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# Topological Aspects of the Dynamics of Fluids and Plasmas

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*My soul's an amphicheiral knot  
Upon a liquid vortex wrought  
By Intellect, in the Unseen residing.  
And thine doth like a convict sit,  
With marlinspike untwisting it,  
Only to find its knottiness abiding...*

From a Paradoxical Ode to Hermann Stoffkraft  
by James Clerk Maxwell  
c. 1873

## Preface

This volume contains papers arising out of the program of the Institute for Theoretical Physics (ITP) of the University of California at Santa Barbara, August–December 1991, on the subject “Topological Fluid Dynamics”. The first group of papers cover the lectures on Knot Theory, Relaxation under Topological Constraints, Kinematics of Stretching, and Fast Dynamo Theory presented at the initial Pedagogical Workshop of the program. The remaining papers were presented at the subsequent NATO Advanced Research Workshop or were written during the course of the program. We wish to acknowledge the support of the NATO Science Committee in making this workshop possible.

The scope of “Topological Fluid Dynamics” was defined by an earlier Symposium of the International Union of Theoretical and Applied Mechanics (IUTAM) held in Cambridge, England in August, 1989, the Proceedings of which were published (Eds. H.K. Moffatt and A. Tsinober) by Cambridge University Press in 1990. The proposal to hold an ITP program on this subject emerged from that Symposium, and we are grateful to John Greene and Charlie Kennel at whose encouragement the original proposal was formulated.

Topological fluid dynamics covers a range of problems, particularly those involving vortex tubes and/or magnetic flux tubes in nearly ideal fluids, for which topological structures can be identified and to some extent quantified. Just as vortex tubes and flux tubes can reconnect as a result of weak diffusion, so it happens that separate strands of scientific inquiry can “reconnect” in a most fruitful and stimulating way as a result of the sort of inter-diffusion that ITP promotes. The separate disciplines of topology, plasma magnetohydrodynamics (MHD) and high Reynolds number fluid dynamics are linked by certain potent analogies that aid this diffusion; and certain results and techniques from knot theory and from MHD have already stimulated fruitful lines of enquiry in the more classical areas of fluid dynamics, particularly Euler flows, instability, Lagrangian chaos and turbulence.

These areas are all represented in the papers collected in this volume, which we hope will provide the reader with a good picture of much current research in this broad field.

We are extremely grateful to Jim Langer and the staff of ITP, which is supported by NSF Grant PHY89-04035, for hosting this program and for providing an ideal research environment. Our thanks also to Lorraine Shallenberger and Anne Braddock who took good care of all practical arrangements in relation to the workshops, and to Darla Sharp-Fitzpatrick and Donna Freet for their skillful preparation of the camera-ready copy for this volume.

The Editors,  
Santa Barbara  
December 1991

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