THE 1993 DIRAC MEMORIAL LECTURE

QUANTUM MECHANICS WITH THE GLOVES OFF

given by Sidney Coleman (Harvard)

Introduction by Professor P. Goddard FRS

This is the 8th Dirac Lecture in a series which began in 1986 with a lecture by Richard Feynman. Today's lecture is given by Sidney Coleman. Professor Coleman took his first degree at Illinois Institute of Technology. The choice was made under the possibly mistaken apprehension that the University of Chicago would be too liberal an institution for him. For his PhD, he moved to Caltech where his thesis advisor, Murray Gell-Mann, described him as the best student for many years - possibly nearly as good as Dick Feynman. Whereas Richard Feynman is reputed to have said that he was the best student for many year, probably better than Murray Gell-Mann. His first position - at least outside the auto wreckers yard - as he puts it, was at Harvard where he has been at the Physics Department since 1961.

In his classic book 'Principles of Quantum Mechanics', Dirac produced an immediate, definitive and, in some ways, profoundly simple treatment of one of the greatest revolutions in physics. Similarly, some decades on, Professor Coleman's biannual lectures at Erice charted with unsurpassed clarity and depth our growing understanding of symmetry in quantum field theory. Today the subject is 'Quantum Mechanics with the Gloves Off.'

The Lecture

When Professor Goddard invited me to give this talk he said it was to be on a subject that would have interested Dirac. It's good that he didn't ask for a talk that would have interested Dirac! I thought about it for a while and decided that I'd always wanted to give a talk on Quantum Mechanics, what a strange thing it is, and exactly *what* strange thing it is. He said

“Fine, that is a wonderful subject,” and he wanted a title and I suggested the title ‘Quantum Mechanics in Your Face’. ‘In Your Face’ is an American locution which derives from the game of basketball, from the playing style of immobilising an opposing player without actually committing violence on his body. It has been generalised to mean a flamboyant, confrontational style, but Professor Goddard said the locution was too American and a British audience might even suspect it was obscene! I said, all to the good since one of the themes of this talk was how people get confused by trying to using classical concepts in a quantum mechanical situation but that title was rejected. I thought I would use ‘It’s Quantum Mechanics - Stupid’ which is a play on the recent American presidential campaign - but again it was rejected as too American! So I decided for something really British ‘And now for something completely different: Quantum Reality’, but that also was rejected and we are left with the title we have now.

What I will do in this talk is first give a quick review of the fundamentals of quantum mechanics. Not the Copenhagen interpretation or the standard interpretation but, when I was trying to think of what to write down the other night, I thought of the fact that when architectural historians describe the sort of housing that developers throw up by the dozen and it’s not really in this style or that style, they call it vernacular architecture. I’m going to give you a version of vernacular quantum mechanics. A very quick review of the fundamental principles of quantum mechanics, just to get us all speaking the same language, even though it’s not a language of great precision, and also as a sheep/goat separator - if you’re hopelessly lost at this stage - politely get up and leave, because the rest will be baffling to you. The next topic I want to discuss is a rerun of the sort of reasoning John Bell used to put paid to the idea of hidden variables 20 odd years ago. There has been a recent improvement that has been widely publicised, but still not widely known, by Greenberger, Horne and Zellinger that makes Bell’s arguments even clearer and more striking. Finally, I will turn to the much vexed problem of the measurement process in quantum mechanics and make a small number of hopefully cogent remarks about that. Absolutely nothing in this lecture is in any way original. Everything I will say, with the exception of slips of the tongue, and little jokes and things like that, will be found in the literature. Of course the opposite of everything I say can also be found in the literature. So my role in this process will not be totally passive. I will also say absolutely nothing about either classical or quantum gravity. The quantum mechanics I discuss will be quantum mechanics in flat space.

Part 1

Now, vernacular quantum mechanics in two minutes. As you will find it in Dirac's book the state of a physical system at a fixed time is described by a vector $|\psi\rangle$ in a Hilbert space normalised to have unit norm $||\,|\psi\rangle||^2 = \langle\psi|\psi\rangle = 1$. It evolves in time, with one exception, which we will come to shortly, according to the famous Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle\,,$$

The time derivative of $|\psi\rangle$ is given by the Hamiltonian operator H, some self-adjoint linear operator, acting on the vector $|\psi\rangle$. These are the two rules of quantum dynamics. The next two rules are, so to speak, interpretational rules. Some self-adjoint operators, maybe all, maybe not, depending on the theory, are 'observables', a brand new word in the theory. If $|\psi\rangle$ is an eigenstate of the observable A with eigenvalue a,

$$A|\psi\rangle = a|\psi\rangle$$

then we say the value of A is certain to be observed to be a. Strictly speaking, this is just a definition, just like Newton's Second Law, $F = ma$, is a definition of F. This is a definition in terms of quantum mechanics of what we mean by the word 'observable' and the words 'the value of A is certain to be observed to be a'. Newton's Second Law of course, although by strict logical standards just a definition, has an implicit promise in it that Newton will then go on when he develops specific dynamical theories like the theory of gravity to tell us how to compute force rather than tell us how to compute something which you will identify with, say, the fourth derivative of particle positions with respect to time. It is a sign that force is going to be important. Here, since observables and words of this kind occur in pre-quantum and classical physics or, even before that, in the language of everyday speech, there is an implicit promise that there will be developed a theory of how, under appropriate approximations, quantum mechanics becomes classical physics. When that theory is developed then these words will be revealed, under the circumstances in which those approximations are valid, to correspond to the normal classical meaning of these words. I'm not going to develop the theory of the classical limit, but I just wanted to make that remark here.

Now the fourth part of the theory is much more violent, because it introduces a totally new kind of process of measurement of which I will say much more later in part three of the lecture. Every measure of the observable A yields as a result one of the eigenvalues of A. For simplicity, I will assume that A has only a discrete spectrum, the generalisations in the case of continuous spectrum are straightforward. Anyway, anyone who is not baffled by this lightning development knows what to do in the case of a continuous spectrum. The probability of finding one eigenvalue or another, a, is the norm of a projection operator $P(A;a)$ for the eigenspace associated with this eigenvalue a acting on the state vector $|\psi\rangle$,

$$||P(A;a)|\psi\rangle||^2 \ .$$

After A has been measured and if you find out the value is a, then the state of the system has been changed to just the sum of eigenvalue eigenvectors with eigenvalue a. Then the wave function, or I should say the state vector, has been renormalized so once again it has norm one

$$\frac{P(A;a)|\psi\rangle}{||P(A;a)|\psi\rangle||} \ .$$

Unlike the previous discussion this cannot be thought of just as a matter of making convenient definitions and postponing the justification of argument that the definitions are reasonable to a later date. This is, uniquely, a second evolution process not the same as time evolution according to the Schrödinger equation. It is not linear, it is not deterministic, it is probabilistic, it is something else, it is wholly other, I think. The Totally Other, was it Marcion who described God that way? Anyway he was a heretic so it doesn't matter. This is something totally other, apparently, from a normal kind of evolution. Nevertheless, it is a standard part of vernacular quantum mechanics and we will accept it for the moment, although, as I promise you, in part three, I will look at things much more closely. Okay, any trivial questions? Deep questions will not be entertained, but if someone asks a question like, "Is that a little a or an α?" I'll be happy to answer it. It's ridiculous to get lost about little things like that when there are so many more places where you can get lost so much more satisfyingly.

Part 2

Now, I'm not going to give a complete bibliography, as I said, the literature on this interpretational stuff is enormous, but the essential papers for our purposes are the following

A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47 (1935) 777.

J.S. Bell, Rev. Mod. Phys. 38 (1966) 447.
Physics 1 (1964) 195.

M.D. Mermin, Physics Today, April 1985, p.38.

D. Greenberger, M. Horne, A. Shimony, A. Zellinger,
Am. J. Phys. 58 (1990) 1131.

N.D. Mermin, Am. J. Phys. 58 (1990) 731.
Physics Today, June 1990, p.9.

We are now going to give a rerun of John Bell's reasoning which, of course, derives from the famous devastating criticism of quantum mechanics launched by Einstein, Podolsky and Rosen. Bell had looked at the question of hidden variables in quantum mechanics, although his paper on this was published second it is, in fact, the predecessor of his paper in Physics. That tells you how long it takes you to get things refereed for Rev. Mod. Phys., I have several manuscripts ageing on my desk at the moment. In the Physics paper he came up with the astonishing Bell inequality and I'll give a version of that later. This was improved on, not philosophically but pedagogically, by a paper that influenced me a lot by David Mermin. Then there came Greenberger, Horne and Zellinger in a rather brief communication that appears in a conference proceeding but you can get a better idea of what is going on from a longer paper that they published in the American Journal of Physics with Shimony. Then Mermin came along again and made everything pedagogically wonderful and what I'm going to give you here is my own polish on Mermin's ideas.

Now, this whole thing occurs within the context of what is sometimes called hidden variables or alternatively the possibility of a realistic interpretation of quantum mechanics. The only way I'll take up all the time that's been allotted to me, with such simple stuff, is by starting at the very beginning and telling you what that is. I like to explain it by imagining a hypothetical physicist, whom I call Dr. Diehard, who neither believes in nor understands quantum mechanics. He has been around for quite a long time now since he was doing physics when quantum mechanics was discovered in the mid 1920's, but although he is old he is vigorous and he just does not believe any of this quantum nonsense. Furthermore, you can't convince him by displaying the marvellous agreements of quantum mechanics with experiment, the Lamb shift or the anomalous magnetic moment of the electron, which agrees with experiments to all those decimal places, because he doesn't understand quantum mechanics. And whenever you try and explain a quantum mechanical calculation to him, his mind shuts off rather like my mind when I find myself in a seminar on string theory, for example. So, if you've got to convince him of something it has to be in his own language, not quantum mechanical language. Well, he feels deep down it's all classical physics and there is some crazy non-linear theory which he or probably some bright young man will think up some time that explains all these quantum effects in terms of classical physics. Possibly a reasonable position in 1920 when he began his scientific career and he holds on to it. Now you say "No, it can't be so", there cannot be a classical theory that makes the same prediction as quantum mechanics, because quantum mechanics is probabilistic. If I prepare an electron in any eigenstate of σ_x and subsequently measure σ_z, a gedanken experiment which we will talk much about later on, I have a 50% probability of finding spin up and a 50% probability of finding spin down and I cannot predict which one I am going to find. That is probabilistic quantum mechanics in contrast to deterministic classical physics. Dr. Diehard said, "That's absolute nonsense." He says that a lot. He says, "After all Cardan founded probability theory in the renaissance and he certainly didn't need to know anything about these silly quanta to do it." When I flip a coin that is a probabilistic event if it is a fair coin, whether it comes up heads or tails. Probability can certainly arise in a completely deterministic theory, it arises simply because you are not good enough at describing the initial state of the system, which in the case of a coin flip is the coin and my thumb and maybe my nervous system and everything, well enough to make an accurate prediction, like these chaos things I hear people talking about sometimes. The result ends up being random just because of ignorance. More precisely, Dr. Diehard believes that those things we call quantum observables, those nonsensical σ_x, σ_z and σ_y are, in reality, thumbprints of some other observables, which we can think of as little gears and wheels inside the electron. And because there are perhaps little gears and wheels inside both the electron and the apparatus we claim is measuring this bit of the electron, whatever that means, there is some probability distribution, some measure for these hidden variables which may, in fact, be huge in number and involve the 'apparatus' as well as the 'system'. I have indicated them by a single letter α and there is some probability measure $d\mu(\alpha)$ here and the probability distribution for a is obtained by integrating over all the α's which gives the desired result

$$\mathrm{Prob}\{A \leq a\} = \int \theta(a - A(\alpha))\, d\mu(\alpha) \ .$$

Here I have written this in terms of the θ function, which I presume is not a Jacobi elliptic function. This is that function which is one when the argument is positive and zero when the argument is negative.

So, that's the fundamental idea of all hidden variables or so called realistic theories. Really, when we measure the electron the variable which we in our quantum delusion call σ_z has some definite value because the α's have some definite value. Just as when I flip the coin, whether it's going to land heads or tails is completely determined the moment it leaves my thumb. Or, if I use a mechanical coin-flipping device, from the moment I start the machine. But, because of our ignorance, we introduce probability. Small errors in the initial condition can lead to large errors in the outcome and therefore we discuss things probabilistically. Now, the second objection people would have to saying that things are really quantum mechanical is not to talk about uncertainty, but to talk about the existence of non-commuting observables. If we had the electron prepared in an eigenstate of σ_x of spin in the x direction which does not commute with spin in the z direction, we find, for example, $\sigma_x = +1$. If we measure it again, assuming no external force has been acting on it, it still has $\sigma_x = +1$ indefinitely. If, however, we measure σ_z randomly we get either $+1$ or -1 and then if we attempt to measure σ_x again we find we get a different answer than our original one. We may get $+1$ but we may get -1. In normal quantum mechanics this is ascribed to the existence of non-commuting observables and, in fact, it was precisely this that lay in the back of John von Neumann's original proof that you couldn't possibly explain quantum effects on the basis of hidden variables. This proof was, in fact, refuted by John Bell in the first of the papers I cited.

Dr. Diehard is not swayed by this at all, he says "Non-commuting observables, that's absolute nonsense!" Of course, we're great clumsy oafs, when we attempt to measure something, we may well interfere with all of those hidden variables in an uncontrolled way and that may effect subsequent measurements. I have heard people, he would say, who know nothing of quantum mechanics talking about this, anthropologists talk about how you disturb a tribe by measuring it. They have some funny name for it I think they call it the uncertainty principle. My friends the social psychologists report that when they conduct a social survey the answers they get may depend on the order of the questions - sounds very much like σ_x and σ_y. You don't need quantum mechanics for that, measurements can interfere with each other. Well, in fact, if Dr. Diehard sticks to this position, then there is no way of refuting him because there are examples of hidden variables theories that function just as he's described, that mimic all the predictions of quantum mechanics. John Bell's brilliant stroke was to explain that there is a third thing that Dr. Diehard might believe and, if he believes this, then we have him. That third thing is that space-like separated measurements, measurements conducted in regions sufficiently widely separated in space and sufficiently close together in time so that light cannot go from one region to another, cannot interfere with each other. If you have space-like separated measurements between two things you really are measuring the simultaneous probability distribution of both of them, not the probability distribution of one as affected by the other. The argument derives from the fact that we do not believe that influence can propagate backwards in time. Here I have two measurements, they are blobs in space and time, here is time and here

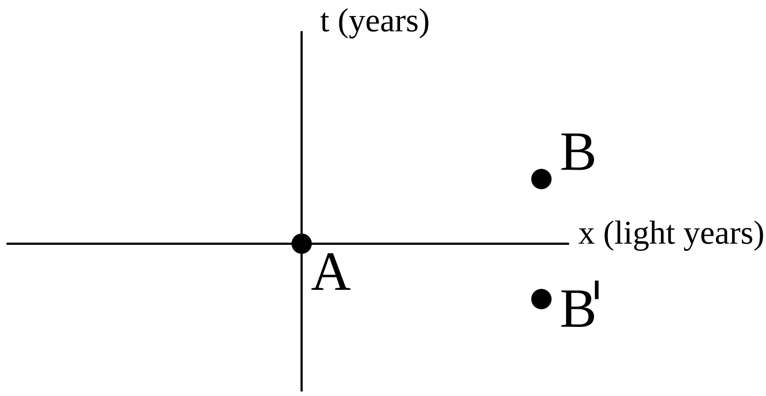

Figure 1: *Space-time diagram for spacelike-separated measurements*

is space, drawn on a scale so that the speed of light is one. One of them is conducted at the origin of coordinates, it's called A the other one is done as a somewhat later time and space-like separated, called B. Dr. Diehard may believe that the measurement of A will effect the subsequent results of measuring B, but he cannot believe, unless he believes that influence can propagate backwards in time, that B can effect A, because B is later than A. On the other hand, by making a Lorentz transformation by changing my coordinates, B put in another Lorentz frame looks like B', another point on the same hyperbola and as you see, B' is thoroughly earlier than A. Therefore we have already established that B cannot effect A. We also know that A cannot effect B', A is unchanged by the Lorentz transformation it's at the origin, but B' is the same as B. So therefore, A cannot effect B and B cannot effect A. You can't send influences backwards in time, you can't send influences faster than the speed of light, if you believe the Lorentz invariance. Therefore, if we do not believe in superluminal transmission of information, if we do not believe in acausal transformation of influence, influence going backwards in time, then we must believe that such measurements do not interfer with each other.

We now have Dr. Diehard. Up to this point I could have been talking about John Bell's original work, because we have a situation where we can demonstrate that the predictions of quantum mechanics for simple systems are in contradiction with the Diehard postulates as I have laid them down. The original Bell inequality was an inequality that was rather complicated to prove. The principles can be upheld experimentally and ten years ago was done experimentally by Alain Aspect, as you probably heard quantum mechanics won! The Greenberger, Horne, Zellinger proposal is even more striking in its consequences but it involves, as I will explain, a three particle correlation which makes for a harder experiment. However, atomic experimenters are very clever, maybe ten years from now it will be done and I am willing to take bets at this moment that, once again, quantum mechanics will win! Let me explain it to you just as a pedagogical experiment. This is from Dr. Diehard's experimental proposal, this is the arrangement of the experimental setup.

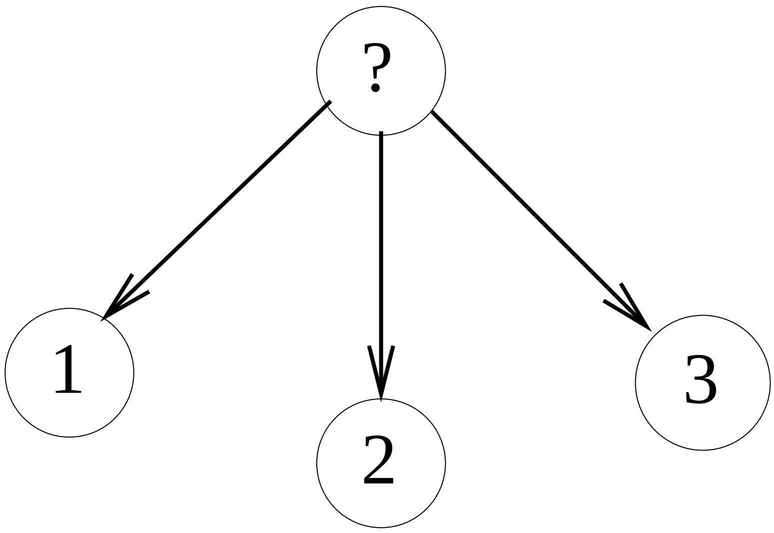

Figure 2: *The Dr. Diehard Experimental Proposal*

There is a region, here called ?, which I and one of my experimental colleagues control, and there are three of Dr. Diehard's collaborators in three experimental regions, one, two and three. The scale is rather large. One inch equals one light minute, so it takes a couple of minutes for light to get from any one experimental region to another. Dr. Diehard is informed that some thing, he knows not what, is going to be sent to these three guys, three particles we shall call them, he may not even believe in particles - three things - are going to be sent to these three guys such that they will arrive every minute on the minute. These three guys are each equipped with the experimental apparatus which is shown here in a masterpiece of twentieth century draughtsmanship, The Acme "Little Wonder" Dual Cryptometer.

The device consists of an on or off switch which can be set to measure A

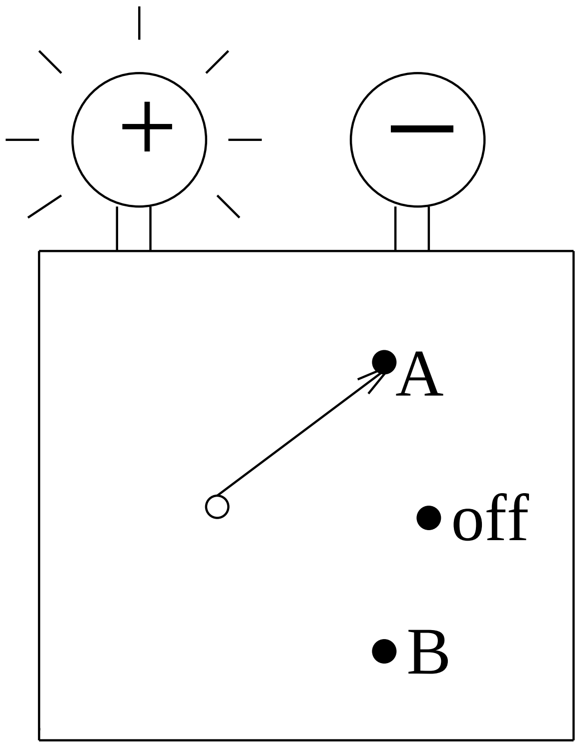

Figure 3: *The Acme "Little Wonder" Dual Cryptometer*

or measure B - it is a cryptometer, you don't know what A or B are. When the event occurs, every minute, one of two of the electric lamps lights up, either the one that is labelled plus or the one that is labelled minus.

A few seconds before the appointed arrival time of the particles these guys, Dr. Diehard's collaborators, decide whether they are going to measure A or B, and then one or the other of the two lamps will light up and then they do the same thing again and again and again. They obtain in this way a sequence of experimental records, about which I will say more shortly.

The experimental records look something like this,

$$\begin{array}{llll} \text{expt. 1} & A_1 = 1 & B_2 = -1 & B_3 = -1\ , \\ \text{expt. 2} & A_1 = 1 & A_2 = -1 & B_3 = -1\ , \\ \text{expt. 3} & B_1 = 1 & B_2 = 1 & A_3 = 1\ . \end{array}$$

In the first set of measurements the first person has chosen to measure A and he has found the value to be $+1$. The second person has chosen to measure B and he has found the value to be -1. The third person has also chosen to measure B and he has found a value of -1. Dr. Diehard recorded the results in this way because he believes he are really measuring something about the system just like the coin has really ended up heads or tails. Whatever has arrived at experimental station one has a value of $A = 1$. Whatever has arrived at experimental station two has a value of $B = -1$ and etc. That is the real value they really have, they don't have to worry about the effects that A and B have on each other. One, two and three are making measurements more or less simultaneously, because they are all space-like separated from each other. So, he obtains a string of these records. I emphasise, he does not know what A and B are, and he does not need to know what A and B are. For all he knows, we are sending bolts of psychic force at those three apparatus or we are sending blood samples and A is measuring high blood glucose and B is measuring high blood chloresterol, two chemical tests which, by the way, would interfere with each other if done in succession. He does the usual statistical tests experimentalists do when they convince themselves that they are measuring the same independent random events drawn from the same statistical sample over time, as we would say if we used quantum language, that they are always getting the same initial state. After having determined that he has taken ten million runs, he looks at what happens to $A_1\, B_2\, B_3$. He finds this product is always plus one, that is to say some of the times these three numbers are plus one, some of the time one or the other of them is plus one and the other two are minus one, but the product is always plus one. Since he believes these are real properties of the system which he is really measuring there is a definite value of A and B associated with whatever is arriving at stations 1, 2 and 3. I'm going to labour the obvious because the unobvious is going to happen shortly, he believes that this product is always therefore $+1$. Likewise, of course, he finds the same thing for $B_1\, A_2\, B_3$ and $B_1\, B_2\, A_3$. The thing is obtained by permuting $1, 2$ and 3. Now, since B is always $+$ or -1 and since he has discovered that these three quantities are always $+1$, he deduces by the magic of modern arithmetic that $A_1\, A_2\, A_3$ is $+1$ simply by taking the product of these three things. Is there a single person in the audience who does not see that the Dr Diehard presented with the experimental situation I have described and believing in the three assumptions which I have elucidated will inevitably be driven to this conclusion by his data? If there is, speak up. Ashamed, huh? Okay. We'll get you when it's time for the Tripos.

Okay, well lets go behind the scenes and find what we who believe in quantum mechanics, believe we have been sending to the Dr. Diehard. Well, I'm sure you will be disappointed to learn it is not a bolt of psychic force or blood samples. We are sending three spin one-half particles to the three stations in a state that has been prepared so it is a linear superposition, with a minus sign, of the state with all three spins up and the state with all three spins down,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[\, |\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle \,\right] \ .$$

A is in fact the observable σ_x for the particle that arrives at the appropriate station and B is the observable σ_y, those are certainly quantities which are always measured to be plus or minus one if quantum mechanics is right. Let's check if the first experimental result of Dr. Diehard is true, that they always get $A_1\, B_2\, B_3 = +1$. This will be true in quantum mechanics if this state $|\psi\rangle$ is an eigenstate of the operator $A_1\, B_2\, B_3$, three commuting operators, with eigenvalue $+1$. That is, of course, if we decode it, $\sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}$. Well, σ_x turns up $|\uparrow\rangle$ into down $|\downarrow\rangle$, σ_y turns up into down with a factor of i or possibly $-i$, I can never remember, but it doesn't matter because $\sigma_y^{(2)}$ does the same thing as $\sigma_y^{(3)}$. $1 \times i \times i = -1$ so that successfully changes the first term in $|\psi\rangle$ into the second term, $\sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}|\uparrow\uparrow\uparrow\rangle = -|\downarrow\downarrow\downarrow\rangle$, with a minus one coefficient and likewise for acting on the three downs,

$$A_1\, B_2\, B_3|\psi\rangle = \sigma_x^{(1)}\sigma_y^{(2)}\sigma_y^{(3)}|\psi\rangle = |\psi\rangle \ .$$

Of course, it's the same thing for any other combination $B_1\, A_2\, B_3$ or $B_1\, B_2\, A_3$, it is a completely symmetric argument. So we do indeed find the first Diehard collaboration experimental result that these products are always $+1$, What, about the second prediction? Well, that's three σ_x's. We know what that does - that turns up into down with no factor of i, I told you that. Therefore, you have $|\uparrow\uparrow\uparrow\rangle$ turned into $|\downarrow\downarrow\downarrow\rangle$ without the i's and this gives -1 as the eigenvalue,

$$A_1\, A_2\, A_3|\psi\rangle = \sigma_x^{(1)}\sigma_x^{(2)}\sigma_x^{(3)}|\psi\rangle = -|\psi\rangle \ .$$

Classical reasoning, hidden variable reasoning, predicts the result is always $+1$. Quantum mechanics predicts the result is always -1. You can't get a better contradiction than that! No probabilities, no correlations, not that it happens this way 70% of the time and quantum mechanics says it should happen 68% of the time. Hidden variable reasoning, the three Diehard postulates which I tried to make explicit, predict $+1$, quantum mechanics predicts -1.

Now, please notice the logical structure of this argument. We have said that the result of this experiment, on which I am prepared to bet a large amount of money, will come out like quantum mechanics predicts, it's just three stupid spin one-half particles. The result of this experiment cannot be explained by little gears and wheels obeying the rules of deterministic evolution unless those little gears and wheels can communicate with each other faster than the speed of light or, equivalently, backwards in time, as they can, for example, in the Bohm-de Broglie hidden variable theory. Or else you don't have Lorentz invariance. In the popular literature, I will not embarrass people, I have met some of them, they're nice guys, they just talk to the wrong people, in the popular literature you will find statements that quantum mechanics implies the transmission of information faster than the speed of light, and that is what John Bell proved. And that's completely wrong! That's got it all upside down. You show if you have quantum mechanics you don't need transmission of information faster than the speed of light. If you have classical mechanics, if you try and explain in terms of these primitive quasi-classical concepts, then you need transformation of the information faster than the speed of light. It's not quantum mechanics implies it - it's *either* quantum mechanics *or* superluminal transmission of information.

The famous spooky action at a distance that Einstein complained about in his famous letter to Max Born is there only if you try and make a classical description of a quantum mechanical process. There aint any spooky action at a distance in quantum mechanics. This makes that absolutely clear, the spooky action at a distance comes about if you try to understand in classical terms something that is inherently non-classical. One of the morals of this is to take quantum mechanics seriously.

Part 3

The next part is about taking quantum mechanics even more seriously and eliminating the measurement process, which seems crazy. For our purposes the critical papers are

J. von Neumann, Mathematische Grundlagen der Quantenmechanik (1932).

H. Everett III, Rev. Mod. Phys. 29 (1957) 454.

J. Hartle, Am. J. Phys. 36 (1968) 704.

E. Farhi, J. Goldstone, S. Gutmann, Ann. Phys. 192 (1989) 368.

The arguments here descend from John von Neumann. A critical step was made by Hugh Everett III who as far as I can find did nothing else and the latter parts of what I talk about are derived from papers by Jim Hartle and by Eddie Farhi, Jeffrey Goldstone and Sam Gutmann. Let's describe the measurement process in accordance with the analysis in the last chapter of von Neumann's famous book.

We prepare a electron in a σ_x eigenstate a linear superposition of spin up and spin down

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow\rangle + |\downarrow\rangle\big)$$

and I choose to measure σ_z and the famous non-deterministic reduction of the wave function, the fourth thing I talked about in my part one, takes place with equal probability. I can't tell which it will be. I measure σ_z, it either goes into a state of pure up $|\uparrow\rangle$ or pure down $|\downarrow\rangle$, correlated with the result of the measurement,

$$|\psi\rangle \longrightarrow \begin{cases} |\uparrow\rangle \\ |\downarrow\rangle \end{cases} \quad \text{with equal probabilities.}$$

Now, in fact, that's an idealisation because I don't have the ability to directly observe the spin of the electron. I have weak eyes. So, actually there is a measuring apparatus involved, something perhaps like the Acme Cryptometer with a little lightbulb that lights up which I can observe. In the first place, we have to consider a larger physical system which not only has the spin of the electron but also the measuring apparatus with states which

I'll call $|M\rangle$. We start out with the same superimposition of electron up and electron down, in either case the measuring apparatus is in its neutral groundstate $|M_0\rangle$ not doing anything,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|\uparrow, M_0\rangle + |\downarrow, M_0\rangle\big)$$

Then the electron interacts. If it's spinning up the measuring apparatus goes into a state $|M_+\rangle$ and the plus lightbulb lights up, if it's spinning down, the measuring apparatus goes into the state $|M_-\rangle$, easy to distinguish. That is what makes a good measuring apparatus. I don't look directly at the electron, I look at the measuring apparatus, but if I find the plus lightbulb on, the whole thing goes into the plus state which means the electron has spin up and correspondingly for spin down, the same thing, with equal probability,

$$|\psi\rangle \longrightarrow \begin{cases} |\uparrow, M_+\rangle \\ |\downarrow, M_-\rangle \end{cases} \quad \text{with equal probabilities.}$$

Now, let's complicate matters. Let's lengthen the chain. I am unfortunately giving the Dirac Lecture so I don't have time to look at the measuring apparatus but I have brought with me as accompanied baggage my ingenious robot, Gort. I say "Gort, go look at the measuring apparatus and see which way the electron is spinning and then come back to me." Well, it's the same story, Gort is just a mechanical system, although he is a lot more complicated than either the electron or the measuring apparatus. So the whole thing ends up in a superimposition of two states $\frac{1}{\sqrt{2}}[\,|\uparrow, M_+, G_+\rangle + |\downarrow, M_-, G_-\rangle\,]$ one state corresponding to electron spin up, the measuring apparatus saying up, a little piece of ram chip in Gort's head has the message in it 'the electron is spinning up' plus the same thing with everything that is up replaced by down. Gort comes and visits me and I am an observer. I say "Hey Gort, what was the electron doing?" and then bang goes the whole wavefunction, electron, measuring apparatus, Gort makes this non-causal leap into the same state. Unfortunately, when Gort comes to the door, he can't get to me because he can't make his way down the stairs, he rolls around on rollers. So he finds a passerby and says "Hey, would you come in and tell Sidney which way is the electron spin". Well, the passerby is awfully complicated, but he is just another mechanical system, if we really believe quantum mechanics all the way down. Therefore the whole thing is electron up, measuring up, Gort up, passerby has in his head, 'Gort told me to give this message to Sidney, "Electron is spinning up" ' plus the same thing down. The passerby reaches me, I say "What's up" he says "The electron". Wango, the whole wavefunction collapses, reduction of the wavepacket etc., just like you read in books. What happens if when he's on the way to the lectureroom I suffer a heart attack, I've been having these chest pains recently, I suffer a heart attack and die? What has happened to the whole thing?

I read about this chain in von Neumann's book when I was a graduate student and I thought a lot about this. I thought the obvious only conclusion, flattering as it is, somehow satisfying, is solipsism, that I am the only person in the world who has the ability to reduce wavepackets, nobody else can do it. I met Aharonov when I was a postdoc, he was also a postdoc, I was at Harvard, he was at Brandeis. I explained this position to him since I knew he was an expert in these matters and, even at that stage, he was smoking these enormous cigars, a sort of quantum George Burns. He said "Tell me, before you were born did your father have the ability to reduce wavepackets?" Clearly, there is something profoundly unsatisfying about this.

I will argue that the resolution to this profound dissatisfaction, and this is in fact Everett's position, is that there is no special process. There is no reduction of the wavefunction, there is no indeterminancy and, in fact, there is nothing probabilistic about quantum mechanics. The indeterminancy and the probability area are just like the superluminal transmission of information in part two. Something that appears when you try and make a measurement is just deterministic evolution according to Schrödinger's equation. Now this position has not been well-received. Here I mention an early and a later reviewer, I could have put in a hundred of them. Schrödinger in 1935 said this was a ridiculous position. Of course, he actually considered it in the famous paper about the cat which ends up in a coherent superimposition of being dead or alive. Same sort of situation, just a little bloodier, and he said we can consider this a ridiculous conclusion. I also found a paper by Zurek in 1991, who has done very good work on interactions of systems with the environment, that is the effect of the measuring apparatus, who said "This is obviously not a tenable position because why do I the observer perceive only one of the outcomes." Now in the time remaining I will explain why I the observer only see one of the outcomes and I will also explain how probability gets into it.

First, I will have to take care of indeterminancy. Now, of course to explain why I the observer only see one outcome, it's got to be quantum mechanics all the way down otherwise you're cheating. So I have to assume there is a mechanical theory of consciousness and there is some state of the observer's head that corresponds to feeling he knows definitely that the electron is spinning in one way or another. Therefore, I introduce a Hilbert space of states $\mathcal{H}_S$ of the observer, S for Sidney, nothing shy. I introduce D a definiteness operator and it is a projection operator, the usual quantum mechanical kind, it says yes or no. Do you feel you are in a definite state, knowing what the spin of the electron is? If you answer yes that is an eigenstate of D with eigenvalue $+1$, while orthogonal states are eigenvectors of D with eigenvalue 0,

$$D|S\rangle = |S\rangle,\ |S\rangle \in \mathcal{H}_S,\ \text{if the observer feels he has perceived a definite outcome,}$$

$$D|S\rangle = 0\,,\ \text{on states orthogonal to these.}$$

Now, let's suppose we start out with an initial state I call $|\psi_i\rangle$ where the electron is spinning up. The electron is spinning up $|\uparrow\rangle$, the measuring apparatus is in its ground state $|M_0\rangle$, the observer is in a passive state the normal condition $|S_0\rangle$. By normal Schrödinger time evolution this evolves into a state where the apparatus says the electron is spinning up $|M_+\rangle$, the observer has looked at it and is in state $|S_+\rangle$. He would certainly say in that case the electron is spinning up, if any case is definite, this must be an eigenvalue of D with eigenvalue $+1$,

$$|\psi_i\rangle = |\uparrow, M_0, S_0\rangle \longrightarrow |\psi_f\rangle = |\uparrow, M_+, S_+\rangle\,, \quad D|\psi_f\rangle = |\psi_f\rangle\ .$$

Now likewise, if the electron is spinning down he also comes to a definite conclusion represented by the state $|S_-\rangle$, that's also an eigenvector of D with eigenvalue $+1$,

$$|\psi_i\rangle = |\downarrow, M_0, S_0\rangle \longrightarrow |\psi_f\rangle = |\downarrow, M_-, S_-\rangle\,, \quad D|\psi_f\rangle = |\psi_f\rangle\ .$$

Now let's get to the problematic case; one where the cat is half dead and half alive. That would be the measuring apparatus, the cat is dead in state plus, alive in state minus. We start out with an initial state where the observer is still passive, the measuring apparatus is passive, the electron is in an eigenstate of σ_x, $\frac{1}{\sqrt{2}}[\,|\uparrow\rangle + |\downarrow\rangle\,]$. Each of these evolve in time according to Schrödinger's equation, by linearity a superposition of initial states gives you the same superposition of corresponding final states,

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}\left[\,|\uparrow, M_0, S_0\rangle + |\downarrow, M_0, S_0\rangle\,\right] \longrightarrow |\psi_f\rangle = \frac{1}{\sqrt{2}}\left[\,|\uparrow, M_+, S_+\rangle + |\downarrow, M_-, S_-\rangle\,\right] .$$

Is $|\psi_f\rangle$ an eigenstate of D with an eigenvalue of $+1$? Sure is, baby! A linear combination of two eigenstates of any operator with the same eigenvalue is an eigenstate of the given operator with the given eigenvalue. Quantum mechanics predicts the observer is sure he knows what state the electron is in. It doesn't predict he is sure he knows it is spin up or predict he is sure it is spin down, but that's not what Zurek asked, because that's not the universal experience, sometimes he sees spin up, sometimes he sees spin down. It's universal that he always feels certain he has seen a definite spin and that is what quantum mechanics predicts. Okay, I'm a funny guy but this is not a joke. I believe this is the truth.

Now, so much for indeterminancy, but I not only deny indeterminancy, I deny probability. Why is this thing seen to be a random sequence? If, for example we take a set of electrons, prepare each of them one after another as eigenstates of x spin and then make a sequence of measurements of the z spin, we seem to get a random sequence of plus ones and minus ones. Absolutely random, totally unpredictable. How can that be the result of deterministic time evolution? Well first, I have to ask the question, "What is a random sequence?" even in classical reasoning, even if I were to ask a nineteenth century mathematician what is a random sequence, or an early twentieth century one, or maybe a contemporary one who has never studied quantum mechanics. So, what is a random sequence? Unfortunately, the discussion has to be phrased in terms of infinite sequences. I know of no way of defining a finite random sequence, it's obviously hard to do with a random sequence of only one zero or one, but it's not much easier with three or four. There is no borderline, only infinity is worthwhile defining.

Suppose we have an infinite sequence of coin flips, of heads or tails, or zero's or ones, or plus ones or minus one's - which I will call σ_r. Then we want to investigate the hypothesis that a series of independent random flips of a fair coin will come up heads or tails, plus or one, with equal probability. The first thing we would do is look at the average value, that is defined by taking the average value of the first N elements of the sequence and letting N go to infinity. If that is zero,

$$\bar{\sigma} = \lim_{N\to\infty} \bar{\sigma}^N = \lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r = 0 \ ,$$

that is certainly a necessary criteria, but, of course, it is not a sufficient one. For example, this would be satisfied by the sequence which is $+1, -1, +1, -1, +1, -1$ which is nobody's idea of random. So I have to look for correlations. I look for, say the product of one entry in the sequence and the next one, the first times the second, the second times the third, the third times the fourth and again I look for the average value of that summed over pairs and that should also be zero. And it cannot be just displaced by one I might look for correlations displaced by two or by three, by any number a so that

$$\bar{\sigma}^a = \lim_{N\to\infty} \bar{\sigma}^{N,a} = \lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r\sigma_{r+a} = 0 \text{ for all } a \ .$$

Alright, then look for correlations among triples, the first times the third times the fifth averaged with the second times the fourth times the sixth etc,

$$\lim_{N\to\infty} \frac{1}{N}\sum_{r=1}^{N} \sigma_r\sigma_{r+a}\sigma_{r+b} = 0 \text{ for all } a, b \ ,$$

all the way down the line. If all these things end up being zero I would say, and my experimental colleagues would say, perhaps a rigorous mathematician would not, this is indeed a random sequence. Some mathematical niceties have been ignored. These limits, of course, do not exist for all sequences. If we had a sequence that was $+1, +1$ then twenty -1's then four hundred $+1$'s and eight thousand -1's, then a mathematician would say "I look at the upper limit and the lower limit and then I would, if they're both equal to each other and both equal to zero, then I would say blah, blah,

blah." All that stuff has been gone into in vast detail in the paper by Farhi *et al.* and I don't want to go into that here.

Now, let's do some quantum mechanics. We start out with the state of an electron, in an eigenstate of σ_x,

$$|{\rightarrow}\rangle \equiv \frac{1}{\sqrt{2}}\big(|\uparrow\rangle + |\downarrow\rangle\big) \ .$$

We consider an infinite string of such electrons that are being fed into our measuring apparatus, one after another, producing a sequence of plus and minus ones. That means we consider the infinite direct product which is, by the way, a perfectly well-defined mathematical concept worked out by von Neumann in 1938, and you'll find references to that in these papers. We consider the infinite direct product of these things,

$$|\psi\rangle = |{\rightarrow}\rangle \otimes |{\rightarrow}\rangle \otimes |{\rightarrow}\rangle \dots \ .$$

It's an infinite string of electrons, just as before. Let each of these interact with a device that measures σ_z as before and with an observer, Sidney, you remember him? Again, we want to ask the question about the observer's head. Does he observe a random sequence? Well, that is equivalent, as we already worked out previously, to how these σ_z's are mapped into states of his head. That's equivalent to asking the question, is $|\psi\rangle$ an eigenstate of the appropriate operators here with eigenvalue 0, i.e.

$$\bar{\sigma}_z|\psi\rangle = 0\,, \quad \bar{\sigma}_z = \lim_{N\to\infty} \bar{\sigma}_z^N\ , \quad \bar{\sigma}_z^N = \sum_{r=1}^{N} \sigma_z^{(r)}\ ,$$

and likewise for $\bar{\sigma}_z^a$ etc.? If it is, it is one hundred percent certain that the observer will say, "This is a random sequence," because what we mean by a random sequence is that all these things are observed to have value zero, and the non-probabilistic part which we are accepting is that, if the state of the system is an eigenstate of an observable with a given eigenvalue then the observable is certain to have that value.

So, that is what we want to do, for all of these guys, I'll just do the first one, $\bar{\sigma}_z$. Once you see the first one, which was originally done by Jim Hartle without some of the mathematical niceties, you will see how all the others go. I have this operator, to show it is an eigenstate with eigenvalue zero, a little mathematics is needed, the only hard mathematics here. I'll just look at the square of the norm of the state where $\bar{\sigma}_z$acts on $|\psi\rangle$. I know what $\bar{\sigma}_z^N$ is, I've written it above. It's a Hermitian operator, so I just apply it twice, here is one of them, here's the other, the sum on r, the sum on s, each of them has a $\frac{1}{N}$ so I've got a $\frac{1}{N^2}$,

$$|| \, \bar{\sigma}_z^N |\psi\rangle||^2 = \frac{1}{N^2} \, \langle\psi| \sum_{r=1}^{N} \sum_{s=1}^{N} \sigma_z^{(r)} \sigma_z^{(s)} |\psi\rangle \ .$$

Every child, at his mother's knee, we shouldn't be sexist, every child at her father's knee, any permutation, doesn't matter, every baby or child knows in this day that if I look at the expectation value of $\sigma_z^{(r)}$ or $\sigma_z^{(s)}$, r and s unequal, the plus and the minus cancel, I get zero. If r equals s then that gives σ_z^2, that's one, so

$$\langle\psi|\sigma_z^{(r)} \sigma_z^{(s)}|\psi\rangle = \delta^{rs} \ .$$

Therefore, this expectation value is trivial, it is just N because there are N terms to sum over and there is a factor N^{-2} so the limit as N goes to infinity $N^{-2} \times N$ is zero!

$$\lim_{N\to\infty} || \, \bar{\sigma}_z^N |\psi\rangle||^2 = \lim_{N\to\infty} \frac{1}{N^2} \, N = 0 \ .$$

This thing, as mathematicians would say, strongly converges to zero. The limit operator exists, applied on the state $|\psi$, and the state is an eigenstate of the limit operator with eigenvalue 0 and likewise for all the higher ones such as $\bar{\sigma}_z^a$, it is always the same story, you only have N non-zero terms with non-zero expectation values but you are dividing by N^2.

This is a definite deterministic state, there appears no mysterious measurement processes. What's happened to it? There is no measurement, only correlations have been established, according to the deterministic Schrödinger equation between the state of the electrons and the state of the observers head. Nevertheless, despite the fact that there is no indeterminancy, no hidden variables, no nothing, everything is known, nothing unknown. Nevertheless, it's a definite deterministic state that definitely defines a random sequence. Something that is completely impossible in classical physics, but this is not classical physics - it's quantum mechanics.

Now, in Tom Stoppard's play "Jumpers", there is an anecdote about Wittgenstein, that is told by the protagonist who is given the jokey name of George Moore by the way. It may be a real Wittgenstein story, I've never been able to find out, here in Cambridge maybe somebody knows. Anyway, Stoppard's character says one day a friend found Wittgenstein standing on the street looking bemused and he says "What's bothering you Ludwig?". And Wittgenstein said "I was just thinking, why did the people say it was natural to think that the sun went around the earth?" And the friend says "Well, because it looks like the sun goes around the earth". Wittgenstein says "I see, what would it have looked like if it had looked like it was the other way round?" The attack people make on Everett's suggestion that there is nothing but time evolution according to the Schrödinger equation and the reason people say that there must be this other process, the reduction of the wave packet, is because they say there are definite outcomes, which are only probabilistically determined and therefore it *looks* like the reduction of the wave packet. I ask you, in the spirit of Stoppard's Wittgenstein, "What would it have looked like if it had *looked* like it was the other way round?" And I have attempted to convince you that what it would look like is *this.*

Welcome home, thank you for your patience.