A Lifelong Affair with Fluid Dynamics

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I always enjoyed mathematical puzzles as a child, stimulated by my father who loved them also. One that I remember from the age of about 12 was this: obviously $8+8+8=24$ and $(9\times\sqrt{9})-\sqrt{9}=24$; the problem is to make up the number 24 in a similar way using each of the integers $1,2,\ldots,9$, in turn exactly three times (no more, no less) and the standard symbols of arithmetic. Try it! Some are harder than others, but all are possible.

I enjoyed maths so much at school that there was no question about wanting to continue with the subject at University also; but I was attracted to physics also, and eventually settled at Edinburgh University in that no-man's-land of Applied Mathematics, which gave equal scope for both. I went on to Trinity College, Cambridge, and after a year in the mysteries of quantum mechanics, discovered that my real spiritual home lay in the classical field of fluid dynamics, a subject of more straightforward mathematical challenge, of immediate and wide-ranging practical applications, and with (for me) the great attraction that you can visualise the flow of a fluid, whether through waves on the seashore or a twig in a turbulent stream or a feather blown in the wind, a luxury that is not similarly available in the quantum realm.

I was fortunate to be supervised in my early research by George Batchelor, one of the great pioneers in the statistical theory of turbulent flow. He taught me the essential art of self-criticism in research—the need to be as critical of one's own ideas and arguments as of any others. I slowly developed what we know in our subject as 'physical intuition', i.e. an instinctive sense of how a complex fluid system will behave in given circumstances; this intuition provides an essential guide to the choice and refinement of appropriate mathematical models which can be subjected to rigorous analysis. The ultimate test lies in the confrontation of theory and experiment, and this usually leads to corresponding refinement of one's physical intuition!

One of the greatest practitioners of theoretical and experimental mechanics (both solid and fluid) of the twentieth century was Sir Geoffrey Taylor, Fellow of Trinity College, best known perhaps for his discovery in 1923 of 'Taylor vortices' in the flow between coaxial rotating cylinders. G.I. (as he was affectionately known to all who knew him) was still very active in research throughout the 1960's, and he provided inspiration to the vigorous young fluid dynamics group in Cambridge. His physical intuition was unsurpassed, and he had the marvellous skill of using mathematics at just the appropriate level for whatever phenomenon he happened to be investigating. I learnt from him that there is no merit in mathematical complexity for its own sake; the true art is in perceiving the simplicity of natural phenomena, and this can often be captured through relatively simple mathematics. But to recognise the key physical effects and to extract the relevant mathematics from the governing principles and equations of fluid dynamics: Ah, therein lies the problem!

My Ph.D. thesis in 1962 was on the subject "Magnetohydrodynamic Turbulence", i.e. turbulence in electrically conducting fluids (ionised gases at one extreme, liquid metals at the other), a subject with important applications in astrophysics, geophysics, thermonuclear fusion physics and engineering. Such fluids are generally permeated by a magnetic field whose lines of force are almost 'frozen' in the fluid, in the sense that they are transported by the fluid like elastic threads, a process that one may easily visualise in the 'mind's eye'. At the same time, they diffuse relative to the fluid, so that lines of force may in effect break and reconnect—their topology may change. These ideas are now very familiar, but they were not so in 1960, and their implications were by no means well understood. The challenges were great, and I was lucky to become involved in this exciting field of interaction between fluid dynamics and electromagnetism at this early stage of its development. I have returned to it regularly ever since.

From the beginning, I was heavily involved in teaching for the mathematics tripos in Cambridge, and this teaching has had an important bearing on my research. It was through posing an examination problem in 1962 that I was led to consider the flow of a viscous fluid in a corner bounded by two rigid planes. This led to the very surprising and counter-intuitive conclusion that an infinite sequence of eddies is in general present in such a corner flow. I published a paper on this subject in 1964; fifteen years later, the experimental evidence for the eddies was provided in Japan by the beautiful flow-visualisation experiments of Taneda (1979). There can be little more satisfying than having a purely theoretical prediction verified in this way after such passage of time.

Although not an experimentalist, I enjoy experimenting in an amateurish kitchen-sink sort of way. My first such published experiment (1977) concerned the complex dynamics of a thick film of viscous liquid (e.g. golden syrup) spread uniformly on the surface of a horizontal cylinder in rotation about its axis. This problem first came to my attention through a lecture given by V. Pukhnachev at an ICTP Summer School of Fluid Dynamics in 1974 (a school that contributed to the modern acceptance of fluid dynamics as a legitimate and reputable branch of theoretical physics!). There is a wonderful surprise awaiting anyone who tries this experiment: the fluid organises itself in a sequence of rings more or less equally spaced along the cylinder; each ring has a single depth discontinuity which rotates with the cylinder, but more slowly. The experiment encapsulates many aspects of viscous flow which are central to fluid dynamics—existence of steady flows, stability of these flows, steepening of waves, formation of discontinuities, and the associated formation of free-surface cusps, a phenomenon of great current interest through its relevance to the mixing (at the microscopic level) of fluids like oil and vinegar. Think of this the next time you prepare a salad dressing, or the next time you stir honey into yogurt; having visualised the mixing process, the product will taste all the better!