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Book Reviews

BOOK REVIEWS

Our section starts with the featured review of Glenn Ledder's book *Mathematical Modeling for Epidemiology and Ecology*. This review is a joint work of 10 authors from Anita Layton's group. This shows that one can efficiently combine a reading course with an introduction to scientific work and the writing of a review. All reviewers are enthusiastic about the book and recommend it to all researchers interested in the topic.

We come from biological aspects to Eric Lauga's book *Fluid Mechanics: A Very Short Introduction*. Reviewer Anita Layton recommends the book especially for the younger generation of researchers. This is followed by Guenther Leobacher's review of *An Introduction to the Numerical Simulation of Stochastic Differential Equations*, by Desmond Higham and Peter Kloeden. The reviewer praises in particular that the authors have made the "not so easy subject accessible." After that Guenter Leugering reviews the book *PDE Control of String-Actuated Motion*, by Ji Wang and Miroslav Krstic. This book is recommended as having only a small gap between mathematical rigor and engineering objectives. In contrast to that, Sean M. Eli and Krešimir Josić note that in the book *Visual Differential Geometry and Forms*, by Tristan Needham, "the intuition gained" is "at the expense of the rigor." The final review is written by Mehrshad Sadria. He praises *Probabilistic Machine Learning: An Introduction*, written by Kevin Murphy, as a "valuable asset for both beginners and experienced readers."

On a final note, after six years of service, I am pleased to pass on the responsibility of the Book Reviews section to Anita Layton, who has been a valued member of our editorial board for some time now. I am delighted to announce that starting in 2024, Anita will be taking the helm of this highly successful segment of *SIAM Review*. I wish her as much joy and fulfillment in this role as I have experienced.

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Book Reviews

Edited by Volker H. Schulz

Featured Review: Mathematical Modeling for Epidemiology and Ecology. Second Edition. *By Glenn Ledder.* Springer Nature, Cham, Switzerland, 2023. $59.99. xx+ 364 pp., hardcover. ISBN 978-3-031-09453-8.

Mathematical Modeling for Epidemiology and Ecology, authored by Glenn Ledder, an Emeritus Professor of Mathematics at the University of Nebraska–Lincoln, presents readers with the necessary mathematical tools to comprehend and utilize mathematical models; it also delves into advanced mathematical biology literature. More specifically, the book explores the application of mathematics in biological contexts, specifically ecology and epidemiology, emphasizing key mathematical concepts and their biological implications through comprehensive explanations. The author assumes no prior mathematical background beyond elementary differential calculus.

The structure of the book is as follows: It begins with an introductory chapter that covers the fundamental principles of mathematical modeling. Subsequent chapters delve into empirical modeling and mechanistic modeling, providing a thorough treatment of essential ideas and techniques often overlooked in mathematics texts, such as the Akaike Information Criterion (AIC). The latter half of the book concentrates on the analysis of dynamical systems, focusing on simplification techniques for analysis, including the Routh–Hurwitz conditions and asymptotic analysis. Instructors have the flexibility to structure courses around either the first or second half of the book, or choose thematically from both sections, such as offering a course on mathematical epidemiology. The biological content within the book is self-contained and encompasses various topics in epidemiology and ecology. Some of this material is presented through case studies that explore detailed examples, while others draw on the author's recent research on vaccination modeling and scenarios from the COVID-19 pandemic.

We will provide comments on individual chapters below. But what's common among chapters is that each contains problem sets that include interconnected problems that present a single biological setting in multistep scenarios, categorized into relevant sections. This approach allows readers to gradually develop comprehensive investigations into topics such as HIV immunology and the sustainable harvesting of natural resources. Some problems incorporate computer programs developed by the author using MATLAB or Octave, complementing more traditional mathematical exercises and equipping students with a comprehensive set of tools for model analysis. Each chapter includes additional case studies in the form of projects, accompanied by detailed instructions. Additionally, appendices provide mathematical details on optimization, numerical solution of differential equations, scaling, linearization, and advanced utilization of elementary algebra to simplify problems.

Chapter 2 provides a comprehensive exploration of *empirical modeling*, a methodology that involves the application of mathematical models to determine the optimal

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parameter values that yield the best fit for the collected data. The chapter gradually progresses from elementary linear mathematical models to more complex nonlinear models. The chapter begins by introducing the method of least squares, and explains how to use it to fit linear models (e.g., $y = mx + b$) and how to linearize selected nonlinear models. While we understand that the author assumes only elementary mathematical background, if a reader is unfamiliar with a straight line, how much of this book can they truly comprehend? The lack of mathematical maturity is often a bigger obstacle than the lack of training itself. Putting that concern aside, we note that the chapter builds upon those foundations and proceeds to develop semilinear models that offer a generalized representation of the exponential and power function models, accommodating any model expressed as $y = A(x;p)$, with p representing the parameter of the nonlinear function $f(x)$. This extension allows for greater flexibility in modeling a wide range of phenomena.

Moreover, the chapter highlights the importance of selecting a model that matches the scenario. That important concept in modeling is explained using simple linear models. We also appreciate the author's discussion of the principle of Occam's razor, which advocates for choosing the simplest model whenever feasible. The author provides comprehensive examples to demonstrate how a simple model can predict better than a complex model. To quantitatively determine the most suitable model, the AIC is employed, providing a rigorous framework for model selection. Here, the author could have included an explanation behind the replacement of the statistical parameter K in the AIC equation with $(k + 1)$, where k denotes the number of parameters in the model.

Lastly, the chapter concludes by providing a case study of different methods used to fit the parameters of the nonlinear Michaelis–Menten model. The author compares different linearization schemes used to fit this model through their AIC scores and explains why some of these methods have inferior performance. This gives the reader a hands-on experience of fitting a model that is extensively used in biological modeling.

In conclusion, Chapter 2 is well designed for beginners in empirical modeling, showcasing a meticulous approach to methodological development through mathematical proofs, MATLAB examples, and graphical illustrations. By incorporating exercises and examples, the author facilitates active learning and comprehension. The use of graphical elements proves to be a valuable tool for conveying complex ideas concisely and intuitively. To further enhance the academic review, the inclusion of a critical analysis of the methods would provide readers with a more nuanced perspective, enabling them to assess the strengths and weaknesses of the presented techniques.

Having developed some elementary modeling tools, in Chapter 3 the author introduces how to formulate mechanistic models. This class of models is based on the fundamental laws of natural sciences, including physical and biochemical principles. By capturing fundamental principles of a system, one may obtain explanatory value from the components of the model. As in other chapters, the author conveys this information in manageable bites, using a simple, clear, and concise writing style. The overall goal of the chapter is to explain how to build mechanistic models with different levels of complexity, using simple tools covered in Chapter 2 and new ones introduced here, and to illustrate these concepts using examples in epidemiology.

One topic that is covered is compartmental analysis, which is a useful approach for providing large-scale structure for some models. Here, the Susceptible-Exposed-Infected-Recovered (SEIR) epidemic model is used as an example. Specifically, the authors introduce the basic reproduction number (a concept that a good fraction of the general public has become familiar with during the pandemic), analytical and numerical methods, early-phase exponential growth, and the end state of such models. The popularity of SIR-type models has skyrocketed during the pandemic, rendering them particularly helpful as examples here. Equivalent forms of mechanistic models are also described and discussed. The author notes that some mechanistic models may be written in different forms based on one's preference and analysis type. The author also shows how the reader may identify equivalent models as well as convert their own mechanistic models into other forms such as a dimensionless form.

We find this chapter is a good starting point for those seeking to understand mechanistic modeling with concrete examples, especially in the context of epidemiology and ecology.

Chapter 4 discusses various types of models describing single population dynamics and the analysis that can be done. The author presents the discrete case, the continuous case, and an example of how to approach the analysis. In the discrete case, the author covers a general seasonal population model, the exponential and logistic situations associated with the model, and the cobweb plot for analysis of stability and fixed points. The author then introduces the continuous equivalents, showing the methods to find equilibrium points, visualize behavior with phase plots, and linearize a model to find local stability. Finally, they give the example of using the above techniques to analyze whale population dynamics, modeled by a growth-consumer. Doing so enables them to create a narrative.

Overall, we find the choice of topics in this chapter suitable, and the author's writing style particularly appealing to first-time learners in ecology modeling. Because the techniques introduced in this chapter are rather general, they can be transferred to many other use cases of population dynamics analysis, such as animals, diseases, and lake algae.

Chapter 5 moves naturally from single to multiple population dynamics; it delves into the dynamics of systems consisting of multiple related quantities changing over discrete time intervals. It expands upon the concepts discussed in Chapter 4 by introducing a more complex model of several subpopulations. The chapter focuses on linear systems, which are used to represent structured populations categorized by age, size, or developmental stage.

The chapter takes a very hands-on approach to learning. It is full of helpful features aimed at improving the reading experience and learning outcomes. It begins with a list of learning objectives; key definitions are highlighted as they emerge; and breakpoints are introduced to prompt the reader to check their understanding. Importantly, the first technical exposition is a practical example of the evolution dynamics of a juvenile and adult population. The author stresses the use of schematic diagrams to represent word problems. The approach of preceding theory by applications can be helpful for understanding mathematical concepts. It can allow students to develop intuition for the methods, which often aids later understanding of the theory. The language is accessible to a wide-range audience, particularly undergraduate students in mathematics, who are the intended target group. The author wisely advises readers on the importance of parameter estimates for useful predictions, particularly in the context of population dynamics.

Overall, Chapter 5 is well illustrated with three cases ranging from ecology to biology; exercises at the end of each chapter build directly upon the applications presented. This chapter reads well as a self-study text for those interested in learning about modeling in ecology and epidemiology. As a result, it is most relevant to applied undergraduate students, epidemiologists, and theoretical biologists, though more seasoned mathematicians may also find some of the content interesting. Indeed, we have found it to be one of the best introductions to discrete linear systems for beginners.

The book ends with Chapter 6, which focuses on nonlinear dynamical systems with multiple variables, expanding upon the foundation laid in the previous chapter of a single variable world.

The initial portion of the chapter introduces readers to phase plane analysis, an essential tool for studying the stability of equilibria in nonlinear systems. The author skillfully breaks down this concept, making it accessible and comprehensible to readers by providing examples, theorems, and problem sets. The use of practical examples and clear explanations enhances the learning experience and ensures that the material is not overly daunting. The subsequent sections of the chapter focus on two methods of analyzing nonlinear systems: the standard eigenvalue-based approach and the more advanced Routh–Hurwitz conditions. The author adeptly presents these methods, offering a comprehensive understanding of their respective strengths and weaknesses. The inclusion of a case study example enriches the chapter further, providing readers with a practical application that reinforces the theoretical concepts discussed. In addition to covering continuous systems, the chapter also delves briefly into discrete systems and their limitations. The author provides valuable insights into the circumstances in which discrete systems are preferable, particularly within a biological setting.

We will end with comments that apply equally well to Chapter 6 and the book in general: We find this chapter a well-structured and informative segment of the book. As we noted before, the author's writing style is clear and engaging, ensuring that readers can follow along and grasp the intricacies of the subject matter. The combination of theoretical explanations, case study examples, and discussions of practical applications creates a comprehensive learning experience. One minor suggestion for improvement would be to include a code section below each example for readers to practice applying the concepts learned. This might allow readers to solidify their understanding and improve their problem implementation skills.

Our conclusion is short: This is a well-written book, highly suitable for applied math undergraduate students.

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Fluid Mechanics: A Very Short Introduction. *By Eric Lauga.* Oxford University Press, Oxford, 2022. $11.95. 144 pp., softcover. ISBN 978-0-198-83100-6.

I love fluid, and that is why I chose Eric Lauga's *Fluid Mechanics: A Very Short Introduction* to review. And the book most certainly did not disappoint! Upon opening the book, I was immediately captivated by the very first sentence in the preface:

> Fluid mechanics is many things to many people.

Indeed! It is fair to say that most readers of this book are already interested in fluid mechanics, but if the author had to "sell" the field, he could list the numerous practical applications of fluid mechanics: engineering and design (it is essential for understanding the behavior of fluids in pipes, channels, pumps, turbines, aircraft, ships and other devices); transportation and energy (key in developing aerodynamic designs for airplanes and vehicles); environmental studies (analyzing water and air pollution); and then there is my own field, biology and medicine, where fluid mechanics plays a role in understanding blood flow, cardiovascular dynamics, and respiratory systems in the human body. Taken together, fluid mechanics is important because it provides the necessary tools and understanding to solve real-world engineering problems, improve efficiency, and advance our knowledge of fluid behavior in various natural and man-made systems. The author did mention research areas in fluids, although those applications aren't used as a hook but instead appear in the very last chapter, after the reader has already gone through most of the book.

Fluid Mechanics: A Very Short Introduction is part of the *Very Short Introduction* series published by Oxford University Press. The books in the series are concise introductions to particular subjects, intended for a general audience but written by experts. Most are under 200 pages long and are meant to be "balanced and complete." The series began in 1995, and by March 2023 there were 730 titles, published or announced. The book series has been commercially successful, and its books have been published in more than 25 languages. I imagine *Fluid Mechanics: A Very Short Introduction* not only lives up to the standards set by the other 700+ titles in the series, but may even exceed them in some aspects (not that I have read many of the books in the series, thus the word "imagine"). I am particularly impressed by the author's exceptional ability to captivate the readers right from the preface and sustain their engagement throughout the entire work.

The book is organized into eight chapters: "Fluids," "Viscosity," "Pipes," "Dimension," "Boundary Layers," "Vortices," "Instabilities," and "Researching Fluids and Flows." These are essential concepts in the study of fluid mechanism. The writing is clear and engaging, and each chapter contains several well-chosen illustrations, for a total of 41 figures; some are photographs, others are line art.

The book highlights a number of well-known scientists in fluid mechanics, ending with two "giants," James Lighthill and Pierre-Gilles de Gennes. The impacts these scientists had on the field are all noteworthy. It is equally noteworthy, albeit not in a positive way, that these scientists are all male. This fact was acknowledged by the author, who also expressed some optimism that "this is now beginning to shift and more women are coming into the field as undergraduates and as researchers." I do hope, as does the author, that "the future of fluid mechanics promises therefore to be more diverse and inclusive." Nevertheless, the lack of role models in publications like the one under review is a discouraging factor for underrepresented groups, not only in gender but in race as well.

This book should make a good read for undergraduate students whose coursework includes elements of fluids. It can help them comprehend and appreciate the inherent beauty of this subject. I would even go as far as to say that even exceptionally talented high school students and anyone with an interest in science should give it a try, as this is undoubtedly one of the ultimate objectives of any book in the *Very Short Introduction* series. In a way, this book is particularly suitable for the younger generation. Measuring only 10.8×17.15 cm, the print is small and not the easiest to read for those who no longer have youthful eyesight. But that is a minor point. Overall, the book should definitely be an individual's go-to source when seeking an authoritative perspective on the field of fluid mechanics. Even for those who are familiar with the field (like myself), they can still benefit and find fresh and insightful perspectives within its pages.

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An Introduction to the Numerical Simulation of Stochastic Differential Equations. *By Desmond J. Higham and Peter E. Kloeden.* SIAM, Philadelphia, 2021. $79.00. xvi+277 pp., hardcover. ISBN 978-1-61197-642-7. https://doi.org/10.1137/1.9781611976434.

This is a book with a very strong focus on making the presentation of a not so easy subject accessible. How do the authors do it? They do it by maintaining an entertaining style that keeps the reader interested, having many small sections instead of long ones, by not dwelling on the many intricacies that tend to occupy (more theoretically oriented) mathematicians. On top of that, every section closes with a (useful and/or interesting) example program in MATLAB, an example section (with difficulty marked), and, as a reward ("Readers are not allowed to look at these until they have done all the exercises"), a small collection of quotes.

The generous approach regarding mathematical detail makes it possible to cover a wide range of topics, which in this sense makes the book much more than an introduction. Not only are the indispensable algorithms for simulation of SDEs (random number generation, the Euler–Maruyama method, higher order methods) covered, but convergence (weak and strong) and stability issues are treated as well. There is also a wide range of examples, in particular from mathematical finance, and modern techniques, such as multilevel Monte Carlo.

The book is directed at practitioners but is certainly also useful for more theoretically inclined researchers who want to get an overview of the field. For proofs and topics that are beyond the scope of the book the reader is pointed to the appropriate references.

The book broadly consists of three parts. The first is an introduction to probability theory, starting with the basic concepts such as events and random variables, but swiftly heading towards stochastic integration and stochastic differential equations. Throughout, the notions are illustrated by computer experiments. Some effort is put into discussing alternative concepts, in particular, the Stratonovich version of the theory.

The next chapters are devoted to numerical methods: The Euler–Maruyama method is introduced, together with the notions of weak and strong convergence of a numerical method for SDEs. The actual proofs of weak and strong convergence are then sketched in separate chapters. A chapter about implicit methods and numerical stability concludes this part.

The third part, comprising half of the book, could be described as the "advanced topics" part. These topics in turn can be divided into concrete applications, such as option pricing or chemical kinetics, advanced numerical methods (higher order methods, multilevel Monte Carlo), and generalizations, such as systems of SDEs (accompanied with numerical methods) and SDEs with jumps. All of the material is (almost) equally as accessible as the earlier parts.

Besides important facts about numerical simulation of SDEs one can learn something equally useful from this book: how to write about mathematics in a way that does not appeal only to mathematicians.

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PDE Control of String-Actuated Motion. *By Ji Wang and Miroslav Krstic.* Princeton University Press, Princeton, NJ, 2022. $165.00. xxiii+488 pp., hardcover. ISBN 978-0-691-23348-2.

This book addresses the problem of observation, adaptive control, and stabilization of hybrid 1-D wave equations coupled to possibly nonlinear dynamics at the boundary involving interior dynamics as well as controls and dynamic observers. The motivation comes from drilling applications in which possibly very long cables or strings are used to elevate masses in a constrained environment. The corresponding mathematical modeling is based on a PDE-ODE setup that may, among other things, affect the length of the cable(s), so that time varying coefficients and moving domains are included in the scope of problems. While each cable or elastic string, originally modeled by a second order hyperbolic equation, can be represented by a 2×2 first order hyperbolic system, multiple string systems are being represented by first order hyperbolic systems of even dimension which, together with corresponding first order two-point boundary couplings and initial conditions, form an inhomogeneous hybrid initial-boundary-value problem. The particular emphasis of the work under review here is the possibly nonlinear and nonlocal-in-time boundary dynamics, modeled by systems of ordinary differential equations with inputs and outputs. Therefore, in principle, a theory of observation, adaptive control, and stabilization is what is needed for general systems of first order hyperbolic balance equations under nonlinear and nonlocal-in-time boundary conditions. In this generality, this is beyond the scope of the current research, and if such a theory were to exist it would be mathematically rather abstract. Indeed, the emphasis of this monograph is on practical realizability. Therefore, and because of the nature of this monograph as a textbook addressing also graduate students and postdocs, the authors have chosen an exemplary approach where the reader is guided along the path of ever increasing complexity of the underlying systems, starting from a single cable with simple boundary dynamics in Chapter 2 to adaptive control of systems of ODE-sandwiched hyperbolic equations with irregular data and disturbances in Chapter 15.

The systems under consideration can be brought into the format

$$
\begin{aligned}
&\dot X(t) = AX(t) + BY(0,t), \quad t \in (0,T),\\
&\frac{\partial}{\partial t}Y(x,t) = \mathcal{A}(x,t)\frac{\partial}{\partial x}Y(x,t) + \mathcal{B}Y(x,t),\\
&\qquad (x,t) \in (0,\ell(t)) \times (0,T),\\
&\mathcal{C}_0 Y(0,t) = \mathcal{D}_0 X(t), \quad t \in (0,T),\\
&\mathcal{C}_l Y(\ell(t),t) = \mathcal{D}_l U(t), \quad t \in (0,T),
\end{aligned}
$$

where $Y(x,t)$ denotes the state governed by the system of hyperbolic first order differential equations parametrized by $(\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D})$, $X(t)$ is the state of the dynamical system (A,B) acting on the boundary at $x=0$, and $U(t)$ is the possibly vectorial control input. We understand that the varying length $\ell(t)$ can be achieved by a change of variables transforming the individual lengths $\ell_i(t)$ to $\ell(t)$ at the cost of having time-dependent coefficients in $\mathcal{A}(x,t)$.

The method that is brought to bear is the backstepping method that M. Krstic has further developed over the past few decades jointly with various coauthors, including the coauthor of this volume. He has published, together with A. Smyshlyaev, a textbook on this method with the title *Boundary Control of PDEs: A Course on Backstepping Designs* (SIAM, 2008) where the method is explained in detail.

The terminology of backstepping goes back to the work of P. Kokotovic in the early 1990s and refers to a recursive method for stabilizing the origin of a system in strict-feedback form. In a sense, it resorts to a subsystem for which the control and stabilization properties are known. In the context of the method developed by Krstic and coauthors, for each particular system, there is a *target system*, again a system with known control and stabilization properties, to which the original system can be transformed by the way of a change of state variables. As for the target system with state $W(x,t)$, which is governed by the same type of PDE, one may want to achieve a homogeneous boundary condition, say, $\mathcal{C}_0 W(0,t) = 0$, and/or one may want to decouple the state equations. Moreover, a state feedback $U(t) = \mathcal{K}W(\ell(t),t)$ and a feedback K at the boundary are implemented such that the target system

$$
\begin{aligned}
&\dot X(t) = (A+BK)X(t) + BW(0,t), \quad t \in (0,T),\\
&\frac{\partial}{\partial t}W(x,t) = \mathcal{A}(x,t)\frac{\partial}{\partial x}W(x,t) + \tilde{\mathcal{B}}W(x,t),\\
&\qquad (x,t) \in (0,\ell(t)) \times (0,T),\\
&\tilde{\mathcal{C}}_0 W(0,t) = 0, \quad t \in (0,T),\\
&\tilde{\mathcal{C}}_l W(\ell(t),t) = \tilde{\mathcal{D}}_l \mathcal{K} W(\ell(t),t), \quad t \in (0,T),
\end{aligned}
$$

has desired stability properties. There is a large variety of modifications, e.g., in order to include dynamic observers and many other design features. Typically, this change of variables is realized by a Volterra integral operator, like

$$
W(x,t) = Y(x,t) + \int_0^x \Gamma(x,s)Y(s,t)dx + \beta(x,X(t)).
$$

Applying the differential operators to this representation transforms it into a Volterra-integrodifferential equation of second type. The parameters, in particular the (vectorial) kernel $\Gamma(\cdot,\cdot)$ of the operator, have to be constructed from the side conditions and the data of the original system and the target system. The constructions are explicit, for constant coefficients, and they need to be explicit in order to be directly applicable, as the control strategies are based on the transformation. In general, a numerical method has to be employed for the solution to the (partial) differential equations for the operator kernel Γ which guarantees the stability of the resulting system. The desire to have computable control laws necessitates the choice of particular 1-D PDEs, even though in principle the method can be used in special 2-D or 3-D examples.

The method of backstepping in its current form can be said to be very rich. There is an abundance of PDE-control problems that have been solved using this method. In this monograph, the authors have selected a particular class which serves perfectly as a guideline through the wealth of results available in the literature, while providing an in-depth analysis in an exemplary fashion.

The introductory chapter addresses exactly and very clearly the scope of the book. It provides very important information and guidelines for the researcher as well as for the teacher who wants to design a course for a particular group of students. Tables with various categorized features of the particular systems make the entire monograph very transparent. For example, one may be interested in chapters where axial, transverse, or torsional vibrations of the cables are addressed, or one may ask for those examples where both ends are subject to extra dynamical systems (A, B)—the *sandwiched* PDE-ODE problems. Another question that the reader may ask concerns the type of data used in the problems. For a more in-depth discussion of these features, the authors provide a thorough description of the particular applications in drilling. The interplay between the depth of modeling and the engineering driven objectives transports a clear message to the reader: Top-down theory, going from the abstract to the concrete, may not lead to actual devices. Rather, one should focus on the desired properties and computable results in order to extend the successful backstepping method to new horizons!

The book is structured in three major parts, where Part I provides the description and the analysis of applications, and Part II extends the scope to sandwiched hyperbolic systems and event-triggered controls including nonlinearities. As a highlight, it is shown that the event-triggered control scheme does not lead to Zeno-effects. Part III is devoted to adaptive hyperbolic PDE-ODE systems. Here the feedback gains are designed to be self-tuning, while projectors guarantee boundedness.

In all chapters, the mathematical analysis is complemented by numerical results and reports on applications to a real plant, thereby closing the cycle from the application, via mathematical modeling, transformation to a predesigned target system and the related mathematical analysis, followed by numerical simulation, and back to its validation in the context of the original application. This is what the metaphor *applied mathematics* is all about. In this field, there is often a gap between pretension and reality, a mismatch between mathematical rigor and engineering objectives. This book is a nice counterexample!

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Visual Differential Geometry and Forms: A Mathematical Drama in Five Acts. *By Tristan Needham.* Princeton University Press, Princeton, NJ, 2021. $132.00. xxviii+501 pp., hardcover. ISBN 978-0-691-20369-0.

Tristan Needham's 1997 book,[1] *Visual Complex Analysis* (VCA), was a masterful, but somewhat divisive introduction to the topic [2]. It offered historical context, and a plethora of diagrams to give visual explanations of ideas that are often taught only abstractly. A common complaint was that the intuition gained was at the expense of the rigor and conciseness offered by a more traditional approach. Needham's recent book, *Visual Differential Geometry and Forms: A Mathematical Drama in Five Acts* (VDGF) unapologetically continues in

[1]The second edition of VCA appeared as we were writing this review

the style of VCA [1].

VDGF is accessible to advanced undergraduates and anyone interested in gaining a deeper understanding of differential geometry and its applications to physics. As in the previous book, the author uses numerous visual aids to explain the geometric intuition behind the main ideas and formulas. Needham has written the book in five "acts," starting from early discussions about the parallel postulate, to metrics and curvature, and finishing with Cartan's differential forms. The text is rich with historical notes, quotes from manuscripts, and original statements of theorems, emphasizing the intuition that is absent from many sources. Each act contains several exercises (some of which involve fruit and other props!); it is clear that Needham had fun writing the book, and it is a pleasure to read.

Act 1, "The Nature of Space," is accessible to the general reader. This act introduces the concepts of curvature and non-Euclidean geometry, and gives practice with Newton's *Ultimate Equality*, a geometric interpretation of limit. This feature adds a "classical geometry" flavor to the proofs in VDGF compared to typical books. Needham introduces geodesics and the concepts of intrinsic and extrinsic geometry. The idea that "straight" and "distance minimizing" lines are equivalent is explored by pulling a string tight across a squash, and peeling off a thin neighborhood of the string. Gauss curvature and the local Gauss–Bonnet theorem are motivated.

Act 2, "The Metric," introduces Riemannian metrics as rules for determining the infinitesimal distance between points, without mentioning tensors and vector fields. Through several worked-out examples Needham demonstrates how metrics give rise to geometry, and how to derive coordinate formulas for metrics. The three model geometries are discussed, including several models of hyperbolic space. Geodesics in the hyperbolic plane are intuitively derived from the form of the metric, and then computed from Snell's law of refraction. The Möbius group is used to construct isometries of the model geometries, as well as the Lorentz group of symmetries of Einstein's spacetime.

Act 3, "Curvature," begins with a discussion on space curves and Frenet frames. Geodesic curvature is introduced, and Clairaut's theorem is proven. Gauss's *Theorema Egregium*, as well as its nonlocal (known secretly to Gauss) predecessor, the "beautiful theorem," are introduced, to be proven later. Half of the act is devoted to the global Gauss–Bonnet theorem and provides three proofs: one via Hopf's *Umlaufsatz*, one via angular excess, and one via the Poincaré–Hopf theorem, which is also proven. The Gauss–Bonnet theorem is motivated with a brilliant story about configurations of bagels. Another gem in the book is the introductory discussion on the Euler characteristic, which includes two proofs of Euler's polyhedral formula.

Act 4, "Parallel Transport," introduces the covariant derivative and parallel transport, without mentioning Christoffel symbols (they are mentioned in Act 5). Holonomy is used to give a simple, geometric proof of the *Theorema Egregium*; the standard proof of the formula for Gauss curvature in terms of the metric is also given. The Gauss–Bonnet theorem is proven again using holonomy. This act makes the jump to n-manifolds and introduces Riemann's and Ricci's curvature tensors; the geometric meaning behind both tensors is explained exceptionally well. Jacobi fields are motivated and the sectional Jacobi equation is proved. The act concludes with a chapter on general relativity, motivating and exploring the connection between curvature and gravity, and deriving Einstein's field equations. The original tests of Einstein's theory are another nice feature of the act.

Act V, "Forms," introduces differential forms (the abstract algebraic "devil") to prove several of the earlier theorems. The computational speed and power gained from using forms, at the cost of geometric intuition, is emphasized. The act introduces abstract tensors and provides geometric meaning for low-dimensional differential forms. The exterior derivative and integration are discussed and the generalized Stokes theorem is proven. A nice addition is de Rham cohomology, which is introduced briefly but clearly. Finally, several of the earlier results are elegantly proven with Cartan's method of moving frames. Highlights include a concise statement of Maxwell's equations using the Hodge star, and a proof of the Hairy Ball theorem in the exercises.

Overall, the book can be viewed as a reaction to the abstraction that is typical of many modern books. VDGF is organized mostly in chronological order, and the interspersed historical remarks give excellent intuition for the main ideas. The book includes most of the topics found in a standard undergraduate book, like the often used textbook of M. P. do Carmo [3]. Indeed, VDGF introduces more advanced topics like de Rham cohomology and curvature forms, and even general relativity.

Given that Needham's goal is to "put geometry back in differential geometry," rigor and analysis are generally not emphasized. This means that technical hypotheses about continuity, differentiability, and proofs of existence of fundamental objects like metrics and geodesics are not the focus of the book. Thus, as Needham claims in the prologue, the book alone is not sufficient to prepare students for research in differential geometry. However, VDGF offers intuition for students who wish to read and understand graduate books, and Needham offers a detailed list of further reading. We believe that VDGF is an excellent companion to traditional textbooks, and a study of both can provide a deeper understanding of differential geometry.

Is this book appropriate for an introductory class in differential geometry? This depends largely on the goals of the instructor and the interests of the students. The second author's experience with VCA is that good students will appreciate the challenge this approach offers and find the resulting insights rewarding. However, average students will struggle, and some may even rebel. Instructors may also want to supplement the material in the book using more traditional and "rigorous" explanations. Unfortunately, all of this may not be possible in a single semester, let alone a quarter. While the book is suitable for self-study, some exercises may be difficult (though rewarding).

VDGF is a book that stands out among its peers. It is fun to read and provides a unique and intuitive approach to differential geometry. The author's passion for the subject is evident throughout the book. Although Needham's approach is unorthodox, it is rewarding, and complements the exposition found in standard textbooks. It will be useful to instructors and students, as well as practitioners.[2]

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Probabilistic Machine Learning: An Introduction. Second Edition. *By Kevin P. Murphy.* MIT Press, Cambridge, MA, 2022. $125.00. 864 pp., hardcover. ISBN 978-0-262-04682-4.

Introduction. Machine learning (ML) and deep learning (DL) have become integral parts of our lives, revolutionizing the way we process and interpret data. To enhance and advance current ML models, it is imperative to have a strong foundation in the fundamentals of ML. Kevin Murphy's *Probabilistic Machine Learning*, in its second edition, is a well-written book that explores the essential foundations of probabilistic ML, providing readers with the

[2]A more detailed review from the perspective of a researcher in differential geometry can be found here: https://www.math.colostate.edu/~clayton/research/papers/VDGF.pdf [4]

necessary knowledge to improve existing models and delve deeper into the field.

Content and Approach. This book encompasses a wide range of topics in ML, making it accessible even to those without a deep background in the subject matter. It begins by laying out the basic definitions of statistics and common mistakes people make, ensuring that readers have a solid understanding of the fundamentals. However, prior knowledge of probabilistic theory proves helpful in fully comprehending the concepts presented. Murphy adeptly elucidates complex probability concepts used in probabilistic ML, explaining them in a clear and understandable manner. His expertise and eloquence shine through, making this book an excellent resource for teaching, explaining, and clarifying concepts for readers of all levels. One of the book's greatest strengths is its coverage of a wide range of probabilistic models and algorithms. Murphy explores various approaches, including Bayesian networks, Gaussian processes, hidden Markov models, and latent variable models, among others. Each topic is presented in a structured manner, with explanations of the underlying theory, accompanied by practical examples and code snippets.

Applications and Vision. *Probabilistic Machine Learning* not only imparts theoretical knowledge but also presents numerous practical applications of this approach. The book vividly illustrates how probabilistic ML can be applied to various domains, enabling readers to envision its promising future. Murphy's inclusion of real-world applications adds a practical aspect to the book, making it relatable and captivating.

Problem Sets and Code Reproducibility. One of the book's standout features is the inclusion of problem sets at the end of each chapter. These exercises encourage readers to apply the concepts learned and reinforce their understanding. Furthermore, the book provides code snippets that allow readers to reproduce the graphs and visualizations presented throughout the chapters. This interactive aspect enhances the learning experience, enabling readers to not only understand the concepts better but also improve their skills in visualizing the concepts.

Conclusion. *Probabilistic Machine Learning* by Kevin Murphy is an exceptional resource for individuals seeking a comprehensive understanding of the foundations of probabilistic ML. The book's clear and concise writing style, coupled with the author's profound knowledge and expertise, makes it a valuable asset for both beginners and experienced readers. With its broad coverage of topics, practical applications, problem sets, and code reproducibility, this book proves to be an indispensable guide. Whether in paper or electronic form, reading this book is undoubtedly informative.

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