Layer formation in stratified turbulence

I. BACKGROUND

Layering of velocity, density, and tracers is often seen in regions of the ocean and atmosphere with stable stratification. Laboratory experiments show that layers can spontaneously emerge from a stable density profile when turbulence is driven by an external force. For example, Figure 1 shows the formation of layers in stratified Taylor-Couette flow driven by two differentially rotating concentric cylinders.

(a) (b)

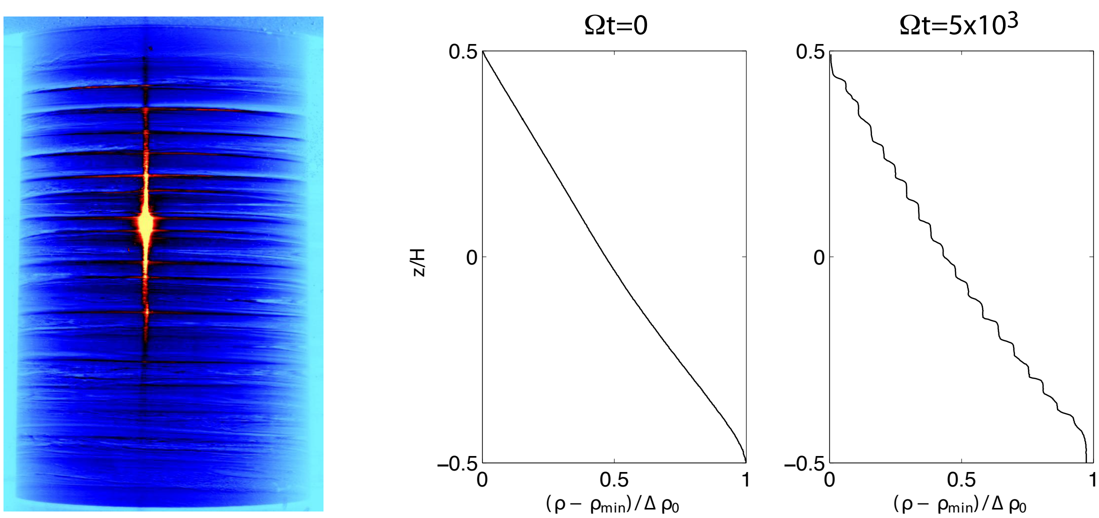

FIG. 1: (a) Shadowgraph of layers in stratified Taylor-Couette flow (image courtesy of Pierre Augier). (b) Density profiles from experiments of stratified Taylor-Couette flow showing spontaneous layer formation (Oglethorpe and Caulfield).

II. INTRODUCTION

A heuristic explanation of stratified layer formation was proposed by Phillips[1]. The papers in the project folder give a detailed explanation of this mechanism, which is summarized briefly here. Consider the horizontal average (denoted by an overbar) of the density equation:

$$\frac{\partial \overline{\rho}}{\partial t} = -\frac{\partial}{\partial z}\overline{\rho' w'}, \tag{1}$$

where primes denote departures from a horizontal average, and molecular diffusion has been neglected. The density flux, $\overline{\rho' w'}$ quantifies the vertical transport of density by turbulent motions. For example, in a stratified fluid with density decreasing with height, if upward motions ($w' > 0$) bring up heavy fluid ($\rho' > 0$) on average, then the density flux will be positive ($\overline{\rho' w'} > 0$), and turbulence will act to reduce the stable stratification and raise the center of mass.

Consider the influence of the density gradient on the density flux in two limiting cases. In a perfectly mixed region with no density gradient, we expect ρ' and hence the density flux to vanish. For very large negative density gradients, stable stratification will inhibit vertical motions and we might expect w' and the density flux to be small. This reasoning implies that the density flux will be maximum at some intermediate value of the density gradient, as illustrated in Figure 2a.

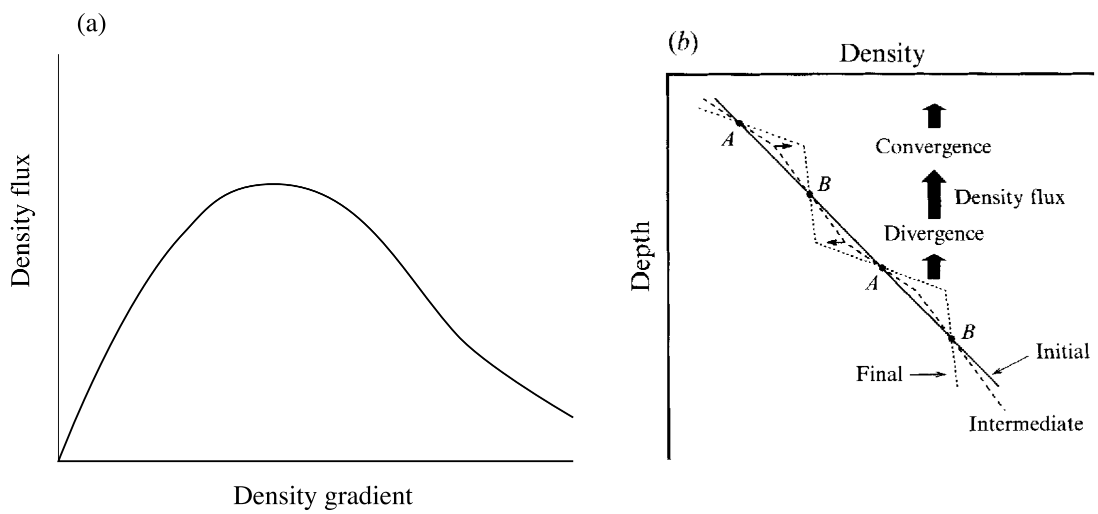

FIG. 2: (a) Cartoon of density flux vs. density gradient, (b) formation of layers via the Phillips [1] mechanism (figure from Park et al. [2]).

Now, consider a fluid with a large density gradient where the density flux locally decreases with increasing stratification. Then perturb the density profile to create small deviations in the density gradient. As illustrated in Figure 2b, the density flux will be smaller in regions of relatively strong stratification and larger in layers of relatively weak stratification. The density itself changes in response to convergences or divergences in the density flux. In the region with weak stratification, we expect to have a convergence in the density flux at the top of the region and a divergence in the density flux at the bottom of the region. From Eq. **??**, we see that this will cause an increase in density at the top of the weakly stratified layers and an increase in density at the bottom of the weakly stratified layers. A similar argument can be made in the regions with strong stratification. In both cases, the density flux will intensify fluctuations in the density gradient.

III. 1D MODEL FOR LAYER FORMATION

Balmforth et al. [3] proposed a one-dimensional model for layer formation in stratified turbulence. The model is based on the following evolution equations for the horizontally averaged buoyancy (b) and turbulent kinetic energy (e):

$$b_t = (\nu_t b_z)_z\,, \tag{2}$$

$$e_t = (\nu_t e_z)_z + \mathcal{P} + \mathcal{B} + \varepsilon, \tag{3}$$

where subscripts denote partial derivatives. The turbulent diffusivity, $\nu_t = le^{1/2}$, from mixing-length theory with a characteristic turbulent length scale, l, and a characteristic turbulent velocity scale $e^{1/2}$. To close the model, we need expressions for the turbulent production, $\mathcal{P}$, buoyancy flux, $\mathcal{B}$, and dissipation, ε. To be consistent with the buoyancy equation, the buoyancy flux should be

$$\mathcal{B} = \nu_t b_z = l e^{1/2} b_z. \tag{4}$$

The turbulent dissipation rate is modeled by assuming that turbulent kinetic energy decays with a timescale $\tau = \alpha^{-1} l/e^{1/2}$ (and α is a constant of proportionality), so that

$$\varepsilon = -\alpha \frac{e}{\tau} = -\alpha \frac{e^{3/2}}{l}. \tag{5}$$

Balmforth et al. [3] model the turbulent production from a grid moving through a stratified fluid with a speed U and suppose that the production takes the form

$$\mathcal{P} = U^2 \tau = \alpha U^2 \frac{e^{1/2}}{l}. \tag{6}$$

Finally, the turbulent length scale is taken to be an interpolation between the buoyancy scale, $l_B = \sqrt{e/b_z}$, and a distance proportional to the spacing between bars of the oscillating grid, d:

$$\frac{1}{l^2} = \frac{1}{d^2} + \gamma \frac{b_z}{e}. \tag{7}$$

Equations (2.9)a-c of Balmforth et al. [3] give a non-dimensional version of these model equations. In the project directory, you will find a matlab code *balmforth.m* that time-steps the non-dimensional Balmforth et al. model. Run this code and verify that layers form.

A. Suggestions

Using *balmforth.m*, plot the buoyancy flux as a function of buoyancy gradient. Does the curve look like the one sketched in Figure 2a? Try varying the input parameters at the top of the script. Can you find a case where layers don't form? How sensitive is the formation of layers to the initial profiles of e and g? Investigate the dynamics of the layers. When do layers merge or disappear? Try changing the form of the various terms in the model (ν_t, $\mathcal{P}$, $\mathcal{B}$, ε). How sensitive is the layer formation to the form of the model equations? Can you understand the changes in terms of the buoyancy flux vs. stratification curve?

IV. LAYER FORMATION IN DIABLO

Try generating stratified turbulence in Diablo. Start with a stable density profile with a random velocity perturbation. Add a forcing term to the momentum equations (in *user_rhs.m*) to mimic turbulent production by an oscillating grid. Do layers form in the velocity and density fields? How does the layer thickness depend on the input parameters? If the forcing is sufficiently strong, can you get the layers to become unstable to a shear instability? What does that do to the layer thickness and lifetime?

V. REFERENCES

[1] O. Phillips, in *Deep Sea Research and Oceanographic Abstracts* (Elsevier, 1972), vol. 19, pp. 79–81.

[2] Y.-G. Park, J. Whitehead, and A. Gnanadeskian, Journal of Fluid Mechanics **279**, 279 (1994).

[3] N. Balmforth, S. Llewellyn Smith, and W. Young, Journal of Fluid Mechanics **355**, 329 (1998).