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Short Communication

# Sedimentation of a rotating sphere in a power-law fluid

Francisco A. Godínez [a], Elsa de la Calleja [b], Eric Lauga [c,*], Roberto Zenit [b,*]

[a] Instituto de Ingeniería, Universidad Nacional Autónoma de México, México D.F. 04510, Mexico
[b] Instituto de Investigaciones en Materiales, Universidad Nacional Autónoma de México, Apdo. Postal 70-360, México D.F. 04510, Mexico
[c] Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom

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ABSTRACT

We measure the sedimenting velocity of rigid spheres in power-law fluids. By imposing a controlled rotation, we can increase the typical shear rate in the surrounding fluid leading to a decrease of the effective fluid viscosity and, consequently, an increase of the sedimentation speed of the spheres. By fitting our experimental measurements to a power-law dependence of the sedimentation speed on the rotation frequency we are able to predict the values of the consistency and power indices for the test fluids. This setup could thus be used as a rheometer for power-law fluids.



## 1. Introduction

The sedimentation of solid spheres is a classical benchmark problem in fluid mechanics and has therefore been studied extensively for both Newtonian [1] and non Newtonian [2] liquids. One important practical aspect of the study of sedimentation is that it can be used to determine fluid properties. For an infinite Newtonian fluid in the creeping flow regime, the sedimentation velocity of a sphere, $U_s$, is obtained analytically [3] as

$$U_s = \frac{W^*}{3\pi D\mu}, \tag{1}$$

where $D$ is the sphere diameter, $\mu$ is the dynamic viscosity and $W^* = \frac{\pi}{6}(\rho_p - \rho)gD^3$ is the effective weight of the sphere ($\rho$ denotes the fluid density while $\rho_p > \rho$ is the density of the sphere). If the size and weight of the sphere are known and the sedimentation velocity is measured, the value of the viscosity can be inferred from Eq. (1). This method has been used extensively to determine the viscosity of Newtonian viscous fluids [1].

For the particular case of inelastic fluids with shear-dependent viscosity, it is common to express their effective viscosity, $\mu_{\rm eff}$, as a power-law

$$\mu_{\rm eff} = m\dot{\gamma}^{n-1}, \tag{2}$$

where $\dot{\gamma}$ denotes the flow shear rate and $m$ and $n$ are the consistency and power indices, respectively [4]. Whereas $m$ characterizes the magnitude of the viscosity, $n$ describes how rapidly the viscosity changes with shear rate.

There have been numerous studies aiming to calculate the drag force on solid spheres moving at constant speed in inelastic shear-thinning fluids, as summarized in Chapter 3 of Ref. [2]. The effect of thinning-viscosity for creeping flow regimes is typically expressed empirically as

$$F_D = 3\pi D U_s \mu_{\rm eff} Y(n), \tag{3}$$

where $Y(n)$ is a drag correction factor defined as $Y(n) = C_D Re/24$, where $C_D$ is the drag coefficient and $Re = \rho D^n U_s^{2-n}/m$. In Eq. (3), the value of $\mu_{\rm eff}$ is to be evaluated at the mean shear rate, $2U_s/D$. Despite many investigations, the reported trends for $Y(n)$ vary widely [2]. With advances in numerical simulations, the situation has improved [5–7] but a general consensus on this issue has not been reached. As a result, very few studies have attempted to use the sedimenting sphere setup to measure non-Newtonian fluid properties [8–11]. One difficulty is that the flow around a sphere is not viscometric, and thus average values of the shear rate and stress need to be considered, and rigorous predictive modeling is not available.

In this note, we study the classical problem of a sedimenting sphere but with a twist – literally. Using a recently developed method we are able to impose a rotation, at a controlled rate, around the axis of sedimentation of a rigid sphere. The flow around the sphere is then the combination of that produced by translation

* Corresponding authors.
*E-mail addresses:* e.lauga@damtp.cam.ac.uk (E. Lauga), zenit@unam.mx (R. Zenit).

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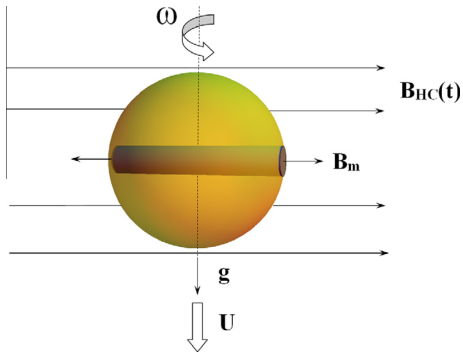


**Fig. 1.** Sketch of the sphere in the sedimentation experiment and its interaction with the magnetic field.

and that produced by rotation of the sphere. As a result of the particle rotation, the effective viscosity around the sphere decreases, resulting in a decrease of the fluid drag and an increase of the terminal velocity of the sphere.

The study presented here can, in fact, be placed in the subject of flow superposition, a topic which has been addressed by many researchers in the past. For the particular case of non Newtonian and complex liquids, the superposing rotation and translation has been addressed by [12–14]. Other authors have studied the superposition of squeezing flows with rotation [15,16]. For most of these investigations, the motivation was to observe the response of a primary flow to a secondary orthogonal flow. In this sense, our study is similar: we observe the changes in the flow around a sphere which result from imposing shear by rotation. However, for the case addressed in the present study the flow is more complex since sedimentation and rotation are not orthogonal and, therefore, they cannot be fully decoupled. Gheissary and van den Brule [17] and Ovarlez et al. [18] have both previously addressed the superposition of these two flows. To our knowledge, however, the idea to use the superposition of rotation and sedimentation to determine the fluid properties has not been proposed before.

We conducted measurements using four different fluids with different values of the consistency and power indices, $m$ and $n$, respectively. We found that the velocity of the sedimenting sphere is proportional to $\omega^{1-n}$, where $\omega$ is its rotational speed. With this new experiment, we are able infer the fluid properties of shear-thinning inelastic fluids, avoiding the unresolved questions arising in the classical sedimentation situation.

## 2. Description of the experiment and test fluids

Plastic spheres with diameters $D = 8.28$ mm were used. One or more small rare Earth rod magnets (Magcraft, models NSN0658) with a diameter of 3.18 mm were inserted into a plastic sphere as shown schematically in Fig. 1. The magnets were aligned horizontally, so as to induce a rotation around the sedimentation axis. For the tests shown in this paper, we used two spheres with weights of 0.33 and 0.42 g.

A sphere was released to sediment freely in the middle of a cylindrical container of height 300 mm and diameter 53.4 mm filled with the test fluid. The container was placed in the middle of a device able to generate a rotating magnetic field of constant intensity. Details of the magnetic setup can be found in Ref. [19]. Briefly, the device produces a uniform magnetic field in the entire container. If the direction of the magnetic moment of the magnet is different from that of the external magnetic field a torque is produced, resulting in rotation of the sphere. As the external magnetic field is rotating around the sedimentation axis, the sphere rotates at the same rate and in the same direction.

**Table 1**
Physical properties of the all fluids tested in this investigation. The ingredients for al fluids (water (W), glucose (G), ethylene glycol (EG), Carbopol (C) and triethyiamine (TEA)) are indicated in percentage by weight. The parameters $n$ and $m$ are the power and consistency indices, respectively, of the power law model. $\mu = m\dot{\gamma}^{n-1}$. The rheological measurements were conducted for $0.1 < \dot{\gamma} < 100$ s$^{-1}$.

| Fluid | Composition | $n$ (–) | $m$ (Pa s$^n$) | $\rho$ (kg/m$^3$) |
|---|---|---|---|---|
| N, (□) | G/W, 91/9 | 0.98 | 0.602 | 1390 |
| ST1, (▼) | EG/C/TEA, 97.91/0.06/2.03 | 0.87 | 0.15 | 1116 |
| ST2, (◆) | EG/C/TEA, 98.88/0.10/0.02 | 0.70 | 1.031 | 1113 |
| ST3, (●) | EG/C/TEA, 99.27/0.70/0.03 | 0.47 | 3.979 | 1110 |

For the test fluids, we fabricated three shear-thinning fluids and a reference Newtonian solution. These fluids are solutions of Carbopol (C) in ethylene glycol (EG). To modify the magnitude of the viscosity and the value of the power index, $n$, the composition of the solutions was varied. Small amounts of triethyiamine (TEA) were also used to modify the pH of the solutions and vary further the fluid properties. The Newtonian reference fluid was obtained from a solution of glucose and water.

The rheological characterization were conducted using a *TA Instruments AR1000N* rheometer with a cone-plate geometry (60 mm, 2°, 65 μm gap). The physical properties of the solutions are summarized in Table 1. All three non-Newtonian fluids follow closely a power-law behavior in steady shear, with negligible elasticity in the range of shear rates attained in the experiments ($\dot{\gamma} < 100$ s$^{-1}$.) The flow curves look very similar to those of [20], where the same type of fluids were used. We further conducted some oscillatory tests to confirm that elastic effects were negligible, following the scheme proposed by [21]. The composition of the Newtonian reference solution was chosen to have a shear viscosity of the same order as that of the shear thinning fluids.

## 3. Analysis

We consider a rigid sphere of radius $a = D/2$ sedimenting under its own weight, denoted $W^*$, along the $z$ direction. In addition to solid-body translation at speed $\mathbf{U} = U_s\mathbf{e}_z$, the sphere is assumed to rotate with angular speed $\mathbf{\Omega} = \omega\mathbf{e}_z,\ \omega > 0$ around the sedimentation axis.

A simple estimate for the sedimentation speed, $U_s$, can be found by balancing the weight of the sphere with a Stokes drag with an effective viscosity $\mu_{\text{eff}}$. Therefore, we can write

$$U_s = \frac{W^*}{3\pi D\mu_{\text{eff}}}. \tag{4}$$

Clearly, the sedimentation velocity will depend on the value of the effective viscosity, $\mu_{\text{eff}}$, which in turn will depend on the rheology of the surrounding fluid:

$$\mu_{\text{eff}} = f(\dot{\gamma}, n, \mu_0, \mu_\infty, N_1, N_2, \ldots), \tag{5}$$

where $\dot{\gamma}$ is the shear rate in the fluid, $n$ is the power index, $\mu_0$ and $\mu_\infty$ are the zero-shear-rate and high-shear-rate viscosities, respectively, $N_1$ and $N_2$ are the first and second normal stress differences, respectively. Of course, these properties will depend on the nature of the fluid rheology and the model used to characterize it; $\mu_{\text{eff}}$ would embed all the fluid effects of the sedimenting motion. A general closed form of the effective viscosity is not readily available.

For the case of shear-dependent viscosity fluids, $\mu_{\text{eff}}$ is given by Eq. (2). Therefore,

$$U_s = \frac{W^*}{3\pi Dm\dot{\gamma}^{n-1}}. \tag{6}$$

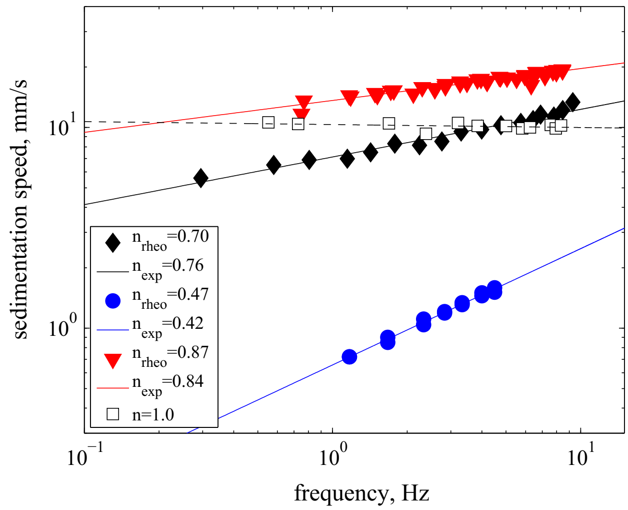


**Fig. 2.** Sedimentation speed, $U_s$ (mm/s), as a function of the frequency of the sphere rotation, $f = \omega/2\pi$ (Hz). Symbols for different fluids refer to the ones in Table 1. The solid lines are least-square fits to the experimental data.

In order to estimate $\dot{\gamma}$ in Eq. (6) we consider that its value is the result of superposing two different flows:

$$\dot{\gamma} = \sqrt{\dot{\gamma}_{sed}^2 + \dot{\gamma}_{rot}^2}, \tag{7}$$

where $\dot{\gamma}_{sed}$ and $\dot{\gamma}_{rot}$ are the shear rates resulting from the flow around a sedimenting sphere and rotation, respectively. It can be readily show that the mean shear around a sedimenting sphere is $\dot{\gamma}_{sed} \approx 2U_s/D$. For a rotating sphere the shear rate is simply $\dot{\gamma}_{rot} \approx \omega$. Therefore,

$$\dot{\gamma} \approx \omega\sqrt{1 + \left(\frac{U_s}{\omega D}\right)^2}. \tag{8}$$

For the experiments reported below, we have that $U_s/(D\omega) < 1$, and thus the typical shear rate in the fluid is dominated by the effect of rotation, $\dot{\gamma} \approx \omega$. Using this scaling in Eq. (6) we then predict the velocity dependence

$$U_s = \kappa\frac{\omega^{1-n}}{m}, \tag{9}$$

where $\kappa$ is constant that depends on $W^*$ and $D$.

If the change of sedimentation velocity is measured for different values of $\omega$, Eq. (9) can be used to infer the value of the power index, $n$. Furthermore, we note that

$$U_s m\omega^{n-1} = \text{constant}, \tag{10}$$

for a given sphere. The constant could be obtained for a given experiment; therefore, having access to the measurements from two different liquids, the value of $m$ could also be determined.

## 4. Results

We show in Fig. 2 the measured sedimentation velocity, $U_s$, as a function of the sphere rotation rate, $\omega$, for the four different liquids tested in this study (log–log plot). As predicted by theory, the sedimentation speed of the rotating sphere is not impacted by the rotation when the fluid is Newtonian ($n = 1$).

For all shear-thinning fluids, we observed a power-law increase of the sedimentation speed with the rotation rate, $U_s \propto \omega^\alpha$, consistent with Eq. (9). By fitting a power-law to the measurements can measure the value of $\alpha_{\text{exp}}$ and compare it to the prediction from the theory, $n_{\text{exp}} = 1 - \alpha_{\text{exp}}$. We find a very close agreement between the indices given by the fit to our data and those measured using a cone-plate rheometer. The results are shown in Table 2.

**Table 2**
Comparison between the properties obtained with the rheometer (*rheo*) and the sedimenting sphere measurements (*exp*) using Eqs. (9) and (10).

| Fluid | $n_{rheo}$ (–) | $m_{rheo}$ (Pa s$^n$) | $n_{exp}$ (–) | $m_{exp}$ (Pa s$^n$) |
|---|---|---|---|---|
| N, (□), reference | 0.98 | 0.602 | – | – |
| ST1, (▼) | 0.87 | 0.15 | 0.841 | 0.454 |
| ST2, (◆) | 0.70 | 1.031 | 0.763 | 0.891 |
| ST3, (●) | 0.47 | 3.979 | 0.421 | – |

It is interesting to note that the numerical results of Gheissary and van den Brule [17], closely resemble those presented here. In that case, an external shear flow was imposed to a sedimenting sphere. The shear caused an increase of the sedimentation velocity, in a similar manner to that shown here.

Finally, we can also deduce the value of the consistency index, $m$ from the measurements of the sedimenting-rotating sphere. Considering Eq. (10), we can write:

$$U_s m\omega^{n-1} = U_{s0} m_0 \omega^{n_0-1}, \tag{11}$$

where the subscript 0 denotes the properties of a reference fluid. Therefore,

$$m = m_0 \frac{U_{s0}}{U_s}\omega^{n_0-n}. \tag{12}$$

If the values of $U_{s0}$, $m_0$ and $n_0$ are known, the value of $m$ can also be determined by the proposed technique. We have considered the Newtonian measurements to be the reference (or calibration) fluid. For this case, we have found a reasonable agreement between the cone-plate measurements and those obtained with the new technique presented here, as shown in Table 2. Note that we could only obtain a comparison for two liquids, since only in these experiments the same sphere was used.

In the best case, the difference between the value deduced and that obtained with the rheometer is only 15%. For the other case, there is a factor of 3 difference (both values are of the same order of magnitude). The differences between the two values may arise, first, from the fact that the reference fluid and the test fluid have slightly different densities, which would make expression (11) not to be exactly true. Also, the constant on the right hand side of Eq. (10) may, in fact, depend on $n$, since the analysis does not include the fitting function $Y(n)$.

Based on these results we can conclude that our experimental setup is thus able to play the role of a rheometer, and we can determine the values of $n$ and $m$ with a precision similar to other instruments. For the particular case of the power index, $n$ a high precision can be obtained from sedimentation measurements.

## 5. Conclusion

In this investigation we conducted experiments measuring the sedimentation speed of rigid spheres in shear-thinning (power-law) fluids. Exploiting a newly developed magnetic method, we were able to impose rotation to the sedimenting spheres. By doing so, the flow around the sphere became rotation-dominated, and the sedimentation velocity increased. The increase of the velocity with the rotation frequency was found to follow a power-law, and a fit to our data allowed us to recover the power index, $n$, for each of our fluids. If the properties of a reference, or calibration, fluid are known, the value of $m$ can also be inferred from the sedimentation measurements. Our experimental apparatus can thus be used as a rheometer for power-law fluids. One may also attempt to generalize the theoretical analysis to consider other fluid rheologies; we feel that it is possible to extend the scope of the present setup to also measure the properties of other rheological behaviors.

There are some uncertainties that need to addressed, in particular to predict the value of the consistency coefficient, $m$. The constant in Eq. (10) may, in fact depend on the value of $n$, to some extent. The functional form of the drag coefficient is not well understood for the case of shear-thinning fluids. The fitting function $Y(n)$, discussed in Section 1, would have to be known to correctly determine $m$ from our measurements. Alternatively, a second experiment should be used (for example the flow generated by a non-translating rotating sphere in a shear-thinning fluid [22]), together with an empirical law of the form of Eq. (3) to determine $Y(n)$ and allow to obtain an estimate for $m$ from the sedimentation measurements.

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