Appendix B. The relationships among w, $\tilde{w}$, and $w_{\rm tr}$.

Recall that w is defined as the vertical advection velocity, $\tilde{w} = w + (K/H) - K_z$ is the quantity found by inverse-solving (4a–c), and $w_{\rm tr}$ is the ascent rate of $\hat{H}$ anomalies, defined graphically from the EOF-reconstructed plot, Plate 1b. We begin by noting that diffusion can change the apparent tape speed much more than dilution can, as discussed in section 3e; variations in K for fixed α had a much bigger effect on the transit time from 100 to 10 hPa than did variations in α for fixed K.

Taking (2) with χ redefined as the $\hat{H}$ *anomaly*, i.e. the departure from the mean, in other words the tape signal alone:

$$\chi_t + \tilde{w}\chi_z = K\chi_{zz} - \alpha\chi - \gamma\chi \tag{B.1}$$

Now, every extremum ($\chi_z = 0$) and zero-crossing ($\chi = 0$) must ascend with velocity $w_{\rm tr}$. The following result can be obtained using either fact, but it is simpler to use the zero-crossings. We have

$$\chi_t + w_{\rm tr}\chi_z = 0 \tag{B.2}$$

at those altitudes and times for which $\chi = 0$. By comparing (B.1) with (B.2), we readily obtain, regardless of the values of α and γ,

$$w_{\rm tr} = \tilde{w} - \left.\frac{K\chi_{zz}}{\chi_z}\right|_{\chi=0} \tag{B.3}$$

whence, using equation 3,

$$w_{\rm tr} = w + K/H + K/h - K_z \tag{B.4}$$

where $h = -\chi_z/\chi_{zz}$; note that no approximations have yet been made. We can calculate h by substituting into the above from (A.1a) and (A.1b), and keeping only the leading order terms in the JWKB sense, yielding

$$1/h \simeq -2(\ln A)_z - (\ln m)_z \tag{B.5}$$

Variations of h about its mean value of 3.6 km are small. Figure 9 compares w, $\tilde{w}$, w_{tr}, and w_{tr} calculated using (B.4) with the approximation for $1/h$. The similarity of the two w_{tr} curves provides a valuable end-to-end check of our method, since the derivation in (B.4) relies on A and m and on $\tilde{w}$ and K found by inverse-solving (4a–c).