Inevitability of a magnetic field in the Sun's radiative interior

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The gas in the convective outer layers of the Sun rotates faster at the equator than in the polar regions, yet, deeper inside (in the radiative zone), the gas rotates almost uniformly[1–3]. There is a thin transition layer between these zones, called the tachocline[4]. This structure has been measured seismologically[1–3], but no purely fluid-dynamical mechanism can explain its existence. Here we argue that a self-consistent model requires a large-scale magnetic field in the Sun's interior, as well as consideration of the Coriolis effects in the convection zone and in the tachocline. Turbulent stresses in the convection zone induce (through Coriolis effects) a meridional circulation, causing the gas from the convection zone to burrow downwards, thereby generating the horizontal and vertical shear that characterizes the tachocline. The interior magnetic field stops the burrowing, and confines the shear, as demanded by the observed structure of the tachocline. We outline a dynamical theory of the flow, from which we estimate a field strength of about 10^{-4} tesla just beneath the tachocline. An important test of this picture, after numerical refinement, will be quantitative consistency between the predicted and observed interior angular velocities.

Two quite different explanations for the near-uniform interior rotation of the Sun have been proposed. The first invokes quasi-horizontal shear-generated turbulence in the (stably stratified) tachocline, producing stresses that are predominantly horizontal[4,5]. The stresses are assumed to act in the manner of a horizontal viscosity that leads to latitudinally invariant angular velocity beneath the tachocline. Therefore, if the rotation of the Sun were steady, the radiative zone beneath the tachocline would indeed be expected to rotate uniformly, though how radial shear would be suppressed in the presence of solar-wind spindown is not adequately explained. The equilibrium angular velocity of the radiative interior Ω_i is predicted to be $0.90\Omega_e$, where $\Omega_e \approx 2.9 \times 10^{-6} s^{-1}$ is the equatorial angular velocity in the convective zone, and is somewhat less than that observed[1−3], namely $\Omega_i \approx 0.93\Omega_e$. The discrepancy seems significant.

As was recognized by Spiegel and Zahn[4], the density stratification of the tachocline is far too strong to permit anything resembling three-dimensional turbulence[6]. In the presence of such stratification, the balance of forces is similar to that in the terrestrial stratosphere, about which there are a wealth of observations and strong theoretical arguments to show that horizontal turbulence controls the distribution of angular momentum in such a way as to drive the system away from, not towards, uniform rotation[7]. Meteorologists once called this negative viscosity[8].

The second proposal[9,10] is that shear-induced differential dissipation of prograde and retrograde gravity waves generated at the interface between the convective zone and the radiative interior acts in such a way as to suppress the shear in the radiative zone. But as already recognized in an earlier discussion of wave dissipation in the Sun[11], torques due to gravity waves do not behave in this way. Broadband gravity waves also tend to drive the system away from uniform rotation[7,12]. There is a large body of theoretical studies coupled with observations of the Earth's atmosphere that bear on the issue of how gravity waves interact with differential rotation[13]. In no known case in which there is a broad spectrum of angular phase speeds is the result of the interaction such as to drive the flow towards a state of uniform rotation. Torques due to acoustic waves appear to be far too weak[14].

Our arguments, strongly reinforced by terrestrial stratospheric observations, suggest that no purely fluid-dynamical mechanism can explain the almost uniform interior rotation: hence the conclusion that shear is suppressed by a large-scale poloidal magnetic field in the interior. The idea that there might be such a field is not new[15]. What is new, however, is the inevitability of its existence. Without it, the tachocline would be much deeper than observed.

As a first step towards a realistic theory of the tachocline we adopt the simplest possible hypothesis: we ignore convective overshoot, shear-generated turbulence and wave-induced stresses, and any magnetic field that is generated in the convection zone. We consider the consequences of a downward-penetrating axisymmetric laminar circulation forced by the turbulent stresses in the convection zone above. This type of circulation has been studied extensively in terrestrial stratospheric models[16,17]. In the solar case, downwelling occurs at both high and low latitudes, with upwelling in middle latitudes, as is illustrated schematically in Fig. 1. The interior magnetic field stops the downwelling branches of the forced circulation from burrowing more and more deeply as time goes on. There is a 'tachopause' in the form of a thin magnetic boundary layer within which the angular-velocity gradient is reduced to zero. The principal balance of forces is summarized below. By hypothesis, there is no azimuthal force on the downwelling flow above the magnetic boundary layer in the body of the tachocline; therefore angular momentum is conserved. In a quasi-steady state, the flow in the body of the tachocline is along surfaces S of constant specific angular momentum, which from helioseismology are known to slant qualitatively as depicted in the figure.

[[Figure 1 near here]]

Only where there is upwelling is there an opportunity for the interior magnetic field to penetrate into the body of the tachocline and thence into the convection zone, where it will be subject to reconnection. The magnetohydrodynamic problem in the upwelling region is severely nonlinear, and its details formidably complicated, perhaps modifying angular-momentum conservation via Lorentz forces and reshaping the upwelling region, which mass conservation demands must still exist.

The dynamics implied by this picture relates tachocline depth to interior field strength. First, in the body of the tachocline, hydrostatic and cyclostrophic balance relates the shear of angular velocity to the temperature field T, or, more precisely, to the aspherical temperature anomaly $\tilde{T}$ induced by the downwelling (warm, compressing) or upwelling (cold, expanding) flow, in a manner that depends on the depth Δ of the tachocline. It proves convenient to introduce spherical polar coordinates (r, θ, ϕ) with respect to the rotation axis, in a frame rotating with the angular velocity Ω_{i} of the radiative interior. In this frame the velocity $\boldsymbol{u} = (u, v, s\tilde{\Omega})$, where $s = r\sin\theta$ and the angular velocity anomaly $\tilde{\Omega}$ depends on r and θ and describes the tachocline shear. The balance is then expressed by the zonal component of the steady axisymmetric inviscid vorticity equation, yielding an appropriately generalized 'thermal-wind'[13] (more aptly 'thermal-shear' equation) which, assuming the perfect-gas law, is

$$2\Omega_{\rm i} s \left(\frac{\partial\tilde\Omega}{\partial z}\right)_s \approx \frac{g}{rT}\left(\frac{\partial\tilde T}{\partial\theta}\right)_r \quad . \tag{1}$$

where $z = r\cos\theta$ and g is the gravitational acceleration.

Second, the thermal-energy equation expresses a balance between radiative diffusion and advection of the background stratification:

$$\frac{N^2 T u}{g} \approx \frac{1}{\rho c_p r^2}\frac{\partial}{\partial r}\left(r^2 K\,\frac{\partial\tilde T}{\partial r}\right) , \tag{2}$$

the partial derivatives being taken at constant colatitude θ, where N is the buoyancy frequency, ρ the mass density, c_p the specific heat at constant pressure and K the radiative conductivity. In the downwelling zones, these equations are to be solved subject to the conditions that the flow is along $\mathcal{S}$ and that $\tilde\Omega$ and $\tilde T$ match smoothly to the corresponding variables in the radiative interior and the convection zone. In particular, $\tilde\Omega \to 0$ and $\tilde T \to 0$ towards the base of the tachocline.

Third, the downwelling in the polar and the sub-equatorial regions is deflected by the interior magnetic field towards the mid-latitude upwelling region. There is a thin magnetic boundary layer in which aspherical deviations vary on a vertical length scale δ; we confirm *a posteriori* that $\delta \ll \Delta$. This boundary layer separates the body of the tachocline from what the model assumes to be a rigidly rotating radiative interior. The location of the boundary layer is determined by a balance between downward advection and upward diffusion of the large-scale, essentially horizontal, poloidal magnetic field: therefore $\eta/\delta \sim |u|$, where η is the magnetic diffusivity.

Equations (1) and (2) continue to be satisfied within the boundary layer (again confirmed *a posteriori*), but now the angular-momentum balance is modified by the Lorentz force. The force balance is expressed by the zonal component of the momentum equation:

$$2\Omega_{\rm i} v\cos\theta \approx \frac{B_0}{\mu_0\rho\, r\sin\theta}\frac{\partial}{\partial\theta}\left(B_\phi \sin\theta\right) , \tag{3}$$

where μ_0 is the vacuum permeability and B_0 is the poloidal magnetic field, which is diffusing upward from the radiative interior and is approximated here as being purely horizontal; B_ϕ is the zonal component of the magnetic field, resulting from the twisting and stretching of the poloidal field by the differential rotation according to the zonal component of the induction equation which, near the bottom of the magnetic boundary layer where the tachocline perturbation is small, is

$$-B_0 \sin\theta \frac{\partial\tilde{\Omega}}{\partial\theta} \approx \eta \frac{\partial^2 B_\phi}{\partial r^2} . \tag{4}$$

Consistent with horizontal-B_0 approximation, we have ignored latitudinal variations of $B_0 \sin\theta$. Equations (1)–(4), coupled with the continuity equation $\mathrm{div}(\rho\boldsymbol{u}) = 0$, can then be reduced to a single linear equation for $\tilde{T}$. The latitudinal variation of $\tilde{T}$ is oscillatory on a scale comparable to the radius $r_\mathrm{c} \simeq 0.7R_\odot$ of the base of the convection zone (here $R_\odot$ is the solar radius). Accordingly, we may analyse the boundary layer locally, introducing a characteristic horizontal wavenumber L/r_c and setting $\partial/\partial\theta = iL$. The boundary-layer equation then reduces to

$$\frac{\partial^6 \tilde{T}}{\partial r^6} - \delta^{-6}\,\tilde{T} \approx 0 , \tag{5}$$

in which the boundary-layer thickness is given by

$$\delta = \left(\frac{2\mu_0 \rho\eta\kappa\Omega_\mathrm{i}^2}{B_0^2 r_\mathrm{c}^2 N^2 L^4}\right)^{1/6} r_\mathrm{c} \tag{6}$$

where $\kappa = K/\rho c_p$ is the radiative diffusivity. In obtaining equation (5) it was possible to neglect the variation of the properties of the unperturbed state because the magnetic boundary layer is thin. The equation has three solutions that decay with depth, all on scales of order $\delta \ll \Delta$, a combination of which solutions must be matched to the solution in the body of the tachocline.

Near the bottom of the magnetic boundary layer the linearization leading to equations (4) and (5) is valid, and shows how the tachocline circulation is prevented by the interior magnetic field from burrowing more deeply. But as the body of the tachocline is approached, with $|u|$ rising to values $\approx \eta/\delta$, advection of B_ϕ by the meridional circulation (not included in equation (4)) and advective modification of B_0 both become important, and the boundary-layer equations become nonlinear. We have not yet solved these equations, but we can make an estimate of a typical value of $|B_0|$ from scaling arguments. We expect the latitudinal variation of $\tilde{T}$ to be similar to that of the spherical harmonic $P_4(\cos\theta)$, and so we adopt the estimate $L \approx 4.5$. We take $\partial/\partial r \approx \beta/\Delta$ for equation (2) in the body of the tachocline, where $\beta \approx \pi$, and we represent the jump in $\tilde{\Omega}$ across the tachocline as $\alpha\Omega_{\rm i}$, and neglect its jump across the boundary layer. Then equations (1) and (2) imply $\tilde{T}/T \approx \alpha r_{\rm c}^2\Omega_{\rm i}^2/gL\Delta$ and $|u| \approx \beta^2\kappa g\tilde{T}/\Delta^2N^2T$, where we have taken $\sin\theta \approx \cos\theta \approx 2^{-1/2}$. We expect these estimates to apply everywhere except near the equator where the $\mathcal{S}$-surfaces fail to traverse the tachocline, and near the poles, where our boundary-layer theory breaks down.

Together with the magnetic advection–diffusion balance, $|u| \approx \eta/\delta$, these estimates imply

$$\delta \;\approx\; \frac{L}{\alpha\beta^2}\,\frac{\eta}{\kappa}\,\left(\frac{N}{\Omega_{\rm i}}\right)^2\left(\frac{\Delta}{r_{\rm c}}\right)^3 r_{\rm c}\;. \tag{7}$$

Equating (6) and (7) relates the local Alfvén speed $|B_0|/\sqrt{\mu_0\rho}$ to the global diffusion speed $\sqrt{\kappa\eta}/r_{\rm c}$:

$$\frac{|B_0|}{\sqrt{\mu_0\rho}} \;\approx\; \frac{\sqrt{2}\alpha^3\beta^6}{L^5}\left(\frac{\kappa}{\eta}\right)^3\left(\frac{\Omega_{\rm i}}{N}\right)^7\left(\frac{r_{\rm c}}{\Delta}\right)^9\frac{\sqrt{\kappa\eta}}{r_{\rm c}}\;. \tag{8}$$

The sign of B_0 is immaterial. Substituting the characteristic values $\alpha \approx 0.07$, $\kappa/\eta \approx 1.4 \times 10^4$, $\Omega_{\rm i}/N \approx 3 \times 10^{-3}$, $\sqrt{\kappa\eta}/r_{\rm c} \approx 2 \times 10^{-8}$ ms^{-1}, $\rho \approx 2 \times 10^2$ kg m^{-3}, $\mu_0 = 4\pi \times 10^{-7}$T^{2}Pa^{-1}, yields

$$|B_0| \;\approx\; 5 \times 10^{-19}\left(\frac{r_{\rm c}}{\Delta}\right)^9 \mathrm{T}\;. \tag{9}$$

This shows how the tachocline thickness Δ depends on the interior field B_0 at the base of the tachocline.

In current standard models of the sun, helium settles under gravity on a timescale $\sim 10^{10}$y, resulting in a negative abundance gradient throughout the radiative interior. Our model implies that material in the tachocline is recirculated into the convection zone in a ventilation time τ_v, which we shall find to be $\ll 10^{10}$y, implying in turn that the helium abundance is uniform down to the tachopause. The upshot is that the mean molecular mass of the tachocline is lower than that in the standard models, with a consequent sound-speed anomaly immediately beneath the convection zone. This phenomenon is observed in the most recent structure inversions of p-mode frequencies obtained from GONG[18] and SOHO/SOI[2]. Seismic calibration of our model[19] yields $\Delta \approx 0.018R_\odot \approx 0.026r_{\rm c}$. Hence, from equation (9), it follows that the intensity of the magnetic field immediately underlying the magnetic boundary layer is $|B_0| \approx 10^{-4}$T. (An earlier report[20] of a much more intense, toroidal, field inferred by seismic inversion could be a misinterpretation of just such a sound-speed anomaly, from which a magnetic field cannot be distinguished by frequency inversion alone[21].)

From equation (7) one can now estimate the characteristic scale of the magnetic boundary layer to be $\delta \approx 10^{-3} r_{\rm c} \approx 4 \times 10^{-2}\Delta$, confirming that $\delta \ll \Delta$. It can be confirmed also that viscous stresses are wholly negligible. Furthermore, using $g \approx 530$ m s^{-2}, we find $\tilde{T}/T \approx 4 \times 10^{-6}$ and, using $\eta \approx 0.07\mathrm{m}^2\mathrm{s}^{-1}$, we find $u \approx \eta/\delta \approx 10^{-7}$ m s^{-1}, from which one obtains the characteristic tachocline ventilation time $\tau_{\rm v} \approx \Delta/u \approx 3 \times 10^6$y. If the magnetic field $B_{\rm i}$ deep in the radiative interior is the remnant of a primordial field, it is presumably stronger than B_0, conceivably by a factor as great as $\sim R_\odot/2\delta$, in which case $B_{\rm i} \approx 0.1$T, which is rather less than a previous rough theoretical estimate of the strength of the protosolar field[22].

The theory that we have outlined is only a preliminary analysis based on linearized boundary-layer equations, which is enough to establish the smallness of the scale δ and, most importantly, the ability of the magnetic boundary layer to stop the downwelling, and suppress the shear.

We acknowledge that our simple model does not take into account several dynamical processes believed to be present in the Sun, such as convective overshoot, shear-generated turbulence and wave-induced stresses. Moreover, because the model assumes a steady flow, we assume there to be no effect from the spindown, and no rectified trace of the 22-year solar-cycle variations that might be carried in the downwelling to affect the tachocline circulation. We estimate that, although any of these processes might actually modify somewhat the magnitude and geometry of the tachocline flow, they are unlikely to alter the model in any serious qualitative respect. The same is true of the flow within about $\sim 15°$ of the equator, where the leading-order dynamics is different because the $\mathcal{S}$-surfaces intersecting the base of the convection zone do not extend to the base of the tachocline.

The only magnetic field in the model is a remnant of the primordial field in the radiative core. Any field from a putative dynamo in the convection zone could be 'dredged' into the tachocline by the meridional flow, and thereby influence the dynamics; but it seems unlikely that the rapidly oscillating field associated with the solar cycle would contribute significantly to the dynamics in the radiative zone[23], particularly in view of the 10^6y tachocline ventilation time $\tau_{\rm v}$.

Seismic calibration of tachocline models having different depths Δ, using our expectation of a near-discontinuity at the thin magnetic boundary layer, estimates Δ much more precisely than the resolving power of any raw inversion. This is fortunate, because the estimate (from equation (9)) for $|B_0|$ varies inversely with the ninth power of Δ, and is therefore sensitive to measurement errors. The calibration reproduces the helioseismic sound-speed inversions extremely well[19].

The picture we have arrived at provides a natural (and simplest possible) consistent view of the dynamical coupling between the convective and radiative zones, and points towards a comprehensive and realistic theory of the interior magnetic field and of spin-down. Such a theory would have far-reaching implications for helioseismological inversion of the differential rotation, because Ferraro's law of isorotation (Ω constant on magnetic field lines[15]) would have to apply, under the simplest hypothesis of no interior torsional oscillation. Moreover, if the nonlinear boundary-layer and tachocline equations were to be solved – a formidable but probably tractable problem – it would be possible to deduce quantitatively the magnetic field at the base of the tachocline.

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Figure caption

Figure 1 Schematic representation of a meridional quadrant of the sun. The arrows represent the tachocline circulation, which follows surfaces $\mathcal{S}$ of constant specific angular momentum in the (green) body of the tachocline (whose thickness has been exaggerated by a factor 5), and is deflected by the magnetic field in the (blue) diffusive boundary layer (whose thickness has been exaggerated by a factor 50). The inclinations of the $\mathcal{S}$-surfaces, which, owing to the exaggeration of the tachocline thickness, are not drawn accurately, follow from the observation that the interior angular velocity $\Omega_{\rm i}$ lies between the angular velocities at the equator and at the poles in the (orange) convection zone. Moreover, the centre of upwelling should be at latitude $\sim 30^\circ$ (where, incidentally, sunspots emerge at the start of a new cycle). We are unable to draw the return flow in the convection zone without knowledge of the Reynolds stresses; details in the midlatitude upwelling region are also uncertain, obeying severely nonlinear dynamics, and may well be unsteady. The red lines represent the magnetic field in the (purple and white) radiative interior, which is assumed to be the dipole relic of a primordial field, arguably the most likely possibility (for simplicity, aligned with the rotation axis); we are unsure of the geometry of the field near the centre of upwelling, where the field lines are either dashed or absent. North-south asymmetry, as seen in the sunspot distributions observed in the Maunder minimum[24], may be related to the non-reversing interior dipole field. At the base of the tachocline the interior field vanishes on the rotation axis, where the magnetic boundary-layer theory suggests a singularity in the tachocline depth. The corresponding physical reality, which would again require nonlinear theory to describe it, would be relatively deep penetration of the tachocline circulation into and out of a 'polar pit', which might, conceivably, extend deep enough for lithium to be destroyed by nuclear reactions. The latest inversions of SOI seismic data[3] suggest such a pit in the angular-velocity variation.