solar context); in the context of the terrestrial dynamo driven by fluid motion in the liquid core the source of energy is more likely to be associated with segregation of light material during the process of solidification of the inner core, a process that provides buoyancy forces of 'compositional' rather than thermal origin. Either way, the need to include consideration of the buoyancy field adds significantly to both the physical and the mathematical complexity of the problem.

The study of fully non-linear dynamos, taking account of the diverse possible sources of energy, is an area of very vigorous current research, and great advances are to be expected over the next 10–20 yr.

Bibliography

Childress S and Gilbert A D 1995 *Stretch, Twist, Fold: the Fast Dynamo* (Berlin: Springer)

Cowling T G 1957 *Magnetohydrodynamics* (New York: Interscience)

Krause F and Rädler K-H 1980 *Mean Field Magnetohydrodynamics and Dynamo Theory* (Oxford: Pergamon)

Moffatt H K 1978 *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge: Cambridge University Press)

Parker E N 1979 *Cosmical Magnetic Fields* (Oxford: Clarendon)

Zeldovich Ya B, Ruzmaikin A A and Sokoloff D D 1983 Magnetic Fields in Astrophysics (London: Gordon and Breach)

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