NEAR-WALL MODELING FOR LES OF AN OCEANIC BOTTOM BOUNDARY LAYER

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Abstract. A stochastic forcing method is proposed to improve the fidelity of high Reynolds number large-eddy simulations of the oceanic bottom boundary layer. The forcing term is added to the wall-normal momentum equation with the amplitude set using a proportional controller. The sign of the forcing is chosen to either increase or decrease the local correlation between the horizontal and wall-normal velocity fluctuations to adjust the mean shear toward a specified logarithmic law. The amplitude and envelope of the forcing function are both significantly smaller than those required by previously suggested stochastic forcing methods. The NWM-LES is tested for smooth and rough channel flow, and for a bottom Ekman layer with and without stratification. In all cases, it is found that the addition of stochastic forcing improves the mean velocity and turbulent stresses compared to results from direct numerical simulations and experimental data.

1. Introduction

When simulating geophysical boundary layers at a large Reynolds number, in addition to the subgrid model for the outer layer, a near-wall model (NWM) is inevitable for handling the inner layer of high-Re geophysical boundary layers. This is due to the fact that the filter scale is typically much larger than the viscous/roughness scales, the generation of turbulence at the wall is not resolved, and consequently the near-wall Reynolds shear stress must be explicitly accounted for through a model. In other words, it is not practical to refine the grid to achieve the small grid spacing, typically on the order of 25 wall units in the wall-parallel directions and 1 wall unit in the wall-normal direction, required to capture the near-wall structures responsible for maintaining turbulence.

Near-wall models for geophysical boundary layers are commonly formulated by applying an approximate boundary condition at the wall with most implementations based on the method of Schumann **?** in simulations of rough channel flow, later adapted by Grotzabch **?**. The instantaneous wall shear stress is related to the instantaneous horizontal velocity at the first grid point,

$$\frac{\tau_{w,i}(x,y,t)}{\rho_0 u_*^2} = \frac{u_i(x,y,1,t)}{U_i(1,t)}, \tag{1}$$

$u_i(x,y,1)$ is the resolved instantaneous horizontal velocity at the first grid point nearest the wall, $U_i(1)$ is the Reynolds-averaged velocity at that point, and u_* is the friction velocity. The first computational point is placed in a location where the logarithmic law may be applied. An explicit relationship between $\tau_{w,i}(x,y,t)$ and the horizontal velocity $u_i(x,y,1,t)$ at the first grid point results, and this serves as the approximate wall boundary condition in the LES. Piomelli *et al.* **?** found that, by displacing the point of calculation of u_i in the streamwise direction by a prescribed distance Δ_s, improved predictions of the velocity field were obtained in channel flow.

Another near-wall model using an approximate boundary condition was proposed by Marusic *et al.* **?** which will be referred to here as the MKP model. The model was intended to improve on the Schumann-Grotzbach-Piomelli models by increasing the level of fluctuations in the local wall stress and to improve the velocity spectra in the logarithmic region**?**. This model has another advantage that it also performs well without modification for geophysical flows with rotation **?** and does not require the specification of the wall-stress angle, γ_0. This model is formulated as:

$$\frac{\tau_{w,i}(x,y,t)}{\rho_0 u_*^2} = \frac{U_i(1)}{U(1)^2 + V(1)^2)^{1/2}} - \alpha \frac{[u_i(x+\Delta_s, y, z(1)) - U_i(1)]}{u_*}, \tag{2}$$

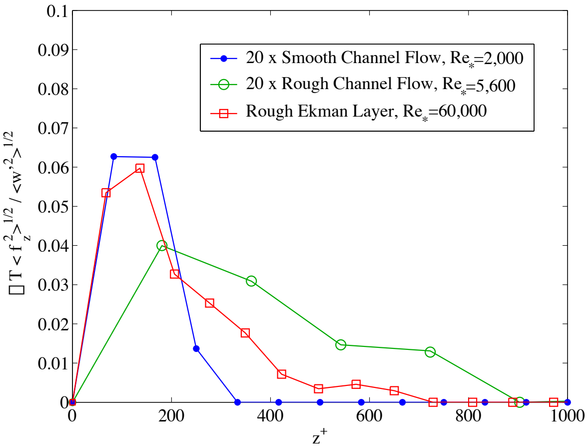

Figure 1. *r.m.s.* of the stochastic forcing function with the amplitude evaluated dynamically for $z/\delta < \Delta_f$. Note that the channel flow cases have been scaled by a factor of 20.

where $z(1)$ is the location of the gridpoint nearest the wall and U(1) and V(1) are the plane averaged horizontal velocity components at the first gridpoint. Marusic *et al.* **?** hypothesized that the constant α should be universal, and was empirically determined to be 0.10. As before, the instantaneous friction velocity and mean wall stress are estimated by assuming that the plane-averaged velocity at the first gridpoint follows the expected log law.

Mason and Thomson (1992) performed simulations of the neutral ABL with a Smagorinsky model. They found that the mean shear at points in the vicinity of the boundary was overpredicted relative to the log-law value and the near-surface value of *rms* streamwise velocity was also larger than measurements. The authors considered stochastic forcing of the momentum and scalar equations with the curl (divergence in the scalar case) of a random vector field with zero mean at every time step. Stochastic forcing has also been used in DES by Piomelli *et al.* (2003) to improve the accuracy of the transition layer between RANS and LES. Keating and Piomelli **?** extended the work of Piomelli *et al.* (2003) by using a simple proportional controller to adjust the amplitude of the stochastic force.

In the present work, we explore the performance of the dynamic eddy viscosity model (DEVM) in the following test problems: (1) a smooth-wall channel flow, (2) a rough-wall channel flow, and (3) an Ekman boundary layer over a rough bottom, both with and without overlying stratification. Since an overlap region with a log law exists in all of the test problems, we employ a NWM that enforces the log law on the mean velocity at the grid point next to the wall. However, as will be shown, this procedure is unable to capture the expected the mean velocity profile at the other grid points and the expected near-wall behavior of the turblent stresses. This observation motivates the NWM model proposed here: stochastic forcing of the wall-normal velocity with proportional control. Unlike the forcing methods employed by previous authors, the forcing amplitude is small and has a thin envelope.

2. Formulation

We have found that the NWM-LES with the DEVM generally underestimates vertical velocity fluctuations and the magnitude of the Reynolds stress near the wall. We therefore propose adding a stochastic forcing term to the wall normal momentum equation that is designed to enhance the vertical velocity fluctuations while modifying the local Reynolds stress according to the expected value from the log law. Since the objective of the stochastic forcing is to improve the representation of the unresolved small-scale motions, the characteristic length and time scales of the forcing are set by the grid-size and time-step, respectively. As it appears on the right hand side of the wall-normal momentum equation, the proposed forcing function can be written

$$f_z(x, y, z) = \pm \mathcal{R} A(z), \tag{3}$$

where $\mathcal{R}$ is a random number between 0 and 1, $A(z)$ is an amplitude function, and the sign is chosen to adjust the turbulent stress in the appropriate direction. The forcing amplitude is evaluated dynamically in order to ensure that the mean velocity follows the logarithmic law near the wall. This choice also has the advantage of eliminating the need to empirically specify the forcing amplitude. The difference between the resolved shear and the expected logarithmic law value is

$$\epsilon(z) = \frac{\kappa z}{u_*}\left(\frac{d\langle u\rangle}{dz}^2 + \frac{d\langle v\rangle}{dz}^2\right)^{1/2} - 1. \tag{4}$$

The forcing amplitude can then be locally adjusted using a proportional controller with

$$A(z)^{n+1} = A(z)^n + \frac{u_*\epsilon(z)}{\tau}, \tag{5}$$

where τ sets the relaxation time and should be large enough to allow the flow to adjust to the forcing, and typically we have used $\tau \approx \delta/u_*$. The sign of the stochastic forcing function is chosen to increase or decrease the u', w' correlation according to the sign of $\epsilon(z)$. Taking u_s to be the velocity in the direction the mean wall stress:

$$Sgn\left(f_z(x,y,z)\right) = -Sgn(\frac{d\langle u_s\rangle}{dz}) * Sgn(\epsilon(z)) * Sgn(u_s'(x,y,z)). \tag{6}$$

If, for example, we consider flow near a lower wall where $d\langle u_s\rangle/dz > 0$ and we find that the mean shear is overestimated so that $\epsilon(z) > 0$, then the sign of the forcing term will be the opposite of the local value of u_s'. In this case, the forcing will effectively act to pull high speed fluid down toward the wall and push low speed fluid away from the wall. This will modify the local Reynolds stress and will act to reduce the mean shear as desired.

We have found that the stochastic forcing method described above is able to effectively adjust the mean velocity toward the expected logarithmic law when active over a small number of gridpoints near the wall. Specifically, the forcing amplitude $A(z)$ is taken to be nonzero only for $z/\delta < \Delta_f$ where Δ_f is the volume-averaged filter size. Before the forcing term is added to the wall-normal momentum equation, the plane-average is removed. After the simulations have been allowed to reach a statistically steady state, the *r.m.s.* forcing amplitudes for each case are shown in Figure **??**.

3. Results

In the following section, we will demonstrate the performance of a NWM-LES using the MKP model with and without stochastic forcing for three flows: smooth channel flow, rough channel flow, and a geophysical boundary layer.

3.1. *Smooth Channel Flow*

Pressure-driven channel flow is a common test case for near wall models and allows comparison with previous simulations and experimental data. We have chosen the friction Reynolds number, $Re_* = 2000$, in order to compare our results to the direct numerical simulation (DNS) of Hoyas and Jimenez **?**. The number of gridpoints used for the NWM-LES after de-aliasing in the x,y, and z directions is 64, 64, and 50, respectively with a uniform grid in all three directions. The first horizontal velocity gridpoint lies within the expected log region, in this case $z^+(1) = 41.7$. For comparison, the DNS of Hoyas and Jimenez uses a 6144 x 4608 x 633 grid, i.e. 87,500 times more gridpoints than the NWM-LES.

Figure **??**(a) shows the mean velocity profiles as a function of $z^+ = zu_*/\nu$ for the smooth wall channel flow simulations. The mean velocity from the DNS follows the expected log law between about $z^+ > 30$ and $z/\delta < 0.2$. It is apparent from Figure **??**(a) that the NWM-LES without stochastic forcing overpredicts the mean

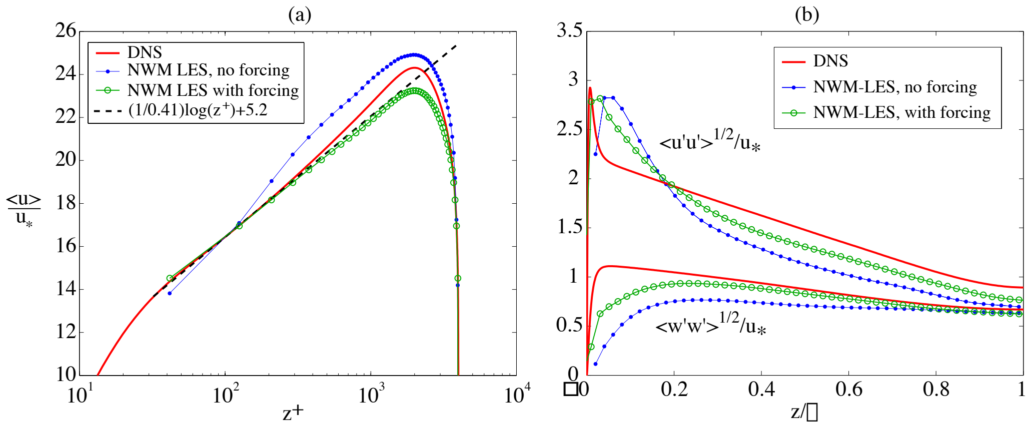

Figure 2. (a) Mean velocity profiles and (b) streamwise and wall-normal rms velocity for smooth wall channel flow with $Re_* = 2000$

shear where a log law is expected. Much better agreement with the DNS in the logarithmic region is achieved when the stochastic forcing term is included. Note that as shown in Figure **??**, the forcing term is nonzero only for the first four grid points, $z^+ < 325$. The NWM-LES appears to be unable to properly capture the deviation from the log law in the so-called 'wake region' seen in the DNS.

The streamwise and wall-normal turbulent velocities are shown for smooth wall channel flow in Figure **??**(b). When the NWM-LES is used, the magnitude of the wall-normal rms velocity is significantly less than the DNS. Adding stochastic forcing increases the wall-normal velocity fluctuations as expected, and results in a better agreement with the DNS. The same conclusion is true for the spanwise velocity fluctuations (not shown for clarity). Without forcing, the streamwise velocity fluctuations compare reasonably well in magnitude to the DNS, but an improvement is still seen when forcing is added, resulting in a shift in the location of the maximum near the wall and an increase in the outer layer fluctuations. We have found that using the dynamic mixed model (DMM) without forcing results in all three components of the rms velocity larger than the DNS (not shown).

3.2. *Rough Wall Channel Flow*

Most boundary layers of geophysical relevance are in the fully rough regime where the roughness elements are large enough to eliminate the buffer layer **?**. In order to evaluate the performance of the near-wall model in response to the presence of roughness, we have compared our simulations to an experimental study at a relatively large Reynolds number. The simulation parameters have been matched to the experiments of Bakken *et al.* **?** where wire mesh was used to roughen the ceiling and floor of a wind tunnel. The friction Reynolds number is 5600, and the roughness length scale is $k^+ = 187$ indicating that the flow is fully rough. By fitting the observed velocity profile to the rough wall log-law:

$$U^+ = \frac{1}{\kappa} log(z^+) + B + \Delta U^+ = \frac{1}{\kappa} log\left(\frac{z}{z_0}\right), \tag{7}$$

it is estimated from the experimental data that $\Delta U^+ = 11.93$ and $z_0/\delta = 2.81 * 10^{-3}$. Our simulations with the NWM-LES use 48x48 gridpoints in the horizontal directions and 64 points in the wall-normal direction with a uniform grid. The first horizontal velocity point away from the wall is located at $z^+(1) = 90$ or $z(1)/z_0 = 5.7$.

Figure **??**(a) shows the mean velocity profiles for the NWM-LES using the parameters given above with and without stochastic forcing. For comparison, the data of Bakken *et al.* **?** are also shown. As in the smooth channel flow, the mean shear in the log region for the NWM-LES is too large. With the addition of stochastic forcing, the mean velocity profile follows the log law very closely. Note that the log

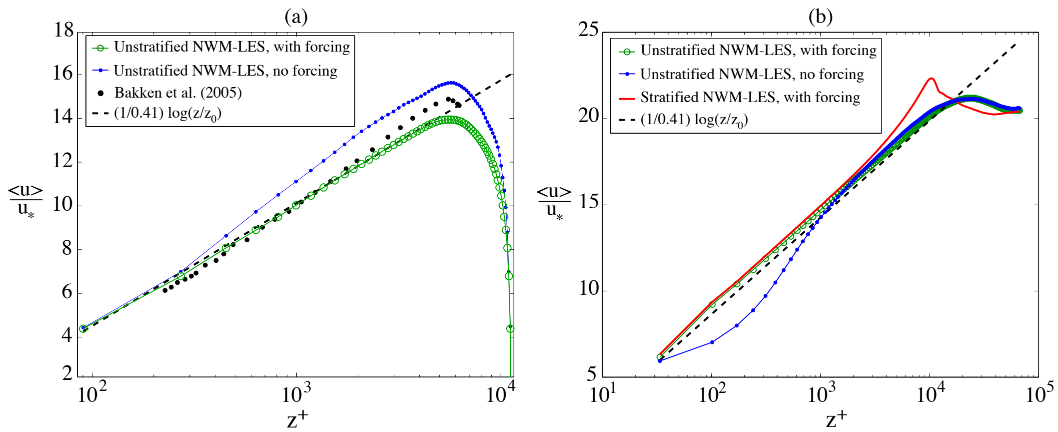

Figure 3. (a) Mean velocity profiles for rough wall channel flow with $z_0/\delta = 2.81x10^{-3}$ and $Re_* = 5600$, (b) Mean velocity profiles for a bottom Ekman layer with $z_0/\delta = 5.7 * 10^{-5}$ and $Re_* = 60,00$.

law used in the near-wall model assumes that $\kappa = 0.41$, while the experimental data matched with a least squares fit gives $\kappa = 0.37$. Using the observed value of κ would have likely yielded an even better agreement between the experimental data and NWM-LES results.

3.3. *Bottom Ekman Layer*

Finally, we have tested the near-wall model in simulations of an oceanic bottom boundary layer. The outer layer flow is assumed to be steady and in geostrophic balance. Unlike a non-rotating boundary layer, the thickness of an unstratified Ekman layer is bounded, and when the flow is turbulent the boundary layer height scales with $\delta = u_*/f$ where f is the Coriolis parameter. While the free stream velocity and the wall stress are not aligned for an Ekman layer, the horizontal velocity magnitude is still expected to follow a log law when the flow is unstratified **?** and when the outer layer is stratified **?**. The friction Reynolds number and the roughness lenthscale have been chosen to match the observations of Perlin *et al.* **?**, $Re_* = 60,000$ and $z_0/\delta = 4.80 * 10^{-3}$, respectively. The NWM-LES uses 85 gridpoints in each horizontal direction (after 2/3 dealiasing) and 201 gridpoints in the wall-normal direction. The domain size is about 3δ in the horizontal directions and 1.5δ in the vertical direction.

Profiles of the mean horizontal velocity magnitude for the NWM-LES are shown in Figure **??**(b). As in the previous test cases without forcing, the NWM-LES does not accurately predict the mean shear in the log region. However, the addition of stochastic forcing corrects the mean velocity in this region. In the ocean, the thickness of the bottom boundary layer is nearly always limited by the presence of a stable stratification in the ocean interior. We have performed simulations with a stratified ambient with a buoyancy frequency $N/f = 75$ chosen to match the observations of Perlin *et al.* **?**. In the stratified simulation, a well-mixed region forms near the wall with a thickness of about 0.12δ where the mean velocity follows the unstratified log law. Above the well-mixed layer, a highly stratified pycnocline forms where the mean shear increases as shown in Figure **??**(b). A similar increase in the mean shear was also observed by Perlin *et al.* **?**. A visualization of the wall-normal velocity near the wall is shown in Figure **??**. It is clear from this figure that the amplitude of the wall-normal velocity fluctuations is increased at this height by the addition of forcing while the characteristic lengthscale of the fluctuations is reduced.

4. Conclusions

We have proposed a stochastic forcing method in an attempt to improve the performance of near-wall model large-eddy simulations for large Reynolds number boundary layers. The amplitude of the forcing term that is added to the wall-normal momentum equation is set using dynamic control. Since the sign of the forcing

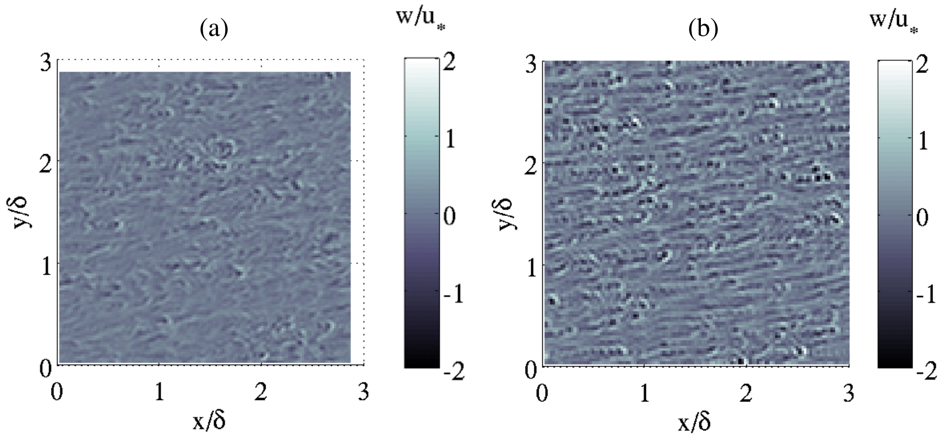

Figure 4. Wall-normal velocity in a bottom Ekman layer at $z/\delta = 2 * 10^{-3}$ (a) without stochastic forcing, and (b) with stochastic forcing.

Simulation type	Re_*	z_0/δ	z_0^+	L_x/δ	L_y/δ	L_z/δ	N_x	N_y	N_z	Δ_x^+	Δ_y^+	$\Delta_z^+(1)$
Smooth Channel	2000			2π	π	2	64	64	50	196	98	41.7
Rough Channel	5600	$2.8 * 10^{-3}$	15.7	2π	π	2	48	48	64	733	367	90
Ekman Layer	$1.08 * 10^5$	$4.78 * 10^{-5}$	5.2	2.2	2.2	1.1	85	85	201	2830	2830	122

term is set to either increase or decrease the correlation between the local wall-normal and horizontal velocity, the forcing amplitude needed to achieve a desired correction in near-wall Reynolds stress is much smaller than previously proposed forcing methods. When used with the dynamic eddy viscosity model, the stochastic forcing is able to improve the estimates of the turbulent stresses and the mean velocity profile compared to direct numerical simulations and experiments. When applied to a turbulent benthic Ekman layer, the NWM-LES with stochastic forcing is able to capture the expected logarithmic scaling of the velocity magnitude as well as the increase in outer layer shear in the presence of stratification.

Acknowledgments

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