

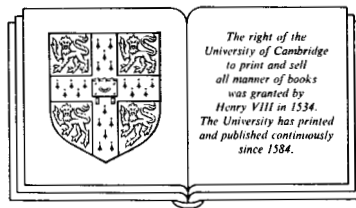
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Proceedings of the IUTAM Symposium

Cambridge, UK 13–18 August 1989

Edited by H. K. MOFFATT and A. TSINOBER



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Contents

Preface	xv
Opening Address	xvii
PAUL GERMAIN	
The Topological (as opposed to the analytical) Approach to Fluid and Plasma Flow Problems	1
H.K. MOFFATT	
I: Flow Kinematics and Lagrangian Chaos	11
Kinematics of Chaotic Mixing: Experimental and Computational Results	13
J.M. OTTINO	
Lagrangian Chaos and Transport of Scalars	23
B. ECKHARDT	
On Chaotic Flow around the Kida Vortex	34
L.M. POLVANI & J. WISDOM	
Fluid Webs with Lagrangian Chaos and Intermittency	45
A.A. CHERNIKOV, R.Z. SAGDEEV & G.M. ZASLAVSKY	
Numerical Simulation of the Lagrangian Flow Structure in a Driven-Cavity	54
K. ISHII & R. IWATSU	
Streamline Coordinates, Moving Frames, Chaos and Integrability in Fluid Flow	64
J.J. FINNIGAN	
The Geometry of Lagrangian Orbits	75
E. DRESSELHAUS & M. TABOR	
Scalar Field Topology in Turbulent Mixing	85
C.H. GIBSON	

Algorithms for Classification of Turbulent Structures A.A. WRAY & J.C.R. HUNT	95
10 Chaotic Advection by a Point Vortex in a Semidisk H. OKAMOTO & Y. KIMURA	105
II: Dynamo Theory	115
Self-Excitation and Reconnections of Magnetic Field in Random Flow S.A. MOLCHANOV, A.A. RUZMAIKIN & D.D. SOKOLOFF	117
Dynamo Action at Large Magnetic Reynolds Number in Spatially Periodic Flow with Mean Motion A.M. SOWARD & S. CHILDRESS	127
Generation of Steady and Oscillatory Magnetic Fields Through Inverse Cascades in Alpha-Effect Dynamos B. GALANTI, A.D. GILBERT & P.L. SULEM	138
The Helical MHD Dynamo A. GAILITIS	147
Relations Between Helicities in Mean-Field Dynamo Models K.-H. RÄDLER & N. SEEHAFFER	157
Boundary Effects on the MHD Dynamo in Laboratory Plasmas Y.L. HO & S.C. PRAGER	164
III: Relaxation and Formation of Discontinuities	177
High Velocities in Quasi-Static Magnetic Singularities E.N. PARKER	179
Current Sheet Formation in Magnetostatic Equilibria E.G. ZWEIBEL & M.R.E. PROCTOR	187
Minimum Dissipation States and Vortical Flow in MHD D. MONTGOMERY, J.P. DAHLBURG, L. PHILLIPS & M.L. THEOBALD	196
20 Island Formation in Magnetostatics and Euler Flows C.C. HEGNA & A. BHATTACHARJEE	206

Steady Motion with Helical Symmetry at Large Reynolds Number S. CHILDRESS, M. LANDMAN & H. STRAUSS	216
General Magnetic Reconnection in 3D Systems A. OTTO, M. HESSE & K. SCHINDLER	225
Spontaneous Formation of Electric Current Sheets in Astrophysical Magnetic Fields B.C. LOW	235
Current Sheets in Line-tied Magnetic Fields with Pressure G.B. FIELD	244
Flux Constraints on Magnetic Energy Release in a Highly Conducting Plasma J.J. ALY	251
IV: Two-dimensional and Quasi-two- dimensional Flows	263
Coherent Structures, Homoclinic Cycles and Vorticity Explosions in Navier-Stokes Flows B. NICOLAENKO & Z-S SHE	265
Isovortical Constraints on the Statistical-Dynamical Behaviour of Strongly Nonlinear Two-Dimensional and Quasi-Geostrophic Flow T.G. SHEPHERD	278
The Survivability of Vortices in a Two-Dimensional Fluid at Extremely High Reynolds Numbers D.G. DRITSCHEL	288
Isovortical Relaxation to Stable Two-Dimensional Flows G.F. CARNEVALE & G.K. VALLIS	294
30 A Simple Hydrodynamic System with Oscillating Topology A.M. OBOUKHOV, F.V. DOLZHANSII & A.M. BATCHAEV	304
Small-Scale Structures of Two-Dimensional MHD Turbulence H. POLITANO, P.L. SULEM & A. POUQUET	315
The Finite-Time Formation of Singularities on the Boundaries of Patches of Constant Vorticity T.F. BUTTKE	325

Steadily Rotating Quasigeostrophic Vortices in a Stratified Fluid S.P. MEACHAM	335
High-resolution Three-dimensional Modelling of Stratospheric Flows: Quasi-two-dimensional Turbulence Dominated by a Single Vortex P.H. HAYNES	345
Nonlinear Vorticity or Potential Vorticity Inversion M.E. McINTYRE & W.A. NORTON	355
V: Topology of Three-dimensional Flows	359
Analysis and Visualization of Flow Topology in Numerical Data Sets J.L. HELMAN & L. HESSELINK	361
Topological Changes of Axisymmetric and Non-axisymmetric Vortex Flows U. DALLMANN, & B. SCHULTE-WERNING	372
On the Topology of Three-dimensional Separated Flow Structures and Local Solutions of the Navier-Stokes Equations P.G. BAKKER & M.E.M. de WINKEL	384
Topological Structures of Separated Flows about a Series of Sharp-edged Delta Wings at Angles of Attack up to 90° WENHAN SU, MOUJI LIU & ZHIZHONG LIU	395
410 A General Classification of Three-dimensional Flow Fields M.S. CHONG, A.E. PERRY & B.J. CANTWELL	408
Construction of Frozen-in Integrals, Lagrangian and Topological Invariants in Hydrodynamical Models R.Z. SAGDEEV, A.V. TUR & V.V. YANOVSKY	421
A Natural Method for Finding Stable States of Hamiltonian Systems G.K. VALLIS, G.F. CARNEVALE & T.G. SHEPHERD	429
Third Order Invariants of Randomly Braided Curves M.A. BERGER	440
Topological Torsion, Pfaff Dimension, and Coherent Structures R.M. KIEHN	449
Steady Confined Stokes Flows with Chaotic Streamlines K. BAJER, H.K. MOFFATT & F.H. NEX	459

VI: Vortex Interaction and Reconnection	467
Collapsing Solutions in the 3-D Euler Equations	496
A. PUMIR & E.D. SIGGIA	
Vortex Nulls and Magnetic Nulls	478
J.M. GREENE	
Topological Aspects of Vortex Reconnection	485
M.V. MELANDER & F. HUSSAIN	
Effects of Incompressible and Compressible Vortex Reconnection	500
R.M. KERR, D. VIRK & F. HUSSAIN	
50 Oblique Collision of Two Vortex Rings and its Acoustic Emission	515
T. KAMBE, T. MINOTA, T. MURAKAMI & M. TAKAOKA	
Reconnection of Two Vortex Rings	525
S. KIDA, M. TAKAOKA & F. HUSSAIN	
Comment on Vortex Ring Reconnections	535
H. AREF & I. ZAWADSKI	
Two Bifurcation Phenomena for Inviscid Fluid Flow	540
M.S. BERGER	
VII: Homogeneous Turbulence	547
Anomalous Helicity Fluctuations and Coherence in Turbulence:	549
Theory and Experimental Evidence	
E. LEVICH	
Some Experimental Results on Velocity-Velocity Gradients	564
Measurements in Turbulent Grid Flows	
T. DRACOS, M. KHOLMYANSKY, E. KIT & A. TSINOBER	
An Experimental Study of Helicity and Related Properties in	585
Turbulent Flows	
J.M. WALLACE & J.-L. BALINT	
Reduction of Nonlinearity and Energy Cascade in Helical and Nonhelical	598
Turbulent Flows	
L. SHTILMAN & W. POLIFKE	
Vortex Filaments and Turbulence Theory	607
A.J. CHORIN	
Distortion of Material Surfaces in Steady and Unsteady Flows	617
N.A. MALIK	

60	Turbulent Stretching of Lines and Surfaces I.T. DRUMMOND & W.H.P. MÜNCH	628
	Entangledness of Vortex Lines in Turbulent Flows W. POLIFKE & E. LEVICH	637
	VIII: Inhomogeneous Turbulent and Convective Flows	647
	Coherent Structures of Mixing Layers in Large-Eddy Simulation P. COMTE, M. LESIEUR & Y. FOUILLET	649
	Topology of Coherent Structures and Flame Sheets in Reacting Mixing Layers R.W. METCALFE & F. HUSSAIN	659
	Topological Magnetoconvection W. ARTER	669
	The Nonlinear Break-up of a Twisted Magnetic Layer F. CATTANEO, T. CHIUEH & D.W. HUGHES	679
	Free Shear Flows: Symmetries and Topological Transitions of the Vorticity Field E. MEIBURG & J.C. LASHERAS	689
	Recirculation Zones in a Cylinder with Rotating Lid K.G. ROESNER	699
	Instabilities of Tertiary Flow in the Taylor-Couette System E. WEISSHAAR, F.H. BUSSE & M. NAGATA	709
	Experimental and Numerical Investigation of Natural Convection in a Cube with Two Heated Side Walls W.J. HILLER, ST. KOCH, T.A. KOWALEWSKI, G. de VAHL DAVIS & M. BEHNIA	717
70	Helicity Associated with Flow around Fluid Lumps and with Inhomogeneous Turbulence J.C.R. HUNT & F. HUSSAIN	727

IX: Special Topics	743
Fluid Networks	745
H. AREF & T. HERDTLE	
Wavelet Analysis of Coherent Structures in Two-dimensional Turbulent Flows	765
M. FARGE, M. HOLSCHNEIDER & J.-F. COLONNA	
Structure Formation in Self-gravitating Flows	777
J. LEORAT, T. PASSOT & A. POUQUET	
Attractors in Problems of Magnetohydrodynamics	782
O.A. LADYZHENSKAYA	
75 Postscript: Knots for Three-dimensional Vector Fields	787
R.S. MACKAY	
List of Participants	789
Author Index	798
Colour Plates	

Preface

The topic of this meeting was chosen in response to the developing interest in aspects of fluid mechanics and of magnetohydrodynamics that can properly be described as topological, rather than exclusively analytical, in character. On the one hand, there are purely kinematic problems such as the classification of possible streamline structures in three dimensions; the deformation of convected lines and surfaces in a prescribed flow field; and the relation between Lagrangian and Eulerian properties for both laminar and turbulent flows. On the other hand, there are dynamical problems such as the treatment of changes in flow topology associated with symmetry-breaking instability; topological invariants (e.g. the helicity invariant) associated with the Euler equations; the manner in which topological constraints may be broken (rapid reconnection, or cut-and-connect mechanisms) in high Reynolds number flow; the influence of helicity fluctuations, or more generally of any departure from reflexional symmetry, in the fundamental dynamics of turbulence; and the manner in which singularities of the Euler equations may develop within a finite time. Underlying all these problems is a desire to identify structures (if any) that are characteristic of fully developed turbulent flows, with a view to constructing an improved statistical theory of turbulence.

Between these kinematic and dynamical aspects lie a fascinating range of problems that arise in the magnetohydrodynamics (MHD) of highly conducting fluids: relaxation to magnetostatic equilibrium under the constraint of conserved field topology; formation of discontinuities during such relaxation; spontaneous magnetic field growth (i.e. dynamo action) due to fluid motion either with or without helicity; and, particularly, the phenomenon of 'fast dynamo action', tantalisingly difficult to describe mathematically, and yet almost certainly the root cause of solar and stellar magnetism.

In focussing on these topics, the Symposium stimulated a fruitful interaction between, on the one hand, fluid dynamicists working on fundamental mechanisms of vorticity dynamics and turbulence, and on the other, plasma physicists with expertise in problems of magnetic field topology and reconnection processes. The power of topological arguments remained at the forefront of the discussion throughout the week, emphasis being placed on global, rather than local, characteristics of field and flow phenomena.

There were 119 registered participants at the Symposium, representing 15 countries. Accommodation was provided at Pembroke College, Cambridge. The scientific programme consisted of Lecture sessions, supplemented by three Poster-discussion sessions. All of the papers presented have been refereed, and where necessary revised, and all are included in this Volume. We apologise for its consequent size, but we believe that it encompasses an impressive body of work which reflects a widespread desire to understand qualitative properties of fluid-dynamical systems as an essential preliminary to quantitative and computational analysis.

H.K.M.

A.T.

30 September 1989