The Birth and Adolescence of MHD Turbulence

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Summary. This essay provides a personal account of the development of the subject of magnetohydrodynamic (MHD) turbulence from its birth in 1950 to its "coming-of-age" in 1971, following the development of mean-field electrodynamics, a major breakthrough of the 1960s. The discussion covers the early ideas based on the analogy with vorticity, the passive vector problem, the suppression of turbulence by an applied magnetic field, and aspects of the turbulent dynamo problem.

1 Birth pangs of the 1950s

The conception of the state of magnetohydrodynamic (MHD) turbulence dates from G.K. Batchelor's seminal paper "On the spontaneous magnetic field in a conducting liquid in turbulent motion", published in the Proceedings of the Royal Society in 1950 [1]. At that time, it was already recognised that, just as vortex lines are, under ideal circumstances, frozen in the fluid (i.e., transported with conservation of flux), so the magnetic lines of force are similarly *frozen* in a conducting fluid (again with conservation of flux) in the ideal perfect-conductivity limit. The evolution equations for magnetic field $\mathbf{B}$ in a perfectly conducting fluid, and for ω in an ideal fluid (with no magnetic effects), are then superficially identical. The word *superficially* is here deliberate, the analogy between $\mathbf{B}$ and ω being imperfect, in that ω is constrained, by its very definition, to be equal to the curl of the velocity field $\mathbf{u}$ that transports it, whereas $\mathbf{B}$ of course suffers no such constraint. There is thus far greater freedom in the choice of initial conditions for the pair of fields $(\mathbf{u}, \mathbf{B})$ than for the pair $(\mathbf{u}, \omega)$. This imperfection was, I believe, recognised by Batchelor, but dismissed as irrelevant; what was important for him was the physical fact that the velocity $\mathbf{u}$ transports both ω and $\mathbf{B}$ in a similar way, and that the statistics of these fields may therefore be expected to evolve in a correspondingly similar way, perhaps after the decay of transients associated with initial conditions. In this regard, as would emerge much later, Batchelor was at best only partially correct.

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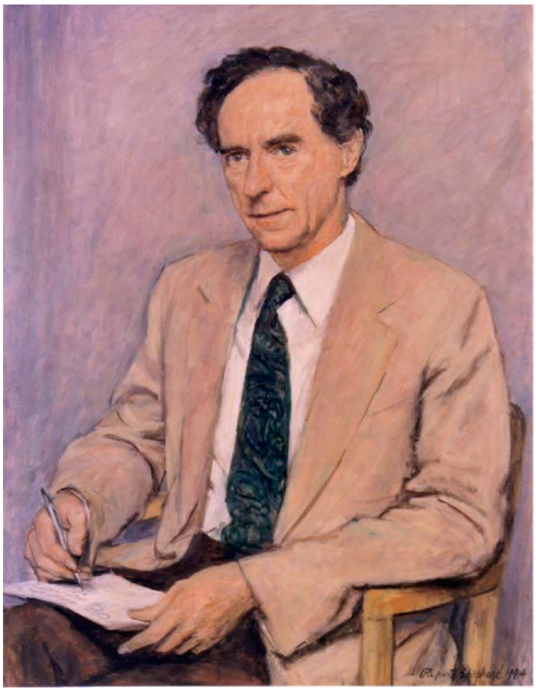

Fig. 1. Portrait of G.K. Batchelor FRS by artist Rupert Shepherd (1984). The portrait hangs in DAMTP, Cambridge

Of course both $\boldsymbol{\omega}$ and $\mathbf{B}$ are subject to diffusive effects, viscous in the case of $\boldsymbol{\omega}$, with diffusivity the kinematic viscosity ν, and resistive in the case of $\mathbf{B}$, with diffusivity the magnetic resisitivity η. However, if $\eta = \nu$, Batchelor's analogy (albeit imperfect for the above reason) persists, and it was this analogy that Batchelor sought to exploit in the context of turbulence. Here, the recognised and very contemporary scenario was that in homogeneous turbulence in an incompressible fluid, the rate of viscous dissipation of mean square vorticity (or *enstrophy* as it is now called) always adjusts itself to be in approximate equilibrium with its rate of production by the all-pervasive process of stretching of vortex lines. If ν were to suddenly decrease (through the agency of some Maxwell demon), then the enstrophy would *increase*, but at the same time, the characteristic length scale of the vorticity field would *decrease* till a new equilibrium at a higher enstrophy level is established. Batchelor argued that, if $\eta < \nu$, the magnetic energy (the analogue of enstrophy) will similarly *increase* through stretching of magnetic lines of force, and he inferred an exponential increase of this energy on the Kolmogorov timescale $(\nu/\epsilon)^{1/2}$ characteristic of the small scales of the turbulence where the enstrophy spectrum is maximal. Batchelor's condition $\eta < \nu$ is satisfied in the interstellar medium where the density is low and the kinematic viscosity ν is correspondingly large.

Batchelor further argued that, when this condition is satisfied and dynamo action occurs, then, as a result of the back-reaction on the turbulence of the growing Lorentz force distribution, the mean magnetic energy density will saturate at a level of order $(\nu\epsilon)^{1/2}$, this being the energy density characteristic of the smallest scales of motion. It was arguable however that, even if saturation is quickly achieved on the Kolmogorov scale, magnetic modes could continue to grow through the familiar stretching mechanism on larger scales l (where the characteristic velocity is $u \sim (\epsilon l)^{1/3}$) provided simply that the local magnetic Reynolds number $R_{\mathrm{m}} = ul/\nu$ is larger than some critical value of order unity. Saturation would then be progressively established at all scales satisfying this criterion, at a level of magnetic energy (*equipartition*) equal to the local (in scale) kinetic energy of the turbulence. This was the alternative turbulent dynamo scenario proposed also in 1950 by Schlüter and Biermann [2], a scenario that was more readily accepted by the astrophysical community. It was a view further developed in the review article of Syrovatsky 1957 [3], which revealed for the first time the high current level of interest and activity in MHD in the former Soviet Union. The two standpoints were considered in Cowling's influential monograph *Magnetohydrodynamics* [4], published in 1957, who however concluded his penetrating discussion with the statement *"These remarks serve to illustrate how unsatisfactory is the present state of the theory of magnetohydrodynamic turbulence. ... Work decisively distinguishing between these standpoints is still to be awaited"*. Within the fluid mechanics community, Batchelor's theory, based on the above *analogy with vorticity*, undoubtedly retained its appeal, but there seemed little prospect of providing convincing proof of its validity by theoretical argument. Nor of course was there at that time any prospect of either numerical simulation or laboratory experiment that could even remotely approach the range of parameters where (on either theory) turbulent dynamo action might be anticipated. Only quite recently (see, e.g., [5]) are numerical simulations at sufficiently high R_{m} becoming possible.

I was fortunate to start my own research in this field, under Batchelor's supervision and guidance, in 1958. Batchelor had just completed his study of the *passive scalar problem*, and he gave me an advance copy of two famous papers on this topic [6, 7] published in JFM one year later. I had been much influenced by Cowling's monograph, and also by lectures on *Cosmical Electrodynamics* given that year by Leon Mestel in Part III of the Cambridge Mathematical Tripos. It seemed to me that the techniques that Batchelor had used for the passive scalar problem might be adapted to the *passive vector problem*, which is of course just the *kinematic dynamo problem* as we now understand it. Batchelor had originally suggested that I work on the problem of the effect of turbulence on the rate of evaporation of droplets in a turbulent airflow; but he readily agreed to this change of focus. Thus, it was that in 1959 I started to think about the detailed nature of the back-reaction of Lorentz forces under the condition $\eta \ll \nu$, when Batchelor's criterion for dynamo growth of magnetic energy is well satisfied. This was to provide the core of my Ph.D. thesis *Magnetohydrodynamic Turbulence* (1962); my paper on this topic appeared 1 year later [8].

2 Marseille 1961: a definitive moment

An important Colloquium *Mécanique de la Turbulence*, sponsored by CNRS, was held in August 1961 on the occasion of the inauguration of the Institut de Mécanique Statistique de la Turbulence in Marseille. The meeting was distinguished by the presence of the great pioneers of the subject, G.I. Taylor, Th. Von Karman, and A.N. Kolmogorov himself. It was the occasion when the first reliable observational evidence in support of the Kolmogorov $k^{-5/3}$ spectrum was first presented by R.W. Stewart (later published by Grant et al. (1962) [9]), only to be followed by Kolmogorov's remarkable contribution *Précisions sur la structure locale de la turbulence dans un fluide visqueux aux nombres de Reynolds élevés* [10] in which he addressed the problem of the intermittency of the local rate of dissipation $\epsilon(\mathbf{x}, t)$, and showed how this could be expected to modify the (-5/3) exponent of the energy spectrum; thus did Kolmogorov undermine the very foundations of the study of turbulence that he had himself laid 20 years previously; it was indeed a revolutionary moment for the subject!

The Colloquium included a section chaired by L.S.G. Kovasznay on *Turbulence in Compressible and Electrically Conductive Media*, to which I was privileged to contribute. It is perhaps an indication of the primitive state of the subject that, apart from myself, only Kovasznay spoke on the subject of turbulence in conducting fluids. He drew attention to the evidence for the presence of turbulence in plasma experiments, and of the need to take account of terms analogous to Reynolds stress in the time-averaged equations of MHD, namely $\langle \mathbf{u} \times \mathbf{B} \rangle$ in the mean induction equation, and $\langle \mathbf{j} \times \mathbf{B} \rangle$ in the mean momentum equation; this seems so absolutely natural now that it is difficult to appreciate how novel, and indeed daring, such a suggestion still appeared at that time. Kovasznay [11] had been primarily concerned with situations typical of plasma experiments in which the source of energy is electromagnetic, and energy flows via MHD instabilities of various kinds to the turbulence with resulting enhancement of the rate of Joule dissipation of energy. In this respect, his approach was complementary to that of the dynamo theoreticians, who were concerned with circumstances when the source of energy was purely dynamic, and the flow of energy was from the resulting turbulence to the magnetic field. I did my best in my contribution [12] to distinguish the main features of these contrasting situations.

3 The low-R_m situation

It was recognised by Golitsyn (1960) [13] that, when R_m is small, an applied magnetic field is weakly perturbed by turbulence, and the induction equation may therefore be linearised in order to obtain the fluctuating field $\mathbf{b}$ in terms of the velocity field $\mathbf{u}$. The spectrum $\Gamma(k)$ of $\mathbf{b}$ may then be obtained in terms of the spectrum of $\mathbf{u}$, with the result that $\Gamma(k) \sim k^{-11/3}$ (a result that may be compared with the corresponding $k^{-17/3}$ law obtained by Batchelor et al. [7] for the passive scalar case).

On the assumption that Batchelor's criterion $\eta < \nu$ for dynamo action was correct, I considered at about the same time [14] the situation of moderate conductivity, when η is large compared with ν but η still small enough that $R_\mathrm{m} \gg 1$; i.e., when the Reynolds number Re of the turbulence satisfies $Re \gg R_\mathrm{m} \gg 1$. I argued the case for a $k^{1/3}$ spectrum (like that of vorticity) up to a *conduction cut-off*, $k_c = (\epsilon/\eta^3)^{1/4}$, and a $k^{-11/3}$ spectrum, like Golitsyn's result (and for similar reasons) above k_c. The $k^{-11/3}$ result has been found by Odier et al. (1998) [15] in experiments on turbulence in liquid gallium, a welcome and long-awaited validation of ideas that were both rudimentary and tentative in those early days of the subject.

4 The high-R_m situation

The situation when $\eta \ll \nu$ is very different. Here, magnetic fluctuations persist on sub-Kolmogorov scales where the velocity gradient may reasonably be assumed to be approximately uniform. Batchelor [6] had exploited this idea to determine a k^{-1} law for the spectrum of a passive scalar. The same arguments applied to magnetic field (treated as a passive vector) led to an unacceptably divergent k^{+1} spectrum, possibly a symptom of dynamo instability. In fact, it had been shown at about the same time by Pearson (1959) [16] that if a weak random vorticity field is subjected to uniform irrotational strain, then the associated enstrophy in general increases exponentially, and this in spite of the effect of viscosity. This result carried over by analogy to the effect of a similar uniform straining motion on a random magnetic field: the mean magnetic energy increases exponentially, despite the effect of Joule dissipation. This surprising result is perhaps attributable to the unphysical assumption of a strain field that is uniform to infinity, but nevertheless it suggested that stretching of field lines could persist until the growing Lorentz force reacted back upon this strain field, in a way that might lead to structures in which the straining process was exactly compensated by this back-reaction. I did indeed find such structures [8], although it seemed inevitable that finite diffusivity would lead to some leakage of magnetic flux, causing slow decay. It was argued at about the same time by Saffman (1963) [17] that the decrease of scale associated with the stretching process during the kinematic phase would ultimately lead to decay of magnetic energy; this prediction has not been substantiated by later developments.

There has been a recent renewal of interest in such “small-scale dynamo action” stimulated by the work of Kulsrud (1999) [18], with reference to processes in the interstellar medium (see also Schekochihin et al. (2004) [19] and the extensive bibliography therein). The availability of high-speed computer power opens up new possibilities for the investigation of this regime. Together with Y. Hattori, I have recently returned to the study of isolated “magnetic eddies” in the perfect conductivity limit [20]. Even without imposed strain, the behaviour of such eddies under the action of the Lorentz force distribution is of interest! It turns out that, in the simplest case, an axisymmetric magnetic eddy can contract towards the axis of symmetry and split into two nearly spherical eddies which propagate away from each other along the axis of symmetry. These are candidate “coherent structures” of MHD turbulence in the high conductivity limit.

5 Suppression of turbulence by a strong applied field

My first research student at Cambridge in the 1960s was Jacques Nihoul from Liège. One of the problems that he worked on was the effect of a suddenly applied magnetic field on a field of homogeneous turbulence at low R_{m}. The fact that a magnetic field could suppress turbulence had been demonstrated experimentally by Murgatroyd (1953) [21]; and it was already recognised from the work of Lehnert (1955) [22] and Shercliff (1965) [23] that vorticity components perpendicular to a uniform applied field tend to be preferentially suppressed; this process is effectively linear, and Nihoul (1965) [24] found that the turbulent energy decays as t^{-3} during this suppression phase. I carried this work somewhat further [25] and showed the manner in which anisotropy develops from an initially isotropic state: in fact, the anisotropy ratio $\langle u^2+v^2\rangle/\langle w^2\rangle$ decreases from the isotropic value of 2 asymptotically to 1 during this phase, where u, v are the velocity components perpendicular to the field, and w is the component parallel to the field. This result holds only insofar as the fluid can be regarded as unbounded; much work has since been done on the nonlinear effects which resist the anisotropisation process (Sommeria and Moreau (1982) [26]) and on the effect of fluid boundaries perpendicular to the applied field (Pothérat et al. (2000) [27]); but the fact that a strong field induces a state of ‘nearly two-dimensional’ turbulence having very weak variation parallel to the field seems to be reasonably well established.

6 Helicity and the α-effect

The great breakthrough in dynamo theory came in the mid-1960s, with the work of Steenbeck et al. (1966) [28] and their subsequent series of papers, work that only became widely known some years later when distributed in English translation by Roberts and Stix (1972) [29], and again much later with the publication of the book of Krause and Rädler (1980) [30]. The key idea of the theory lay in recognition of the fact that, within the framework of the kinematic dynamo problem for which the statistics of the velocity field are regarded as "given", the mean electromotive force (or *emf*) $\langle \mathbf{E} \rangle = \langle \mathbf{u} \times \mathbf{b} \rangle$ is linearly related to the mean magnetic field $\langle \mathbf{B} \rangle$, assumed to vary on a length-scale large compared with that of the turbulence. At leading order in the ratio of these scales, and assuming isotropic turbulence, this gives the famous relationship $\langle \mathbf{E} \rangle = \alpha \langle \mathbf{B} \rangle$. This astonishing result, the appearance of a mean emf parallel, rather than perpendicular, to the mean field, was somewhat arbitrarily described by Steenbeck et al. [28] as the *α-effect*, a description that is now firmly established. It is an effect that appears only when the turbulence *lacks reflexional symmetry* (such turbulence may be described as *chiral*), and, as shown by Steenbeck et al., it is responsible for the exponential growth of the mean field in a variety of planetary and stellar circumstances.

This discovery completely superseded, and rendered almost irrelevant, the previous divergence between the points of view that had been advocated by Batchelor [1] and Biermann and Schlüter [2], as described above. The focus from this point on was to be on length-scales large (rather than small) compared with the scale of the energy-containing eddies of the turbulence; this was entirely appropriate as far as the problem of explaining the observed existence of stellar and planetary fields was concerned.

One of my research students in the late 1960s was Glyn Roberts, who worked on dynamo action associated with space-periodic velocity fields; this work, contained in his 1969 Ph.D. thesis, was published 1 year later [31]. (Roberts's subsequent paper [32] developed this theme further, and provided the basis for the Karlsruhe experiment of Müller et al. [33] which, some 30 years later, was to be one of the first experiments successfully demonstrating dynamo action in a fluid.) I had difficulty initially in understanding Glyn's arguments, and I finally succeeded only through carrying out a parallel treatment for homogeneous turbulence [34]; at this stage, it became clear that turbulent dynamo action could occur even when the magnetic Reynolds number $R_{\mathrm{m}}(l)$ based on the scale l of the turbulence is *small* compared with unity; the sole requirement was indeed that the turbulence should lack reflexional symmetry; a magnetic field similarly lacking reflexional symmetry would then grow on scales L large enough for the associated magnetic Reynolds number $R_{\mathrm{m}}(L)$ to be of order unity or greater. Batchelor's theory [1] therefore turned out to be wrong for the reason that it failed to take account of large-scale modes that are available to the magnetic field but not to the vorticity field. Its relevance to reflexionally symmetric turbulence is still however a matter for debate.

By sheer coincidence, I had just 1 year earlier in 1969 [35] published a paper concerning *the degree of knottedness of tangled vortex lines* and relating this to a new invariant, which I named the *helicity*, of Euler flows. (This was the analogue of Woltjer's (1958) [36] invariant for ideal MHD, which henceforth became known as *magnetic helicity.*) A physical interpretation of the *cross-helicity* $\langle \mathbf{u} \cdot \mathbf{b} \rangle$ as the *degree of mutual linkage* of vorticity and magnetic fields emerged at the same time. I learnt some years later that the helicity invariant had been previously discovered by J.-J. Moreau (1961) [37]; but I think it is fair to claim that it was my 1969 paper that firmly established the bridge between topology and the dynamics of ideal fluids which has since proved so fruitful. In any event, non-zero helicity provided the simplest symptom and measure of the required lack of reflexional symmetry, and has played a central role in the understanding of the dynamo process ever since.

The mean-field electrodynamics of Steenbeck et al. had a precursor in the work of Parker (1955) [38], who had considered the effect of what he described as *random cyclonic events* (i.e., helical upwellings) acting on a locally uniform magnetic field $\mathbf{B}$. The aggregate effect of these upwellings was to provide a mean current parallel to $\mathbf{B}$. The role of magnetic diffusivity in this process remained obscure however, and the theory, as presented by Parker, though physically appealing, lacked the mathematical foundation that followed only 10+ years later. A second precursor lay in the work of Braginskii (1964) [39], who had developed a theory of *nearly axisymmetric dynamo action*; here again, an α-effect, whose origin was to be greatly clarified later by Soward (1972) [40], was the main outcome of the theory; but the sheer complexity of Braginskii's treatment meant that for many years it had less impact on the MHD community than it undoubtedly deserved.

With mean-field electrodynamics, turbulent dynamo theory had come of age. The two-scale technique opened the way to dynamic, as opposed to purely kinematic, models of dynamo action, particularly in circumstances where Coriolis, as well as Lorentz, forces played a dominant role [41]. The next decade was to be a period of consolidation, and of reaping the fruits of the great advances of the 1960s, the 'teenage years' of the subject. This in turn would lead into the modern era when high-powered numerical simulation would play a role of ever-increasing importance. The situation as it appeared to me in 1978 may be found in my research monograph *Magnetic Field Generation in Electrically Conducting Fluids* [42], which treated dynamo theory in both its kinematic and dynamic aspects, as then understood.

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