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A Note on the General Concept of Wave Breaking for Rossby and Gravity Waves

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Abstract—A recently proposed general definition of wave breaking is further discussed, in order to deal with some points on which misunderstanding appears to have arisen. As with surface and internal gravity waves, the classification of Rossby waves into 'breaking' and 'not breaking' is a generic classification based on dynamical considerations, and not a statement about any unique 'signature' or automatically recognizable shape. Nor is it a statement about passive tracers uncorrelated with potential vorticity on isentropic surfaces. A strong motivation for the definition is that proofs of the 'nonacceleration' theorem of wave, mean-flow interaction theory rely, explicitly or implicitly, on a hypothesis that the waves do not 'break' in the sense envisaged.

The general definition refers to the qualitative behaviour of a certain set of material contours, namely those, and only those, which would undulate reversibly, with small 'slopes', under the influence of the waves' restoring mechanism, in those circumstances for which linearized, nondissipative wave theory is a self-consistent approximation to nonlinear reality. The waves' restoring mechanism depends upon the basic-state vertical potential density gradient in the case of gravity waves, and upon the basic-state isentropic gradient of potential vorticity in the case of Rossby waves. In the usual linearized theory of planetary scale Rossby waves on a zonal shear flow, the relevant material contours lie along latitude circles when undisturbed.

Key words: wave propagation; linear Rossby wave theory; stratospheric planetary waves; potential vorticity; passive tracers; critical layers; wave breaking; wave mean-flow interaction; reversibility

Introduction

The paper 'A critical analysis of the concept of planetary wave breaking' (Rood, 1986, hereafter R) questions both the meaning and usefulness of our definition of wave breaking (McIntyre and Palmer, 1983, 1984, hereafter MP83, MP84, collectively MP). The purpose of the definition was to generalize the idea of breaking, as commonly applied to surface and internal gravity waves, in such a way that it is also useful in connection with planetary scale Rossby waves and with the ideas and general theorems of wave, mean-flow interaction theory.

In R it is suggested, *inter alia*, that the definition 'incorrectly classifies' certain

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stratospheric phenomena, and that 'since the theoretical basis of Rossby wave breaking is not well established, it is not at all clear that there are Rossby wave analogues to breaking surface gravity waves.' Also *implicitly* called into question in R are the ideas of 'wave', 'wave propagation', and 'reversibility', as commonly used in theoretical physics. These fundamental ideas of physics formed the conceptual background to our definition. It is possible that a misunderstanding about them may underlie the way in which the definition is discussed in R§2bff., where a crucial point is missed (e.g. 3, 11 below).

We do not wish to suggest that the concepts of wave propagation and wave breaking are indispensable to understanding the dynamics and chemistry of the middle atmosphere, even though a number of investigators have found both concepts useful for this purpose. However, there are some fundamental points, including those already mentioned, which call for clarification.

Specific comments on some of the points raised in R

1. The discussion in R places great emphasis on passive-tracer fields, regardless of their correlation, or lack of it, with isentropic distributions of Rossby–Ertel potential vorticity (IPV distributions). In each of the first two examples presented in R§4a, the tracer and IPV distributions are very different. But it should hardly need saying that the concept of wave breaking is primarily a *dynamical* concept, not a passive-tracer concept. In the case of surface and internal gravity waves it has to be viewed in the context of gravity-wave dynamics, and in the case of Rossby waves, in the context of Rossby-wave dynamics.

2. The dynamical mechanism to which Rossby waves owe their existence—and which makes it possible for such waves to propagate, for instance, up from the troposphere into the wintertime stratosphere—depends upon IPV distributions. That is one reason why IPV distributions were given such emphasis in MP. As with other types of wave propagation in material media, the dynamical mechanism can be thought of as a 'restoring' mechanism, capable of counteracting small displacements of certain material contours. In the usual linearized, nondissipative theory of planetary-scale Rossby waves on a zonally symmetric basic state, the relevant material contours lie along latitude circles in the absence of waves. The restoring mechanism depends on the existence of a latitudinal isentropic gradient of potential vorticity, and comes into play when the material contours in question, which coincide with IPV contours in the absence of dissipation, undulate sideways. This well known fact was recalled briefly at the beginning of MP83; a fuller account is given in the recent review by Hoskins *et al.* (1985, §§6a, c).

Note that, in order for the linearized wave theory to be a self-consistent approximation to nonlinear reality—along with associated concepts such as the principle of superposition—the relevant material contours must, in general, undulate only gently. More precisely, the angle through which their local orientation fluctuates must, in general, stay small. In the gravity-wave literature this is sometimes referred to as the small 'slope' or 'wave steepness' condition; the reasons for it are recalled in the Appendix below. The discussion in R makes very strong claims for the applicability of linearized wave theory (see for instance comment 6 below), but makes no reference to the small 'slope' condition, nor to any other way of expressing the conditions for validity of that theory.

3. The restatement in R§2b of our wave-breaking definition correctly quotes MP83, but the subsequent discussion, and selection of quotations from MP84, miss a crucial point. The point is that the definition refers only to those material contours which are relevant, in the sense already indicated. They are characterized in general as those material contours which would undulate, and not deform irreversibly, 'under the conditions assumed by linear, nondissipative wave theory' (MP84§2). The first and second examples in R§4a refer to material contours which do not even approximately qualify under the definition.

4. The definition distinguishes situations, called 'wave breaking', in which the relevant material contours deform rapidly and irreversibly, from 'pre-breaking' situations (R§1) in which the wave motion causes the same contours to undulate reversibly, as happens, for instance, under the conditions assumed by linear, nondissipative wave theory. This answers the question posed in R§1 as to whether 'a fundamental change in the state of the wave' is envisaged.

(There are, of course, further questions such as when, and where, the circumstances assumed by linear, nondissipative wave theory might comprise a useful approximate model of physical reality. In trying to make sense of observational data one may have to think in terms of more than one phenomenon at once, the various theoretical models or paradigms, e.g. linear, nondissipative wave theory, serving as self-consistent reference points but not, it should hardly need saying, as complete and detailed descriptions of the reality underlying the observations.)

5. The wave-breaking definition is consistent with accepted usage for surface gravity waves and for internal gravity waves. In particular, the definition says that the waves are breaking whenever common usage, and expert opinion, says they are (e.g. Banner and Phillips, 1974; Longuet-Higgins and Cokelet, 1976; New *et al.*, 1985). It could be added that the definition seems conceptually simpler than some of the alternative definitions quoted in R§2a.

Details of how the relevant material contours deform (and how rapidly) will naturally depend on the circumstances, including the kind of wave involved (and the associated time scales, MP84§2). For instance laboratory and field observations indicate that both surface and internal gravity wave breaking lead to the generation of three dimensional turbulence (implying that exceedingly complicated, three dimensional contour shapes will develop), whereas what we have called Rossby-wave breaking leads to motions which, if anything, may be more closely compared to two dimensional 'turbulence' (HAYNES, 1985, 1986a, 1986b; JUCKES and MCINTYRE, 1986).

6. R§4b states that quasi-linear models which retain only one zonal harmonic component reproduce the 'formation of potential vorticity tongues', and seems to regard this as showing that the physical process being modelled is essentially linear. The point was dealt with in MP84 (Appendix B), but it is worth repeating here that, in the region where those model tongues form (usually outside the region of strongest mean zonal winds) the model equations are liable to be quite inaccurate, for the reasons indicated in comment 2 and in the Appendix below. It is suggested in R§4b that interactions among zonal harmonics (due to nonlinear terms neglected in linearized wave theory) will not qualitatively alter the results. Quite the contrary: it is just such nonlinear interactions that lead to the fluid-dynamical irreversibility which we discussed in MP84 (§2 and Appendix B)—the irreversibility characteristic of both 'wave breaking' and 'turbulence'. These nonlinear interactions may well, in reality, involve a vast range of spatial scales, many of them invisible to observational networks, and many of them incapable of being accurately described by any quasi-linear model.

It may sometimes be possible to parametrize crudely some of the unresolved nonlinear effects by introducing a linear eddy viscosity into a linearized model. But even though nonlinear terms are not explicitly present in such models, it does not follow that the actual physical processes being modelled are inherently linear. It is pertinent to note also the complete failure, in certain cases, of ideas suggested by the use of such eddy-viscosity parametrizations. For instance the standard idea of a 'viscous critical layer' may often fail to represent nonlinear reality, as was recently shown by Killworth and McIntyre (1985, §1.3, q.v.).

It is true, and interesting, that quasi-linear models, which neglect nonlinear interactions among zonal harmonics but retain mean flow evolution, do appear to succeed in capturing certain gross features of the incipient stages of some Rossby wave breaking events (and also some aspects of 'main-vortex, surf-zone structure'). The reasons for this were indicated in Appendix B of Dunkerton *et al.* (1981), and are examined in more detail in a forthcoming paper by Haynes and McIntyre (1986). But, again, one cannot infer that the basic physical process being modelled is itself essentially linear.

7. A further point about the use of linearized wave theory is that a completely linear model, which neglects the evolution of the mean state, cannot produce a true main-vortex, surf-zone structure in the sense envisaged in MP. In particular, the attempt in R§4b to demonstrate such a structure in a completely linear model, by plotting the meridional distribution of vorticity *at a single longitude* (R Figure 10), is wholly inappropriate.[3] A better measure of main-vortex, surf-zone structure (when applied

[3] For what it is worth, the mathematical expressions presented in support of the structure shown in Figure 10 do not, in any case, satisfy the linearized equations. In particular, the disturbance velocity field disagrees with the disturbance vorticity field nearly everywhere, including the neighbourhood of 30°N where attention is drawn to 'the most apparent feature in Figure 10b'.

to the real atmosphere or to fully nonlinear models of it) is the area diagnostic advocated in MP and further developed by Butchart and Remsberg (1986). This expresses the structure in terms of the functional relationship between potential-vorticity values and areas enclosed within potential-vorticity contours on coarse-grain IPV maps.

8. R speaks of the Rossby-wave critical 'line' interaction as if, in the real stratosphere, it were a phenomenon qualitatively distinct from wave breaking as we defined it. It is true that the term 'critical line interaction', more appropriately 'critical layer interaction', is usually reserved for that subclass of cases in which the 'surf zone' surrounding the critical line is very narrow.[4] It is also true that the corresponding subclass of parameter values is not the one of greatest immediate interest in connection with observations of the stratosphere, at the spatial resolutions available today. This is all the more so when further parameter restrictions are imposed, as in the SWW (Stewartson-Warn-Warn) theory, as we were careful to mention in MP84§2. However, one can imagine a continuous sequence of cases, as suggested schematically in Figure 1, of which the first one or two are self-consistently describable by nonlinear critical-layer theory, and the subsequent ones not. Where exactly is one to draw the line between critical 'line' interactions and 'wave breaking'? We feel that it makes sense to call them all examples of 'wave breaking', and to regard those cases describable by nonlinear critical-layer theory as simply a subclass, albeit an interesting one, of all possible cases. It is a subclass, moreover, for which some self-consistent analytical theory is available, providing us with theoretically justifiable thought-experiments—described by the SWW solution (and its instabilities; see Killworth and McIntyre, 1985; Haynes, 1985, 1986b)—which serve to illustrate the *concept* of Rossby-wave breaking as we defined it.

It can be added that recent theoretical work has made some progress toward elucidating which predictions of critical-layer theory are likely to carry over to other cases (Killworth and McIntyre, *op. cit.*). But we did not intend to suggest that nonlinear critical-layer theory in itself, let alone the special case studied by SWW, provides a quantitatively accurate model of major stratospheric wave-breaking events. We agree with R that it does not do so.[5] Still less does any *linear* critical-layer theory.

With regard to Warn and Warn's diagram showing that 'the width of the critical

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[4] Or shallow, if it happens that the mean zonal flow depends on height alone as in the Matsuno–Nakamura model—but still, it should be noted, unlikely in either case to be 'viscous' in the standard sense of critical layer theory (at least in the real stratosphere), if only for the reasons already referred to in comment 6.

[5] One difference which can undoubtedly be important is the fact that, in major stratospheric planetary-wave breaking events which rearrange large amounts of potential vorticity, the developing IPV pattern may induce wind fields which are locally far stronger than in the SWW parameter regime. Recent investigations at the U.K. Meteorological Office have provided some excellent illustrations of this point (A. O'Neill and V. D. Pope, personal communication), and have also thrown more light on the question of the interaction between wave breaking and diabatic processes. On the diabatic effects, see also the important work of Butchart and Remsbergh (1986).

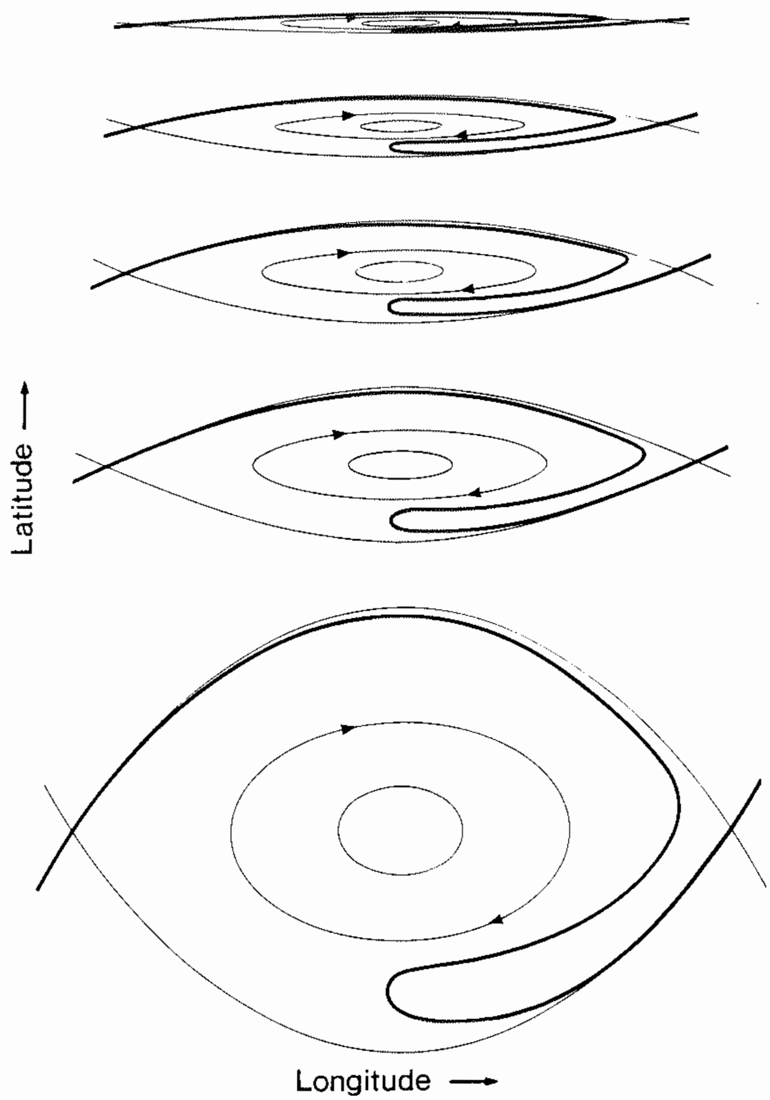

Figure 1

Sketches schematically depicting an idealized sequence of cases of the 'simplest kind' (MP84§2) of Rossby wave breaking event in a shear flow. The width of the wave-breaking region is an increasing function of wave amplitude, as suggested by nonlinear critical-layer theory (SWW; RITCHIE, 1984), as well as by general kinematical considerations. The light contours are streamlines and the heavy contours IPV contours, with high potential vorticity to the north and low to the south. The sketches suggest only gross features of the early stages of wave breaking; in reality, there would be many differences from case to case, depending strongly on the parameter regime, initial conditions, importance of diabatic processes, and so on (MP84; HAYNES, 1985, 1986b; BUTCHART and REMSBERG, 1986). See text, comments 8 and 9 and footnote 5.

layer decreases as the perturbation progresses from a linear to a nonlinear situation' (R§5), it is important to realise that this refers to the width of the transient region in the initial, linear stage, and not at all to the width of the Kelvin cat's eyes. The latter is an increasing function of wave amplitude, as suggested in Figure 1. It is within and near the cat's eyes that the small 'slope' condition is most strongly violated as nonlinearity develops. RITCHIE (1984) and HAYNES (1985) have recently presented fully nonlinear simulations illustrating, in different ways, the basic fact that when the disturbance amplitude grows in time, the cat's-eye width increases correspondingly.

9. With the bottom case of Figure 1 in mind we did, indeed, suggest that the Aleutian anticyclone in a large-amplitude planetary-wave event can be thought of, in the words of R§5, as a 'global analogue to a Kelvin cat's-eye' involving a 'large-scale organized circulation'. But our wave-breaking definition deliberately left open the question of how strongly, or otherwise, such a region might be 'dominated by nonlinear mixing processes' (*ibid.*), since that would be bound to depend on the circumstances. An ocean breaker of the 'tube' variety, of which side views are sometimes seen in surfing movies, also has a coherent 'large-scale organized circulation' including a prominent, quasi-permanent core of air. But that would be no argument against including it among examples of breaking waves.

We agree wholeheartedly that time variability (R§4a) is an extremely important aspect of 'the circumstances'. Some interesting dynamical-systems insights into this point may be found in the work of Aref (1984).

10. The term 'reversible' is used in R in a way which is understandable in relation to practical concerns about tracer transport (albeit based largely on results from quasi-linear models, whose limitations have already been commented on in 6). But it should be noted that this differs from the statistical-mechanical concept of reversibility which was our concern. For example, the discussion near the end of R§5 would seem by implication to classify as 'reversible' a dynamical process in which an ocean surface gravity wave breaks violently enough to produce foam and spray, but in which all the spray subsequently falls back into the ocean, and all the underwater bubbles return to the atmosphere, without significant evaporation.

As remarked in R§5, the paper by CLOUGH *et al.* (1985) points to an observed case which at first sight appears to comprise a Rossby-wave analogue of the gravity-wave situation just described. High potential vorticity air previously ejected from the main stratospheric vortex, as a result of wave breaking, subsequently appears to merge back into the main vortex. Near-perfect vortex merging may be possible dynamically, albeit not likely (e.g. DRITSCHEL, 1986). Irrespective of this, however, the observed phenomenon is clearly not one in which all *relevant material contours* undulate reversibly, let alone a process self-consistently describable by linear Rossby wave theory. There can be little doubt that relevant material contours would have been distorted into extremely complicated shapes during the process, and that the small 'slope' condition for the general validity of linear wave theory would have been grossly violated.

High-resolution barotropic models of the fully nonlinear Rossby wave-breaking situation, in parameter regimes far closer to the real stratosphere than the SWW parameter regime, have recently produced an example which appears comparable to the one noted by CLOUGH *et al.*, and which provides detailed support for the scenario just described (JUCKES and MCINTYRE, 1986). Interestingly, it was found (a) that perfect merging does not, in fact, occur in this model example, but (b) that it would have *appeared* to have occurred if the potential vorticity field had been viewed at much lower resolution, comparable to the resolution of the satellite observations.

11. A potential source of confusion in talking about 'planetary waves' may come from the two senses in which the word 'wave' can be used. The first is the purely mathematical sense of a zonal-harmonic or other Fourier component, regardless of the nature of the field being Fourier analyzed, and regardless of whether linear theory describes its dynamics self-consistently. This seems to be the predominant sense in which the word 'wave' is used in R's discussion of planetary waves (as, for example, in 'the waves within the [fully nonlinear] critical layer', R§5, and 'the planetary wave circulation fields', R§6 and Figure 12, the latter phrase being used to refer to flow fields like the Aleutian anticyclone as the discussion in R§§5, 6 clearly shows).

The second sense in which the word 'wave' can be used is the theoretical-physics sense explained in 2 above—to which the discussion in R§§5, 6, by implication, attaches little significance. It is to 'waves' in this latter sense that the physical concepts of 'propagation' and 'breaking' apply.

12. There is a subtlety concerning the third example in R§4a, the example taken from Mied's paper. This is a typical vortex-interaction situation of the kind extensively studied in fluid dynamics. Unlike the first example, and contrary to what is said in R, it can, actually, be regarded as involving Rossby wave breaking. Like the stratospheric main vortex, any vortex has a Rossby restoring mechanism against departures from axisymmetry. It is possible, therefore, to speak of breaking Rossby waves in such vortex-interaction problems, although Mied's example reproduced in Figure 8 of R is not a particularly clear one, possibly because of the effect of viscosity on the vorticity field in the numerical simulation. Also, the close proximity of the second vortex, exciting the breaking Rossby wave on the first, makes it questionable whether one should regard the disturbance on the first vortex as a freely propagating wave, even when it first begins to break. Much clearer examples are given for instance by DRITSCHEL (1986). The phenomenon seems to have been first noted in this context, and called 'breaking', by DEEM and ZABUSKY (1978).

Concluding remarks

We agree with R (a) that there is no complete, detailed mathematical description of all cases of wave breaking, (b) that wave breaking (like wave propagation) has no unique 'signature' or shape whereby one can always be sure of recognizing it automatically, and (c) that 'the outcome of the phenomenon may vary with the circumstances', including the effects of overt dissipation, if present (CLOUGH *et al.*, 1985; LEOVY *et al.*, 1985; BUTCHART and REMSBERG, 1986). We also agree (d) that wave breaking does not act to limit, or clip, the wave amplitude at any precise threshold, although it may well limit, in a vaguer sense, the *order of magnitude* of a suitably-defined local wave amplitude.

As the discussion in R§2a clearly shows, points (a)–(d) all apply to the familiar case of surface gravity waves. They also apply to internal gravity waves. The idea of breaking is nevertheless widely accepted, and manifestly useful, in connection with both types of gravity waves. For example, most dynamicists interested in the mesosphere would probably now regard the ideas that internal gravity waves can propagate and break as almost, if not quite, indispensable to making sense of a large variety of mesospheric observations (WMO 1986, & refs.), including the former enigma of the cold summer mesopause, and, more recently, the seasonal variation in 557.7 nm green-line airglow emissions (Garcia and Solomon, 1985). It seems to us illogical to regard (a)–(d), and the 'generic' character of the concept, as fatal objections to using the idea of breaking in connection with Rossby waves, when they are not so regarded in connection with either surface or internal gravity waves.

A strong reason for making the distinction between breaking and non-breaking waves in the way we did is its direct relevance to the theory of wave, mean-flow interaction. That theory, used in conjunction with the idea that various kinds of waves may propagate from the troposphere into the middle atmosphere, has played a key role in accounting for previously unexplained phenomena such as the quasi-biennial oscillation and the cold summer and warm winter mesopauses. One of the general theorems of wave, mean-flow interaction theory is the so-called 'non-acceleration theorem', which makes useful statements about the circumstances in which a large class of waves have no quasi-permanent effect upon the mean state. In order to prove the theorem, it is necessary to assume *inter alia* that the waves do not 'break' in the sense envisaged. Conversely, if the waves do 'break', in this sense, then the effects of wave 'transience' upon the mean flow, can become quasi-permanent.[6] While we are not complacent about the need for more theory, and detailed numerical simulations, we think that the direct relevance of our definition to a basic theorem of wave, mean-flow interaction theory is in itself a sufficient answer, for the time being, to the charge that the definition has no well established theoretical basis.

Regarding point (b) above, we wish to emphasise that we did not *define* the concept of wave breaking in terms of any particular 'signature'. (Nor can the breaking of internal or surface gravity waves be defined in terms of any unique signature; e.g.

[6] The fact that the behaviour of the relevant material contours is involved can be seen from the way in which material displacements, such as the transverse displacement η', appear in proofs of the non-acceleration theorem. A careful analysis is given in ANDREWS and MCINTYRE (1978, §5 and Figure 1); for further discussion see §2 of DUNKERTON (1980) and §3 of MCINTYRE (1980). It should be noted that in some proofs of small-amplitude versions of the theorem η' appears in the approximate form $v'/ik(U_0 - c)$ valid for monochromatic waves whose local amplitudes are well below breaking. Here v' is the transverse disturbance velocity, U_0 the basic-state longitudinal velocity, k the longitudinal wavenumber, and c the longitudinal phase speed of the monochromatic wave.

BANNER and PHILLIPS, 1974; R§2a.) For the purpose of interpreting satellite observations—bearing in mind the previous conventional wisdom about satellite data being inadequate to estimate IPV distributions—we did make judgements about IPV patterns likely to be typical of the early stages of the 'simplest kind' of Rossby-wave breaking event (MP84). We did so in the light of the theoretical paradigms then available. All of these, together with general kinematical considerations, suggested the gross features sketched in Figure 1 (now, in fact, being confirmed by high-resolution numerical simulations). Such judgements about typical IPV patterns will no doubt continue to be made in future observational work; and we agree with R that like other scientific judgements they must be made with care. But these considerations were not intended as part of the fundamental definition of the wave-breaking concept itself. Nor was the circumstance that a well-developed 'main vortex, surf zone' structure appeared to exist in January–February 1979; cf. R§4.

Regarding point (a), it has to be said that a complete, detailed theory of wave breaking is unlikely to be developed soon, if ever—at least in an analytical, as opposed to a numerical, sense. The reasons for this are much the same as the reasons why there are no complete analytical theories of two and three dimensional 'turbulence'. In reality we are dealing with complicated kinds of fluid motion to which no analytical theory is likely to be self-consistently applicable over significant spans of time, particularly linear theory. However, the lack of a complete theory need not deter one from using a concept, especially one which has been found to be useful as a heuristic organizing concept when trying to make sense of very complicated phenomena. This is exactly what Osborne Reynolds did when he classified laminar and turbulent fluid flow just over a century ago—well in advance of any complete and detailed theory of 'turbulence'.

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Appendix: the small slope condition for the validity of linear wave theory

If the small 'slope' condition is violated somewhere, then the terms neglected in linearizing the equations will not generally be smaller than the terms retained, in the locality concerned. In the linear theory of Rossby waves, for instance, a term of the type $v'\partial Q'/\partial y$ is neglected against one of the type $v'\partial Q_0/\partial y$, where Q_0 and Q' are the basic and disturbance potential vorticities, v' is the latitudinal disturbance velocity, and the latitudinal gradient $\partial/\partial y$ is taken in isentropic surfaces (or in isobaric surfaces if the quasi-geostrophic potential vorticity is used in place of the Rossby–Ertel potential vorticity). If the small slope condition is violated, then the ratio of $\partial Q'/\partial y$ to $\partial Q_0/\partial y$ and therefore of $v'\partial Q'/\partial y$ to $v'\partial Q_0/\partial y$ is not generally small. There are, to be sure, a few special wave solutions in which all the neglected terms happen to cancel one another. But the most important general principle of linear theory, the principle of superposition (which says that the sum of any set of solutions is also a solution), fails whenever the small slope condition is violated, assuming of course that the basic gradient giving rise to the wave restoring mechanism does not vanish.

If the basic IPV gradient is weak in some band of latitudes, as might occur in a theoretical model of a 'surf zone' formed by earlier wave-breaking events, then an interesting situation arises: even when linearization, and the predictions of linear theory, fail locally in the band of latitudes concerned, it is possible that linear theory may still give approximately correct predictions in other latitudes. The analysis by KILLWORTH and MCINTYRE (1985) illustrates this point in a context for which a fully nonlinear theory is available, together with some relevant general theorems. It suggests why, and in what circumstances, linearized theory may model some planetary-wave phenomena better than could have been expected *a priori* (e.g. Salby, 1984). But, as remarked elsewhere in this discussion, such examples cannot be taken as implying that the actual physical processes being modelled are inherently linear.

REFERENCES

ANDREWS, D. G. and MCINTYRE, M. E. (1978), *An exact theory of nonlinear waves on a Lagrangian-mean flow.* *J. Fluid Mech. 89*, 609–646.

AREF, H. (1984), *Stirring by chaotic advection.* J. Fluid Mech. *143*, 1–21.

BANNER, M. L. and PHILLIPS, O. M. (1974), *On the incipient breaking of small scale waves.* J. Fluid Mech. *65*, 647–656.

BUTCHART, N. and REMSBERG, E. E. (1986), *The area of the stratospheric polar vortex as a diagnostic for tracer transport on an isentropic surface.* J. Atmos. Sci. *43* 1301–1321 in press.

CLOUGH, S. A., GRAHAME, N. S. and O'NEILL, A. (1985), *Potential vorticity in the stratosphere derived using data from satellites.* Quart. J. R. Meteor. Soc. *111*, 335–358.

DEEM, G. S. and ZABUSKY, N. J. (1978), *Vortex waves: stationary 'V-states', interactions, recurrence, and breaking.* Phys. Rev. Lett. *40*, 859–862.

DRITSCHEL D. G. (1986), *The nonlinear evolution of rotating configurations of uniform vorticity,* J. Fluid Mech. *172*, 157–182

DUNKERTON, T. J. (1980), *A Lagrangian mean theory of wave, mean-flow interaction with applications to nonaceleration and its breakdown*, Revs. Geophys. Space Phys. *18*, 387–400.

DUNKERTON, T., HSU, C.-P. F. and MCINTYRE, M. E. (1981), *Some Eulerian and Lagrangian diagnostics for a model stratospheric warming.* J. Atmos. Sci.,*38*, 819–843.

EDMON, H. J., HOSKINS, B. J. and MCINTYRE, M. E. (1980), Eliassen-Palm cross-sections for the troposphere. J. Atmos. Sci. *37*, 2600–2616. (See also *Corrigendum*, J. Atmos. Sci., *38*, 1115, especially second last item.)

GARCIA, R. R. and SOLOMON S. (1985), *The effect of breaking gravity waves on the dynamics and chemical composition of the mesophere and lower thermosphere.* J. Geophys. Res. *90D*, 3850–3868.

HAYNES, P. H. (1985), *Nonlinear instability of a Rossby-wave critical layer.* J. Fluid Mech. *161*, 493–511.

HAYNES, P. H. (1987), *On the instability of sheared disturbances.* J. Fluid Mech., ~~to appear.~~ 175, 463–478.

HAYNES, P. H. (1989), *The effect of barotropic instability in the nonlinear evolution of a Rossby-wave critical layer.* J. Fluid Mech., ~~to be submitted.~~ 207, 231–266.

HAYNES, P. H. and MCINTYRE, M. E. (1986) 7, *On the representation of planetary-wave critical layers and wave breaking in zonally truncated models.* J. Atmos. Sci., [redacted] 44, 2359–2382.

HOSKINS, B. J., MCINTYRE, M. E. and ROBERTSON, A. W. (1985), *On the use and significance of isentropic potential-vorticity maps.* Quart. J. Roy. Meteor. Soc. *111*, 877–946 & 113, 402–404. one-layer model

JUCKES, M. N. and MCINTYRE, M. E. (1986), 7 High-resolution~~, barotropic simulations~~ of breaking planetary waves in the stratosphere ~~to be submitted to Quart. J. Roy. Meteorol. Soc.~~ Nature 328, 590–6.

KILLWORTH, P. D. and MCINTYRE, M. E. (1985), *Do Rossby-wave critical layers absorb, reflect, or over-reflect?* J. Fluid Mech., *161*, 449–492.

LEOVY, C. B., SUN, C.-R., HITCHMAN, M. H., REMSBERG, E. E., RUSSELL, J. M., GORDLEY, L. L., GILLE, J. C.,

LYJAK, L. V. (1985), *Transport of ozone in the middle stratosphere: evidence for planetary wave breaking.* J. Atmos. Sci., *42*, 230–244.

LONGUET-HIGGINS, M. S. and COKELET, E. D. (1976), *The deformation of steep surface waves on water, I, A numerical method.* Proc. Roy. Soc. London, *A350*, 1–26.

MCINTYRE, M. E. (1980), *Towards a Lagrangian-mean description of stratospheric circulations and chemical transports.* Phil. Trans. Roy. Soc. London, *A296*, 129–148.

MCINTYRE, M. E. and PALMER, T. N. (1983), *Breaking planetary waves in the stratosphere.* Nature *305*, 593–600.

MCINTYRE, M. E. and PALMER, T. N. (1984), *The 'surf zone' in the stratosphere.* J. Atmos. Terrest. Phys. *46*, 825–849.

NEW, A. L., MCIVER, P. and PEREGRINE, D. H. (1985), *Computations of overturning waves.* J. Fluid Mech. *150*, 233–251.

RITCHIE, H. (1984), *Amplification of forced Rossby waves in the presence of a nonlinear critical layer.* J. Atmos. Sci. *41*, 2012–2019.

ROOD, R. B. (1986), *A a critical analysis of the concept of planetary wave breaking.* Pure Appl. Geophys. *123* (5), in press.

SALBY, M. L. (1984), *Survey of planetary-scale travelling waves: the state of theory and observations.* Revs. Geophys. Space Phys. *22*, 209–236.

WMO (1986), *Atmospheric ozone 1985: Assessment of our understanding of the processes controlling its present distribution and change. Geneva, World Meteorological Organization, Global Ozone Research and Monitoring Report No. 16.* Available from Global Ozone Research and Monitoring Project, World Meteorological Organization, Case Postale 5, CH 1211, Geneva 20, Switzerland.

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