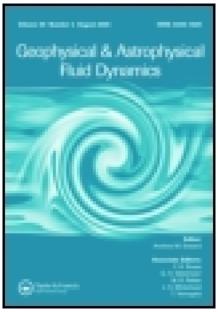

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# The Role of the Helicity Spectrum Function in Turbulent Dynamo Theory

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It is shown that, for general homogeneous turbulence, the anti-symmetric part of the spectrum tensor can be expressed in terms of a single scalar function $H(\mathbf{k},\omega)$ (the helicity spectrum function). Under the first-order smoothing approximation, the coefficients $\alpha_{ij}$, $\beta_{ijk}$ in the expansion of the mean electromotive force in terms of the mean magnetic field are determined; $\alpha_{ij}$ is a weighted integral of $H(\mathbf{k},\omega)$, and $\beta_{ijk}$ contains a part $\beta^{(a)}_{ijk}$ which is likewise a weighted integral of $H(\mathbf{k},\omega)$. When the turbulence is axisymmetric, $\beta^{(a)}_{ijk}$ contains Rädler's (1969a) "$\mathbf{\Omega}\wedge\mathbf{J}$-effect". It is shown that when the turbulence is statistically symmetric about a plane perpendicular to the axis of symmetry, then $\alpha_{ij}=0$ but the Rädler effect is non-zero. Explicit expressions for $\alpha_{ij}$ and $\beta_{ijk}$ are given when the velocity field is generated by random forcing in a rotating medium. Finally, it is shown by means of a local analysis that the Rädler effect, in conjunction with uniform mean shear, can give rise to non-oscillatory dynamo action, and it is argued that this effect may be significant in the well-mixed interior of a stellar convection zone, where by symmetry the $\alpha$-effect may be weak.

## 1. INTRODUCTION

When turbulence acts upon a magnetic field in a conducting fluid, certain effects (e.g. eddy diffusivity) arise which depend only on the reflexionally symmetric ingredients of the turbulence, and certain other effects arise [in particular the $\alpha$-effect of Steenbeck, Krause and Rädler (1966), and the so-called $\mathbf{\Omega}\wedge\mathbf{J}$-effect of Rädler (1969a)], which depend only on the reflexionally *a*symmetric ingredients of the turbulence. Rädler (1969b) has shown that dynamo excitation of a large-scale magnetic field can arise through the combined action of the $\mathbf{\Omega}\wedge\mathbf{J}$-effect and differential rotation. However, as pointed out by Moffatt (1978, p. 156), when the $\mathbf{\Omega}\wedge\mathbf{J}$-effect is present, it is to be expected that the more potent $\alpha$-effect will in general be present also, and that this will then swamp any influence of the $\boldsymbol{\Omega} \wedge \mathbf{J}$ ingredient of the mean electromotive force.

It nevertheless appears possible that under particular additional symmetry conditions, the $\alpha$-effect may vanish while the coefficient $R$ appearing in the $\boldsymbol{\Omega} \wedge \mathbf{J}$-effect remains non-zero, for the simple reason that $\alpha$ and $R$ are both expressible as weighted integrals of the helicity spectrum function of the turbulence (and possibly higher order spectral functions) but with *different* weight functions. The main purpose of the present paper is to explore this possibility, and to clarify the circumstances (if any) under which the $\boldsymbol{\Omega} \wedge \mathbf{J}$-effect may be of dominant importance.

In order to focus attention on the problem, certain idealisations are in order. To begin with, we shall adopt a strictly kinematic point of view, the statistical properties of the velocity field $\mathbf{u}(\mathbf{x}, t)$ being assumed known. (Dynamical effects associated with the action of Coriolis forces in a rotating system will be considered later in Section 7.) We shall suppose that these statistical properties are homogeneous and stationary in time, and that $\langle \mathbf{u} \rangle = 0$, the angular brackets representing an ensemble average. We suppose that the dominant length-scale of $\mathbf{u}(\mathbf{x}, t)$ is $l_0$, and we consider the evolution of a mean magnetic field $\mathbf{B}(\mathbf{x}, t)$ on a much larger length-scale $L(\gg l_0)$. If the total field is $\mathbf{B}+\mathbf{b}$ where $\langle \mathbf{b} \rangle = 0$, then the mean electromotive force generated by the turbulence is

$$\mathscr{E} = \langle \mathbf{u} \wedge \mathbf{b} \rangle, \tag{1.1}$$

and the mean field evolves according to the equation

$$\partial \mathbf{B}/\partial \mathbf{t} = \nabla \wedge \mathscr{E} + \eta \nabla^2 \mathbf{B}. \tag{1.2}$$

It is also well-known that $\mathscr{E}$ is linearly related to $\mathbf{B}$ by an equation of the form

$$\mathscr{E}_i(\mathbf{x}, t) = \int d\boldsymbol{\xi} \int_0^\infty d\tau \, K_{ij}(\boldsymbol{\xi}, \tau) \, B_j(\mathbf{x} - \boldsymbol{\xi}, t - \tau), \tag{1.3}$$

where $K_{ij}(\xi, \tau)$ is a tensor (or, more precisely, a pseudo-tensor) determined (in principle) by the statistical properties of the turbulence and the magnetic diffusivity of the fluid $\eta$.

The assumption that the scale $L$ of $\mathbf{B}$ is large (with the consequence—see (1.2) above and (1.4) below—that its time variation is slow) allows us to expand $\mathbf{B}(\mathbf{x} - \boldsymbol{\xi}, t - \tau)$ about $\boldsymbol{\xi} = 0, \tau = 0$, and to integrate (1.3) term by term. This results in a series of the form

$$\mathscr{E}_i = \alpha_{ij} B_j + \beta_{ijk} \partial B_j / \partial x_k + \alpha_{ij}^{(1)} \partial B_j / \partial t + \ldots, \tag{1.4}$$

where

$$\begin{pmatrix} \alpha_{ij} \\ \beta_{ijk} \\ \alpha_{ij}^{(1)} \end{pmatrix} = \int d\xi \int_0^\infty d\tau \begin{pmatrix} 1 \\ \xi_k \\ \tau \end{pmatrix} K_{ij}(\xi, \tau), \tag{1.5}$$

etc. It is generally believed that the series (1.4) converges rapidly provided $l_0/L$ is sufficiently small.

In the special case of isotropic turbulence, the coefficients $\alpha_{ij}, \beta_{ijk}, \alpha_{ij}^{(1)}, \ldots$ must also be isotropic, i.e.

$$\alpha_{ij} = \alpha\delta_{ij}, \quad \beta_{ijk} = \beta\varepsilon_{ijk}, \quad \alpha_{ij}^{(1)} = \alpha^{(1)}\delta_{ij}, \ldots \tag{1.6}$$

and (1.4) becomes

$$\mathscr{E} = \alpha\mathbf{B} - \beta\nabla \wedge \mathbf{B} + \alpha^{(1)}\partial\mathbf{B}/\partial t + \ldots. \tag{1.7}$$

Correspondingly, Eq. (1.2) then becomes

$$\partial\mathbf{B}/\partial t = \alpha\nabla \wedge \mathbf{B} + (\eta + \beta)\nabla^2\mathbf{B} + \alpha^{(1)}\partial(\nabla \wedge \mathbf{B})/\partial t + \ldots. \tag{1.8}$$

The term $\alpha\mathbf{B}$ in (1.7) represents the $\alpha$-effect of Steenbeck, Krause and Rädler (1966), and the $\beta$-term provides an eddy-diffusivity effect. The $\alpha^{(1)}$-term was discussed by Rädler (1968) and interpreted as a "capacitative" effect.

If the scale of $\mathbf{B}$ is so large that

$$|\beta\nabla \wedge \mathbf{B}| \ll |\alpha\mathbf{B}| \tag{1.9}$$

in (1.7), then the time derivatives in (1.7) may be replaced iteratively by space derivatives using (1.8); this procedure gives

$$\mathscr{E} = \alpha\mathbf{B} - (\beta - \alpha\alpha^{(1)})\nabla \wedge \mathbf{B} + O(L^{-2}), \tag{1.10}$$

where the $O(L^{-2})$ terms include a contribution $\alpha^{(1)}(\eta + \beta)\nabla^2\mathbf{B}$. It would appear from (1.10) that when $\alpha$ and $\alpha^{(1)}$ are non-zero, we have a modified turbulent diffusivity

$$\beta^{(M)} = \beta - \alpha\alpha^{(1)}. \tag{1.11}$$

The corresponding modification of the tensor $\beta_{ijk}$ in (1.4) is given by

$$\beta_{ijk}^{(M)} = \beta_{ijk} + \varepsilon_{pkq}\alpha_{ip}^{(1)}\alpha_{qj}. \tag{1.12}$$

We shall find that determination of $\alpha_{ij}^{(1)}$ is coupled very naturally with determination of $\beta_{ijk}$, so that the modifications (1.11) and (1.12) are easily evaluated. However, as pointed out by a referee, the procedure by which (1.11) and (1.12) are obtained is somewhat suspect if (as is frequently the case in dynamo models) the terms $\alpha\mathbf{B}$ and $\beta\nabla\wedge\mathbf{B}$ of (1.7) are of the same order of magnitude. We shall not therefore pursue the implications of (1.11) and (1.12) in this paper.

## 2. KINEMATICS OF REFLEXIONALLY NON-SYMMETRIC TURBULENCE



We adopt the usual Fourier representation for $\mathbf{u}(\mathbf{x},t)$, viz.

$$\mathbf{u}(\mathbf{x},t)=\int \hat{\mathbf{u}}(\mathbf{k},\omega)\, e^{i\varphi}\, d\Lambda, \tag{2.1}$$

where $\varphi=\mathbf{k}\cdot\mathbf{x}-\omega t$ and $d\Lambda=d\mathbf{k}\,d\omega$, the integral being a four-fold integral with inverse

$$\hat{\mathbf{u}}(\mathbf{k},\omega)=(2\pi)^{-4}\int \mathbf{u}(\mathbf{x},t)\ e^{-i\varphi}\, d\mathbf{x}\, dt. \tag{2.2}$$

Reality of $\mathbf{u}$ implies that

$$\hat{\mathbf{u}}(-\mathbf{k},\,-\omega)=\hat{\mathbf{u}}^*(\mathbf{k},\omega), \tag{2.3}$$

where * denotes the complex conjugate. We restrict attention to incompressible velocity fields for which $\nabla\cdot\mathbf{u}=\mathbf{0}$, so that

$$\mathbf{k}\cdot\hat{\mathbf{u}}(\mathbf{k},\omega)=0 \quad (\text{all } \mathbf{k},\omega). \tag{2.4}$$

The spectrum tensor $\Phi_{ij}(\mathbf{k},\omega)$ of the turbulence is given by the well-known relation

$$\langle\hat{u}_i^*(\mathbf{k},\omega)\hat{u}_j(\mathbf{k}',\omega')\rangle=\Phi_{ij}(\mathbf{k},\omega)\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'), \tag{2.5}$$

and it satisfies the conditions [corresponding to (2.3) and (2.4)]

$$\Phi_{ij}(\mathbf{k},\omega)=\Phi_{ji}(-\mathbf{k},\,-\omega)=\Phi_{ji}^*(\mathbf{k},\omega), \tag{2.6}$$

and

$$k_i\Phi_{ij}(\mathbf{k},\omega)=0,\quad k_j\Phi_{ij}(\mathbf{k},\omega)=0 \quad (\text{all } \mathbf{k},\omega). \tag{2.7}$$

Let us now decompose $\Phi_{ij}$ into its symmetric and antisymmetric parts with respect to $i \leftrightarrow j$ [superscripts $(s)$ and $(a)$],

$$\Phi_{ij}(\mathbf{k}, \omega) = \Phi_{ij}^{(s)}(\mathbf{k}, \omega) + \Phi_{ij}^{(a)}(\mathbf{k}, \omega), \tag{2.8}$$

where, by virtue of (2.6), $\Phi_{ij}^{(s)}$ is real, and $\Phi_{ij}^{(a)}$ is pure imaginary. Writing

$$\Phi_{ij}^{(a)} = i\varepsilon_{ijk} P_k(\mathbf{k}, \omega), \tag{2.9}$$

where $\mathbf{P}(\mathbf{k}, \omega)$ is a real pseudo-vector, the conditions (2.7) imply that $\mathbf{k} \wedge \mathbf{P} = 0$, i.e.

$$\mathbf{P} = \tfrac{1}{2}\mathbf{k} H(\mathbf{k}, \omega)/k^2 \tag{2.10}$$

for some real pseudo-scalar function $H(\mathbf{k}, \omega)$. Hence

$$\Phi_{ij}^{(a)}(\mathbf{k}, \omega) = \tfrac{1}{2} i\varepsilon_{ijk} k_k H(\mathbf{k}, \omega)/k^2, \tag{2.11}$$

and, reciprocally,

$$H(\mathbf{k}, \omega) = -ik_m \varepsilon_{ijm} \Phi_{ij}(\mathbf{k}, \omega). \tag{2.12}$$

This is the *helicity spectrum function* of the turbulence, with the property

$$\langle \mathbf{u} \cdot \nabla \wedge \mathbf{u} \rangle = \int H(\mathbf{k}, \omega)\, d\Lambda. \tag{2.13}$$

It is noteworthy that (without any assumption of isotropy or axisymmetry) the single scalar function $H(\mathbf{k}, \omega)$ is sufficient to determine the asymmetric part of the spectrum tensor.

The energy spectrum function $E(\mathbf{k}, \omega)$ is given by

$$E(\mathbf{k}, \omega) = \tfrac{1}{2}\Phi_{ii}(\mathbf{k}, \omega) \quad (\geqq 0), \tag{2.14}$$

and has the property

$$\tfrac{1}{2}\langle \mathbf{u}^2 \rangle = \int E(\mathbf{k}, \omega)\, d\Lambda. \tag{2.15}$$

The function $H$ satisfies an important inequality which follows from Cramer's (1940) theorem

$$\Phi_{ij}(\mathbf{k}, \omega) X_i X_j^* \geqq 0 \quad \text{(all complex } \mathbf{X}). \tag{2.16}$$

Taking $\mathbf{X} = \mathbf{p} + i\mathbf{q}$, where $\mathbf{p}$ and $\mathbf{q}$ are real unit vectors such that $(\mathbf{k}, \mathbf{p}, \mathbf{q})$

forms an orthogonal triad, we have

$$\Phi_{ij}X_iX_j^* = 2E(\mathbf{k},\omega) \pm k^{-1}H(\mathbf{k},\omega), \tag{2.17}$$

and it follows from (2.16) that

$$|H(\mathbf{k},\omega)| \leqq 2kE(\mathbf{k},\omega). \tag{2.18}$$

This inequality is well-known for the case of isotropic turbulence, but does not seem to have been recognized previously for the general homogeneous turbulence considered here. From (2.6), (2.12) and (2.14), it is evident that $E(\mathbf{k},\omega)$ and $H(\mathbf{k},\omega)$ satisfy the symmetry conditions

$$E(-\mathbf{k}, -\omega) = E(\mathbf{k},\omega), \quad H(-\mathbf{k}, -\omega) = H(\mathbf{k},\omega). \tag{2.19}$$

It follows that, for any function $A(\mathbf{k},\omega)$ with the property

$$A(-\mathbf{k}, -\omega) = -A(\mathbf{k},\omega), \tag{2.20}$$

we have

$$\int A(\mathbf{k},\omega)H(\mathbf{k},\omega)\,d\Lambda = 0, \tag{2.21}$$

(and similarly with $H$ replaced by $E$). Repeated use will be made of this type of result in the following sections.

## 3. CALCULATION OF $\alpha_{ij}$ UNDER THE FIRST-ORDER SMOOTHING APPROXIMATION

In calculating $\alpha_{ij}$ in (1.4), it is clearly legitimate to assume that $\mathbf{B}$ is strictly uniform, and therefore steady, since $\mathscr{E}$ [in (1.2)] is then also uniform. The following calculation is well-known, but is reproduced briefly here as an essential preliminary to the subsequent calculation of $\beta_{ijk}$.

The perturbation field $\mathbf{b}$ satisfies (Moffatt 1978, Section 7.5)

$$\partial\mathbf{b}/\partial t = \nabla \wedge (\mathbf{u} \wedge \mathbf{B}) + \nabla \wedge \mathscr{E}' + \eta\nabla^2\mathbf{b}, \tag{3.1}$$

where $\mathscr{E}' = \mathbf{u} \wedge \mathbf{b} - \langle \mathbf{u} \wedge \mathbf{b} \rangle$. The first-order smoothing approximation, which we adopt here, consists in neglecting the term $\nabla \wedge \mathscr{E}'$ in (3.1), which then becomes

$$\partial\mathbf{b}/\partial t - \eta\nabla^2\mathbf{b} = \mathbf{B}\cdot\nabla\mathbf{u}, \tag{3.2}$$

with Fourier transform

$$(-i\omega+\eta k^2)\hat{\mathbf{b}}=i(\mathbf{B}\cdot\mathbf{k})\hat{\mathbf{u}}. \tag{3.3}$$

The mean electromotive force $\mathscr{E}$ is then given by

$$\mathscr{E}_i=\langle\mathbf{u}\wedge\mathbf{b}\rangle_i=\varepsilon_{ijk}\iint\langle\hat{u}_j^{*\prime}\hat{b}_k\rangle\, e^{i(\phi-\phi')}d\Lambda\, d\Lambda', \tag{3.4}$$

where $\varphi'=\mathbf{k}'\cdot\mathbf{x}-\omega' t$, and we use the shorthand notation $\hat{\mathbf{u}}'=\hat{\mathbf{u}}(\mathbf{k}',\omega')$. Hence, using (3.3) and (2.5), and carrying out the trivial integration over $\Lambda'$, we obtain $\mathscr{E}_i=\alpha_{ij}B_j$ where

$$\alpha_{ij}=\int\frac{k_j i\varepsilon_{ipk}\Phi_{pk}(\mathbf{k},\omega)}{-i\omega+\eta k^2}d\Lambda=-\int\frac{k_i k_j H(\mathbf{k},\omega)}{k^2(-i\omega+\eta k^2)}d\Lambda$$

or

$$\alpha_{ij}=-\eta\int\frac{k_i k_j}{\omega^2+\eta^2k^4}H(\mathbf{k},\omega)\,d\Lambda, \tag{3.5}$$

where we have used (2.21) to discard the imaginary part—of course the result *must* be real from the definition of $\alpha_{ij}$. The result (3.5), which has been obtained previously in very similar form by Soward and Roberts (1976), holds for general homogeneous turbulence, with no assumptions concerning directional symmetry. Note that $\alpha_{ij}=\alpha_{ji}$ under first-order smoothing. An antisymmetric part of $\alpha_{ij}$ (and an associated field pumping effect) appears only at higher orders, involving cubic velocity spectra (Krause and Rädler, 1980, Section 7.2).

## 4. CALCULATION OF $\beta_{ijk}$ AND $\alpha_{ij}^{(1)}$

This calculation proceeds in a similar manner, but with the assumption that **B** has a uniform gradient, and in consequence a uniform time-rate-of-change, i.e.

$$B_j(\mathbf{x},t)=B_{0j}+B_{j,l}x_l+\dot{B}_j t, \tag{4.1}$$

where $\mathbf{B}_0=\mathbf{B}(0,0)$, and $B_{j,l}$ and $\dot{B}_j$ are constants. From (1.4) this gives

$$\mathscr{E}_i=\alpha_{ij}B_j+\beta_{ijk}B_{j,k}+\alpha_{ij}^{(1)}\dot{B}_j, \tag{4.2}$$

and substitution of (4.1) and (4.2) in (1.2) gives

$$\dot{B}_i = \varepsilon_{ijk}\alpha_{km}B_{m,j}. \tag{4.3}$$

Since the time and space gradients of **B** are related in this way, determination of $\beta_{ijk}$ is necessarily coupled with a simultaneous determination of $\alpha_{ij}^{(1)}$.

Again neglecting the term $\nabla \wedge \mathscr{E}'$, (3.1) now becomes

$$\begin{aligned}(\partial/\partial t - \eta\nabla^2)b_k &= B_m \partial u_k/\partial x_m - u_m \partial B_k/\partial x_m \\ &= (B_{0m} + B_{m,l}x_l + \dot{B}_m t)(\partial u_k/\partial x_m) - B_{k,m}u_m,\end{aligned} \tag{4.4}$$

with Fourier transform

$$\hat{b}_k = G(k,\omega)\left[\left(B_{0m} + iB_{m,l}\frac{\partial}{\partial k_l} - i\dot{B}_m\frac{\partial}{\partial\omega}\right)ik_m\hat{u}_k - B_{k,m}\hat{u}_m\right], \tag{4.5}$$

where

$$G(k,\omega) = (-i\omega + \eta k^2)^{-1}. \tag{4.6}$$

Substitution in (3.4) gives

$$\mathscr{E}_i = \varepsilon_{ijk}\int\int G\left\langle \hat{u}_j^{*\prime}\left[\left(B_{0m} + iB_{m,l}\frac{\partial}{\partial k_l} - i\dot{B}_m\frac{\partial}{\partial\omega}\right)ik_m\hat{u}_k - B_{k,m}\hat{u}_m\right]\right\rangle e^{i(\phi-\phi')}\,d\Lambda\,d\Lambda'. \tag{4.7}$$

Consider first the term involving the operator $\partial/\partial k_l$. We have

$$\begin{aligned}&\int\int G\left\langle \hat{u}_j^{*\prime}\frac{\partial}{\partial k_l}k_m\hat{u}_k\right\rangle e^{i(\phi-\phi')}\,d\Lambda\,d\Lambda' \\ &= \int\int G\frac{\partial}{\partial k_l}\{k_m\Phi_{jk}(\mathbf{k},\omega)\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\}\,e^{i(\phi-\phi')}\,d\Lambda\,d\Lambda' \\ &= -\int\int k_m\Phi_{jk}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega')\frac{\partial}{\partial k_l}\{Ge^{i(\phi-\phi')}\}\,d\Lambda\,d\Lambda' \\ &= -\int k_m\Phi_{jk}[(\partial G/\partial k_l) + ix_lG]\,d\Lambda.\end{aligned} \tag{4.8}$$

The term involving $\partial/\partial\omega$ in (4.7) may be manipulated in a similar way.

Gathering the various terms together, we find

$$\mathscr{E}_i = i\varepsilon_{ijk}\int G(B_{0m}+B_{m,l}x_l+\dot{B}_m t)k_m\Phi_{jk}\,d\Lambda$$
$$+\varepsilon_{ijk}\int[(\partial G/\partial k_l)k_m B_{m,l}\Phi_{jk}-(\partial G/\partial\omega)k_m\dot{B}_m\Phi_{jk}-GB_{k,m}\Phi_{jm}]\,d\Lambda. \quad (4.9)$$

The first integral here is just $\alpha_{im}B_m(\mathbf{x},t)$, where $\alpha_{im}$ is as determined in Section 3 and $B_m(\mathbf{x},t)$ is given by (4.3). The second integral gives $\beta_{iml}$ and $\alpha_{im}^{(1)}$ directly in the form

$$\beta_{iml}=\varepsilon_{ijk}\int(\partial G/\partial k_l)k_m\Phi_{jk}\,d\Lambda-\varepsilon_{ijm}\int G\Phi_{jl}\,d\Lambda, \quad (4.10)$$

and

$$\alpha_{im}^{(1)}=-\varepsilon_{ijk}\int(\partial G/\partial\omega)k_m\Phi_{jk}\,d\Lambda. \quad (4.11)$$

These expressions may be further simplified. The decomposition (2.8) of $\Phi_{ij}$ into its symmetric and antisymmetric parts provides a similar decomposition of $\beta_{iml}$, viz.

$$\beta_{iml}=\beta_{iml}^{(s)}+\beta_{iml}^{(a)}. \quad (4.12)$$

The first ingredient here is given by

$$\beta_{iml}^{(s)}=\varepsilon_{imp}D_{pl} \quad \text{with} \quad D_{pl}=\int\eta k^2(\omega^2+\eta^2k^4)^{-1}\Phi_{pl}^{(s)}\,d\Lambda. \quad (4.13)$$

$D_{pl}$ is a symmetric positive-definite tensor (and it is actually the symmetric part of the diffusion tensor that acts on a convected *scalar* field with molecular diffusivity $\eta$, under first-order smoothing). The corresponding contribution to $\mathscr{E}$ is

$$\mathscr{E}_i^{(s)}=\beta_{ijk}^{(s)}\partial B_j/\partial x_k=\varepsilon_{ijm}D_{mk}\partial B_j/\partial x_k, \quad (4.14)$$

and so

$$(\nabla\wedge\mathscr{E}^{(s)})_i=\varepsilon_{ijk}\varepsilon_{kpl}D_{lm}\partial^2B_p/\partial x_j\partial x_m=D_{jm}\partial^2B_i/\partial x_j\partial x_m, \quad (4.15)$$

since $\nabla\cdot\mathbf{B}=0$, so that we have here also a pure eddy-diffusivity contribution in the mean-field equation (1.2).

The second ingredient of (4.12) simplifies, using (2.11), to the form

$$\beta_{ijk}^{(a)}=\int\frac{\omega}{\omega^2+\eta^2k^4}\left(\frac{4\eta^2k_ik_jk_k}{\omega^2+\eta^2k^4}+\frac{k_i\delta_{jk}-k_j\delta_{ik}}{2k^2}\right)H\,d\Lambda, \quad (4.16)$$

which, like $\alpha_{ij}$, is a weighted integral of the helicity spectrum function. This term has the property

$$\varepsilon_{ijk}\beta_{ijk}^{(a)}=0, \tag{4.17}$$

and therefore makes no contribution to eddy diffusivity (see Section 5 below). It does however contain the Rädler "$\mathbf{\Omega} \wedge \mathbf{J}$-effect", although of course dependence on $\mathbf{\Omega}$ (the rotation rate of the frame of reference) appears only when we take account of the dynamical influence of Coriolis forces on the function $H(\mathbf{k},\omega)$—see Section 7 below.

Finally, the expression (4.11) for $\alpha_{im}^{(1)}$ may be simplified to the form

$$\alpha_{im}^{(1)}=-\int \frac{(\omega^2-\eta^2k^4)k_ik_m}{(\omega^2+\eta^2k^4)^2k^2}H\,d\Lambda, \tag{4.18}$$

again a weighted integral of the helicity spectrum function.

## 5. THE CASE OF ISOTROPIC TURBULENCE

In the case of isotropic turbulence (i.e. turbulence whose statistical properties are invariant under rotations, but not necessarily under reflexions, of the frame of reference), we have

$$E=E(k,\omega), \quad H=H(k,\omega), \tag{5.1}$$

where $k=|\mathbf{k}|$. From (3.5), we then have

$$\alpha_{ij}=\alpha\delta_{ij}, \tag{5.2}$$

where

$$\alpha=\tfrac{1}{3}\alpha_{ii}=-\tfrac{1}{3}\eta\int k^2(\omega^2+\eta^2k^4)^{-1}H(k,\omega)\,d\Lambda, \tag{5.3}$$

with $d\Lambda=4\pi k^2\,dk\,d\omega$. Similarly, from (4.13),

$$D_{ij}=D\delta_{ij}, \tag{5.4}$$

where

$$D=\tfrac{1}{3}D_{ii}=(2\eta/3)\int k^2(\omega^2+\eta^2k^4)^{-1}E(k,\omega)\,d\Lambda. \tag{5.5}$$

It follows from (4.13) that

$$\beta_{ijk}^{(s)}=\beta^{(s)}\varepsilon_{ijk} \tag{5.6}$$

where

$$\beta^{(s)} = \varepsilon_{ijk}\beta^{(s)}_{ijk}/6 = \tfrac{1}{3}D_{ii} = D. \tag{5.7}$$

The integral (4.16) for $\beta^{(a)}_{ijk}$ vanishes, by symmetry, in the isotropic situation, but (4.18) becomes

$$\alpha^{(1)}_{im} = \alpha^{(1)}\delta_{im} \quad \text{where} \quad \alpha^{(1)} = \frac{1}{3}\int \frac{\eta^2 k^4 - \omega^2}{(\omega^2 + \eta^2 k^4)^2} H \, d\Lambda. \tag{5.8}$$

## 6. THE CASE OF AXISYMMETRIC TURBULENCE

Suppose now that the turbulence is statistically axisymmetric about an axis defined by the unit vector **e**. Let

$$\mu = \cos\theta = \mathbf{k}\cdot\mathbf{e}/k, \tag{6.1}$$

so that

$$d\Lambda = 2\pi k^2 \, d\mu \, dk. \tag{6.2}$$

Then

$$E = E(k, \mu, \omega), \quad H = H(k, \mu, \omega), \tag{6.3}$$

and the symmetry conditions (2.19) become

$$E(k, -\mu, -\omega) = E(k, \mu, \omega), \quad H(k, -\mu, -\omega) = H(k, \mu, \omega). \tag{6.4}$$

From (3.5), $\alpha_{ij}$ is now given by

$$\alpha_{ij} = \alpha\delta_{ij} + \alpha_1(\delta_{ij} - 3e_ie_j), \tag{6.5}$$

where, as before,

$$\alpha = \tfrac{1}{3}\alpha_{ii} = -\tfrac{1}{3}\eta \int k^2(\omega^2 + \eta^2 k^4)^{-1} H(k, \mu, \omega) \, d\Lambda, \tag{6.6}$$

and

$$\alpha_1 = \tfrac{1}{2}(\alpha - \alpha_{ij}e_ie_j) = \frac{\eta}{6}\int \frac{k^2(3\mu^2 - 1)}{\omega^2 + \eta^2 k^4} H(k, \mu, \omega) \, d\Lambda. \tag{6.7}$$

Similarly, from (4.13),

$$D_{ij} = D\delta_{ij} + D_1(\delta_{ij} - 3e_ie_j) \tag{6.8}$$

where

$$D=\tfrac{2}{3}\eta\int\frac{k^2}{\omega^2+\eta^2k^4}E(k,\mu,\omega)\,d\Lambda,\quad D-2D_1=\eta\int\frac{k^2}{\omega^2+\eta^2k^4}\Phi_{ij}e_ie_j\,d\Lambda,\tag{6.9}$$

and

$$\beta^{(s)}_{ijk}=D\varepsilon_{ijk}+D_1(\varepsilon_{ijk}-3\varepsilon_{ijm}e_me_k).\tag{6.10}$$

The expression (4.16) for $\beta^{(a)}_{ijk}$ takes the form

$$\beta^{(a)}_{ijk}=-R(e_i\delta_{jk}+e_j\delta_{ki}+e_k\delta_{ij})+R_1(e_i\delta_{jk}-e_j\delta_{ki}),\tag{6.11}$$

where

$$R=-\frac{2\eta^2}{5}\int\frac{k^3\mu\omega H}{(\omega^2+\eta^2k^4)^2}d\Lambda,\quad R_1=\frac{1}{2}\int\frac{\mu\omega H}{(\omega^2+\eta^2k^4)k}d\Lambda,\tag{6.12}$$

and the corresponding contribution to $\mathscr{E}$ is

$$\mathscr{E}^{(a)}=R\mathbf{e}\wedge(\nabla\wedge\mathbf{B})-(R_1+2R)\nabla(\mathbf{e}\cdot\mathbf{B}).\tag{6.13}$$

The second term, being the gradient of a scalar, makes no contribution to the mean-field equation. The first term does make a contribution, however, and in fact it is the contribution that Rädler (1969) described as the "$\mathbf{\Omega}\wedge\mathbf{J}$-effect". (We shall refer to $R$ as the Rädler coefficient.) The derivation given here however should make it clear that this effect (like the $\alpha$-effect) will arise in general when (and only when) the helicity spectrum $H(\mathbf{k},\omega)$ is non-zero. *Both* effects depend indirectly on the rotation vector $\mathbf{\Omega}$ of the frame of reference only insofar as Coriolis forces are responsible for the generation of a non-zero $H(\mathbf{k},\omega)$.

The expressions (6.6) and (6.12a) for $\alpha$ and $R$ differ only in the weighting factors in the integral. It is clear that if $H$ satisfies the condition

$$H(k,-\mu,\omega)=-H(k,\mu,\omega),\tag{6.14}$$

[note the distinction between this and the *automatic* condition (6.4b)] then, in general,

$$\alpha=0\quad\text{and}\quad R\neq0.\tag{6.15}$$

Physically, the condition (6.14), which may be described as the condition of "up-down symmetry" means that positive helicity associated with waves for which $\omega\mu>0$ is compensated by equal negative helicity associated with waves for which $\omega\mu<0$. The net helicity density $\langle \mathbf{u}\cdot\nabla\wedge\mathbf{u}\rangle$ is then zero, and $\alpha_{ij}$ is zero likewise. It is the extra factor $\mu$ in the integrand defining $R$ which nevertheless gives $R\neq 0$. The need for a lack of up-down symmetry (as well as a lack of reflexional symmetry) in generating an $\alpha$-effect has been noted previously [see e.g. the discussion of Moffatt (1978, Section 10.2)]. The fact that "the $\mathbf{\Omega}\wedge\mathbf{J}$-effect ... does not require a predominance of right- or left-handed helical motions" is noted by Krause and Rädler (1980, p. 93); the above expression for $R$ directly in terms of the helicity spectrum is however new, and makes it clear in what sense the above quotation from Krause and Rädler (1980) is to be understood.



## 7. DYNAMIC RESPONSE TO RANDOM FORCING IN A ROTATING FLUID

Suppose now that the turbulence is forced by a random force field $\mathbf{f}(\mathbf{x},t)$ (with $\nabla\cdot\mathbf{f}=0$) in a fluid rotating with uniform angular velocity $\mathbf{\Omega}$. We suppose that the magnetic field is weak and the Lorentz force negligible. The linearised response is then easily determined (Moffatt, 1978, Section 10.3) and is given by

$$\hat{\mathbf{u}}(\mathbf{k},\omega)=A^{-1}\{2(\mathbf{k}\cdot\mathbf{\Omega})\mathbf{k}\wedge\hat{\mathbf{f}}+i\sigma k^2\hat{\mathbf{f}}\}, \tag{7.1}$$

where

$$\sigma=\omega+i\nu k^2,\quad A=\sigma^2k^2-4(\mathbf{k}\cdot\mathbf{\Omega})^2. \tag{7.2}$$

We shall suppose that the force field $\mathbf{f}$ is itself reflexionally symmetric, so that $\langle i\hat{\mathbf{f}}\wedge\hat{\mathbf{f}}^*\rangle=0$; then from (7.1), we find

$$\langle i\hat{\mathbf{u}}\wedge\hat{\mathbf{u}}^*\rangle=-4|A|^{-2}(\mathbf{k}\cdot\mathbf{\Omega})\mathbf{k}\omega k^2\langle|\hat{\mathbf{f}}|^2\rangle, \tag{7.3}$$

and the equivalent relation between the spectrum function $F(\mathbf{k},\omega)$ of $\mathbf{f}$ and the helicity spectrum function $H(\mathbf{k},\omega)$ of $\mathbf{u}$ is evidently

$$H(\mathbf{k},\omega)=-4|A|^{-2}(\mathbf{k}\cdot\mathbf{\Omega})\omega k^4F(\mathbf{k},\omega). \tag{7.4}$$

Note that the pseudo-scalar character of $H$ is here associated with the pseudo-scalar $\mathbf{k}\cdot\mathbf{\Omega}$ on the right-hand side.

The corresponding expressions for $\alpha_{ij}$ and $\beta_{ijk}^{(a)}$ may now be obtained (in terms of $F$) by straightforward substitution of (7.4) in (3.5) and (4.16). We focus attention here on the situation in which $F(\mathbf{k},\omega)$ is axisymmetric about the direction of $\mathbf{\Omega}$, i.e.

$$F = F(k, \mu, \omega) = F(k, -\mu, -\omega), \tag{7.5}$$

where $\mu = \cos\theta = \mathbf{k}\cdot\mathbf{e}/k$ as before, $\mathbf{e}$ being the unit vector in the direction of $\mathbf{\Omega}$. Then the axisymmetric forms obtained in Section 6 are relevant; in particular from (6.6),

$$\alpha = (4\eta\Omega/3)\int k^7 \mu\omega(\omega^2 + \eta^2 k^4)^{-1} |A|^{-2} F(k, \mu, \omega)\, d\Lambda, \tag{7.6}$$

and from (6.12)

$$R = (2\eta^2\Omega/5)\int k^8 \mu^2\omega^2(\omega^2 + \eta^2 k^4)^{-2} |A|^{-2} F(k, \mu, \omega)\, d\Lambda, \tag{7.7}$$

where $\Omega = \mathbf{\Omega}\cdot\mathbf{e}$ (a pseudo-scalar). It is now clear that if

$$F(k, -\mu, \omega) = F(k, \mu, \omega), \tag{7.8}$$

i.e. if the forcing is symmetric with respect to the directions $\pm\mathbf{e}$, then $\alpha = 0$ but, in general, $R \neq 0$.

The above results are consistent with the discussion of Krause and Rädler (1980, Section 7.5) who however restrict attention to a situation in which the anisotropy is entirely due to the influence of Coriolis forces, assumed *weak*. In the above treatment [as in Moffatt (1972)] there is no such limitation, and indeed the linearised model on which (7.1) is based becomes *more* justifiable when $|\Omega|$ is large. The limiting form of the integrals (7.6) and (7.7) as $|\Omega| \to \infty$ (keeping all other parameters fixed) is of particular interest. Since

$$|A|^2 = (4k^2\Omega^2\mu^2 - \omega^2 + \nu^2 k^4)^2 + 4\omega^2\nu^2 k^4, \tag{7.9}$$

the dominant contributions to both integrals come from the neighbourhood of $\mu = 0$—a reflection of the strong anisotropy associated with the Taylor–Proudman theorem. Putting $\xi = 2k\Omega\mu$, and $d\xi = 2k\Omega\, d\mu$, the limits $\mu = \pm 1$ become (for $\Omega \to \infty$) $\xi = \pm\infty$; integration over $\xi$ in both (7.6) and (7.7) then involves the integral

$$I(k, \omega) = \int_{-\infty}^{\infty} \xi^2 [(\xi^2 - \omega^2 + \nu^2 k^4)^2 + 4\omega^2\nu^2 k^4]^{-1}\, d\xi, \tag{7.10}$$

which may be evaluated by contour integration to give

$$I(k, \omega) = \pi/2\nu k^2. \tag{7.11}$$

Using this result, the asymptotic forms of (7.6) and (7.7) are

$$\alpha \sim \frac{\pi^2 \eta}{3\nu\Omega^2} \int\int \frac{k^4 \omega F_\mu(k,0,\omega)}{\omega^2+\eta^2 k^4} dk\, d\omega, \tag{7.12}$$

and

$$R \sim \frac{\pi^2 \eta^2}{10\nu\Omega^2} \int\int \frac{k^5 \omega^2 F(k,0,\omega)}{(\omega^2+\eta^2 k^4)^2} dk\, d\omega, \tag{7.13}$$

as $|\Omega| \to \infty$. Both $\alpha$ and $R$ decrease like $\Omega^{-2}$ as $|\Omega| \to \infty$. Note that $\alpha$ involves $\partial F/\partial \mu$ evaluated at $\mu=0$; of course this vanishes under the condition of up-down symmetry (7.8). By contrast, $R$ involves only the function $F$ itself evaluated at $\mu=0$.

Having established the formal circumstances in which the Rädler effect may be expected to occur in the absence of an $\alpha$-effect, the question still remains as to whether such circumstances can ever arise in a natural physical system. Turbulent thermal convection in a stellar convection zone is frequently modelled by Boussinesq convection in a plane layer $-z_0 < z < z_0$ rotating with angular velocity $\Omega$ about the vertical. The fact that heat is transported upwards by the turbulence does not in itself break the up-down symmetry, because as far as the velocity field is concerned, a rising hot blob and a falling cold blob are quite equivalent—e.g. they entrain fluid and conserve angular momentum in much the same manner. In stability studies, it is in fact found that the helicity $\langle \mathbf{u} \cdot (\nabla \wedge \mathbf{u}) \rangle$ (the average being over horizontal planes) is an antisymmetric function of $z$, and that when a magnetic field is present, an antisymmetric $\alpha(z)$ appears also [see, e.g. Soward (1974), Moffatt (1978, Section 12.3)]. The coefficient $R$ will also be $z$-dependent in these circumstances, but there is no reason to believe that it will vanish on $z=0$. If the turbulence is approximately homogeneous in a core region (the inhomogeneity being concentrated in thermal boundary layers on $z=\pm z_0$) then it is at least conceivable that the $R$-effect may dominate over the $\alpha$-effect throughout the core region.

This situation is still of course somewhat idealised. In a stellar convection zone, the spherical geometry in itself is enough to break the up-down symmetry; moreover compressibility plays an important part in the process by which a rising blob acquires helicity, and the fact that the ambient pressure increases with depth is clearly responsible for a further strong departure from up-down symmetry. The assertion of Moffatt (1978, p. 156) that "it seems almost inevitable that whenever the Rädler-effect is operative, it will be dominated by the $\alpha$-effect" is hard to refute in the stellar context.

## 8. FREE MODES ASSOCIATED WITH THE RÄDLER EFFECT IN CONJUNCTION WITH MEAN SHEAR

Despite the above reservations, it seems worthwhile to investigate whether the Rädler effect (without any $\alpha$-effect) can, in conjunction with mean shear (or differential rotation), give rise to dynamo excitation of a mean field. Numerical results for spherical models of this type are reported by Krause and Rädler (1980, Section 16.5) and it appears that both steady and oscillatory dynamos are possible. Here we investigate local solutions of the mean field equation

$$\partial \mathbf{B}/\partial t = \nabla \wedge (\mathbf{U} \wedge \mathbf{B}) + \nabla \wedge \mathscr{E} + \eta \nabla^2 \mathbf{B}, \tag{8.1}$$

with $\mathscr{E}_i = \beta_{ijk} \partial B_j / \partial x_k$, following the free-mode approach of Parker (1955). We suppose that $\mathbf{B} = \mathbf{B}(x, z, t)$ and that

$$\mathbf{U} = (0, Gz, 0), \tag{8.2}$$

where $G = \text{constant}$. Then

$$[\nabla \wedge (\mathbf{U} \wedge \mathbf{B})]_i = G_{ij} B_j, \tag{8.3}$$

where $G_{ij} = G\delta_{i2}\delta_{j3}$. Equation (8.1) admits solutions of the form

$$\mathbf{B} = \mathbf{B}_0 \exp(pt + i\mathbf{K} \cdot \mathbf{x}), \quad \mathbf{K} \cdot \mathbf{B}_0 = 0, \tag{8.4}$$

where

$$(p + \eta K^2) B_i + A_{ij} B_j = 0, \tag{8.5}$$

with

$$A_{ij} = \varepsilon_{ipk} \beta_{kjm} K_p K_m - G_{ij}. \tag{8.6}$$

The possible values of $p$ are then determined by

$$\| A_{ij} + (p + \eta K^2) \delta_{ij} \| = 0. \tag{8.7}$$

For simplicity, consider the particular case when $\mathbf{K} = (K, 0, 0)$ (which corresponds to horizontal field variation in the presence of a vertically sheared horizontal velocity; the result below that growing modes are non-oscillatory is unaffected by this simplification). Then (8.7) becomes

$$\begin{vmatrix} p' & 0 & 0 \\ A_{21} & p' + A_{22} & A_{23} \\ A_{31} & A_{32} & p' + A_{33} \end{vmatrix} = 0, \tag{8.8}$$

where $p'=p+\eta K^2$; the root $p'=0$ is spurious (since by (8.4b), $B_{0x}=0$); the other two roots are given by

$$p'=-\tfrac{1}{2}(A_{22}+A_{33})\pm\tfrac{1}{2}\{(A_{22}-A_{33})^2+4A_{23}A_{32}\}^{1/2}. \tag{8.9}$$

Now from (8.6) we have

$$A_{22}=-K^2\beta_{321},\quad A_{33}=K^2\beta_{231},\quad A_{23}=-K^2\beta_{331}-G,\quad A_{32}=K^2\beta_{221}, \tag{8.10}$$

and, from (4.12), (4.13) and (4.16),

$$\beta_{321}=\beta_1-D_{11},\quad \beta_{231}=\beta_1+D_{11}, \tag{8.11}$$

where

$$\beta_1=4\eta^2\int\omega k_1k_2k_3(\omega^2+\eta^2k^4)^{-2}H\,d\Lambda. \tag{8.12}$$

Also

$$\beta_{331}=4\eta^2\int\frac{\omega k_3^2k_1}{(\omega^2+\eta^2k^4)^2}H\,d\Lambda,\quad \beta_{221}=4\eta^2\int\frac{\omega k_2^2k_1}{(\omega^2+\eta^2k^4)^2}H\,d\Lambda. \tag{8.13}$$

Note that, by the Schwartz inequality,

$$\beta_1^2\leqq\beta_{221}\beta_{331}. \tag{8.14}$$

Substitution of (8.10) and (8.11) in (8.9) gives

$$p=-(\eta+D_{11})K^2\pm K^2\{(\beta_1^2-\beta_2\beta_3)-G\beta_2/K^2\}^{1/2}, \tag{8.15}$$

where we have written $\beta_2$ for $\beta_{221}$ and $\beta_3$ for $\beta_{331}$. If $G\beta_2>0$, both roots (8.15) have negative real parts and represent decaying solutions. If $G\beta_2<0$ however, one of the roots (8.15) is real and positive provided

$$K^2<-G\beta_2/[(\eta+D_{11})^2+(\beta_2\beta_3-\beta_1^2)], \tag{8.16}$$

i.e. provided the wavelength of the field is sufficiently large. Note that, in the case of turbulence that is axisymmetric about the direction of the unit vector **e**, from (6.10) and (6.11).

$$\beta_2=\beta_3=-Re_1, \tag{8.17}$$

where $R$ is the Rädier coefficient. The result (8.16) therefore confirms that non-oscillatory dynamo action can occur as a result of an interaction between mean shear and the Rädler effect.

The possibility of *oscillatory* dynamo action does *not* emerge from this simplified local analysis. The existence of oscillatory modes in a spherical geometry (Rädler, 1976; Krause and Rädler, 1980, Section 16.5) must therefore be associated with the global geometry, or with spatial variation of the shear rate $G$ and the Rädler coefficient $R$.



## 9. CONCLUSIONS

The main aim of this paper has been the elucidation of circumstances in which the $\alpha$-effect due to a turbulent velocity field vanishes, but the Rädler "$\mathbf{\Omega} \wedge \mathbf{J}$"-effect does not. The two effects are closely related, since they both depend on the helicity spectrum function $H(\mathbf{k},\omega)$ alone, even for fluctuating fields with no isotropy or symmetry. In spite of this, the integrals (3.5) and (4.16) from which the coefficients $\alpha_{ij}$ and $\beta_{ijk}^{(a)}$ are calculated are invariant under different symmetry operations, with the result that there are circumstances for which $\alpha_{ij}$ vanishes and $\beta_{ijk}^{(a)}$ is non-zero. Two examples are the propagation of helical inertial waves in a uniformly rotating fluid if there is no net flux of momentum in the direction of the rotation axis $\mathbf{\Omega}$, and the well-mixed turbulent core that occurs far from boundaries in convection in a rotating layer. Although the precise symmetries that are needed to make $\alpha_{ij}$ vanish will not be exactly satisfied in any real system, one would expect that if they are approximately true, then the effect of $\alpha_{ij}$ will be no more than comparable with the Rädler effect (of nominally smaller order in the expansion). Thus the apparenrly contradictory stances taken by Moffatt (1978) and Krause and Rädler (1980) on the importance of the Rädler effect can be reconciled. Some properties of growing solutions of the mean field equations due to the interaction of shear and Rädler effect are also discussed.

An interesting secondary point to emerge from the development concerns the necessity to calculate the coefficient of $\partial\mathbf{B}/\partial t$ in the ansatz (1.4). Here we have assumed, consistent with the spatial form of $\mathbf{B}$, that $|\mathbf{B}| \propto t$. More generally, one would expect that a more accurate description can be obtained by using the form $\mathbf{B} \propto \exp(i\mathbf{k}\cdot\mathbf{x}+\mathbf{p}t)$. Although the results would be unaffected to the order we have taken them, it is possible to produce a 'renormalized' theory, taking effects at all orders into account. The results of this promising new approach will be presented elsewhere.

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