Non-linear Forchheimer corrections in acoustic scattering

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2021 AIAA AVIATION Forum and Exposition

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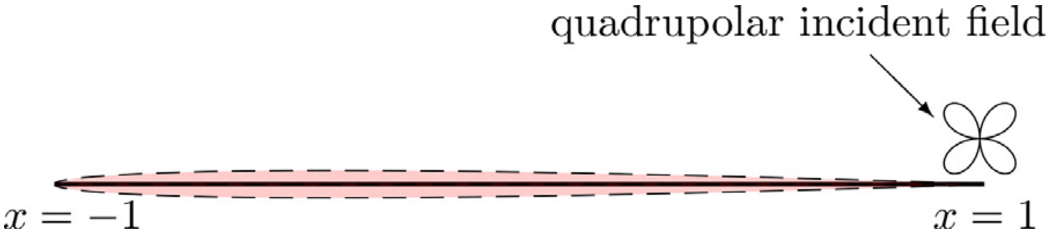
Related paper: M. Colbrook, L. Ayton, "Do we need non-linear corrections? On the boundary Forchheimer equation in acoustic scattering", JSV (2021)

The Problem

Modeling of porosity for materials (e.g. metal foams) with microscopic void spaces.



Low Reynolds number: $K\frac{[p]}{h} = -\mu v$ Darcy's law

High Reynolds number: $K\frac{[p]}{h} = -\mu v - \beta\rho\sqrt{K}v|v|$ Forchheimer inertial correction

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Mathematical Model

scattered field pressure

2D Helmholtz: $$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + k_0^2\right) p = 0,$$

incident field pressure

Kinematic condition: $$\frac{\partial p}{\partial y}\bigg|_{y=0} + \frac{\partial p_{\mathrm{I}}}{\partial y}\bigg|_{y=0} = C_0(x)\eta_a|_{y=0}, \qquad |x| < 1$$

$$\eta_a = \text{average fluid displacement normal to the plate}$$

Forchheimer condition: $$[p] = C_1(x)\eta_a + C_2(x)\eta_a|\eta_a|, \qquad |x| < 1$$

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Numerical Method

Goal: A fast, semi-analytical model that incorporates such non-linear effects without requiring a full turbulent simulation.

Idea: Separation of variables in elliptic coordinates:

$$x = \cosh(\nu)\cos(\tau), y = \sinh(\nu)\sin(\tau)$$

$$p(\nu,\tau) = \sum_{m=1}^{\infty} a_m \mathrm{se}_m(\tau)\mathrm{Hse}_m(\nu)$$

Expand η_a in a Chebyshev series.

Collocation yields a nonlinear system

Solve with Newton's method

$$A\mathbf{v} + (B\mathbf{v}) \circ |C\mathbf{v}| = \mathbf{c}$$

unknown coefficients

component-wise multiplication

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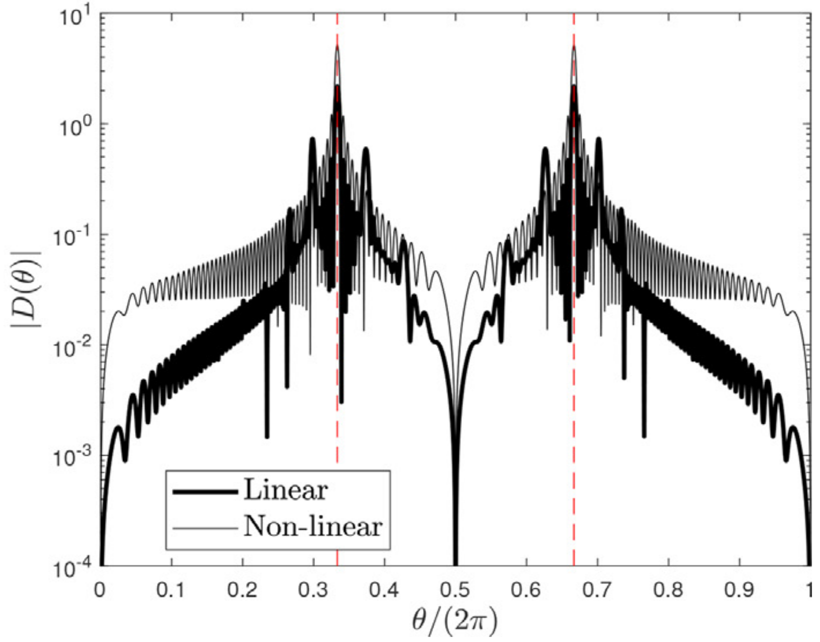
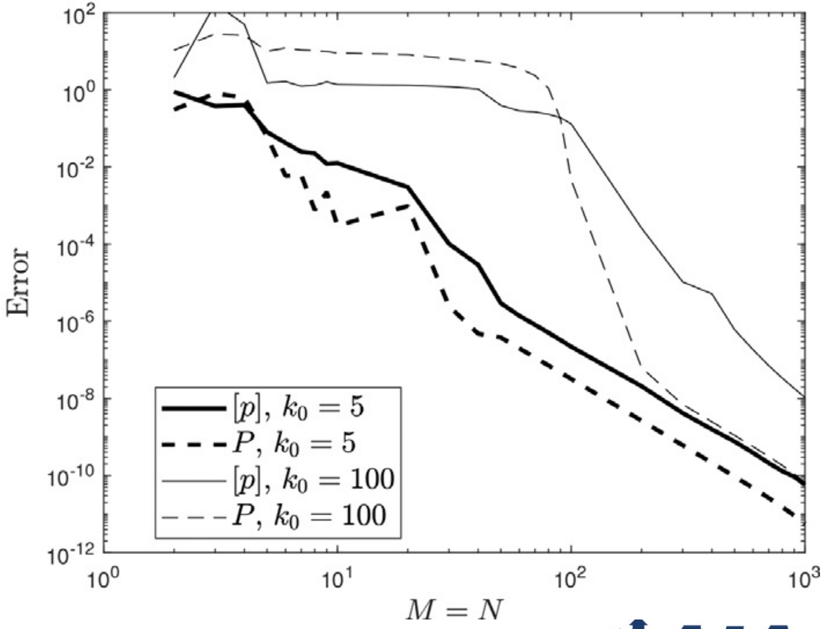
Convergence

$$C_0(x) = k_0^2, \quad C_1(x) = \mathrm{i}k_0(1.2 + \sin(20x)), \quad C_2(x) = \mathrm{i}20k_0^2(x^2+1)$$

$$k_0 = 100$$





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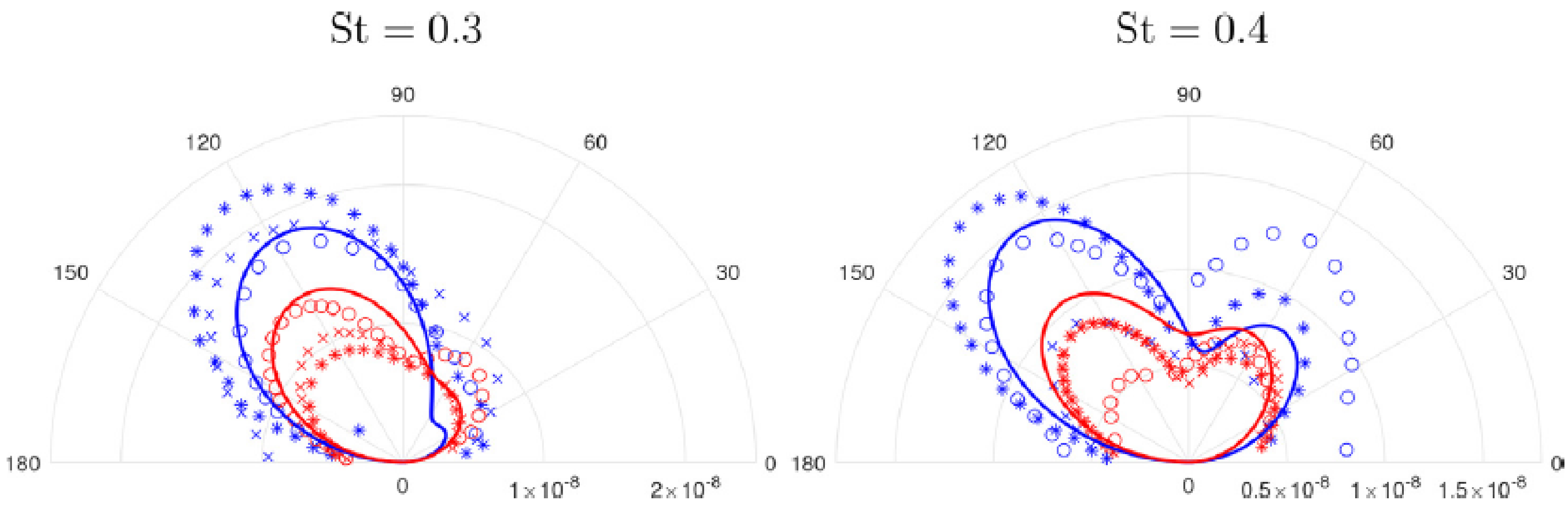
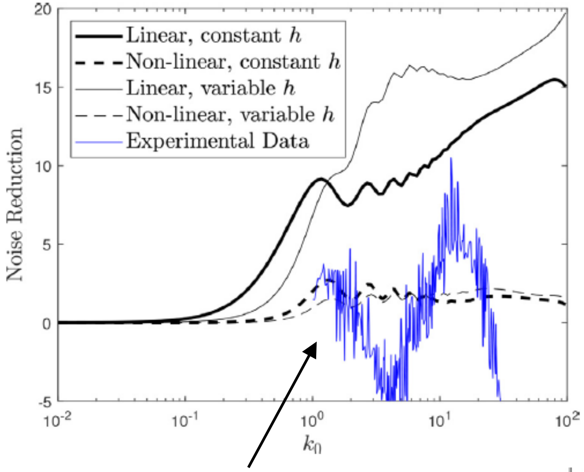
Comparison to LES [S. Koh, M. Meinke, W. Schröder (2018)]



Comparison to Experiments

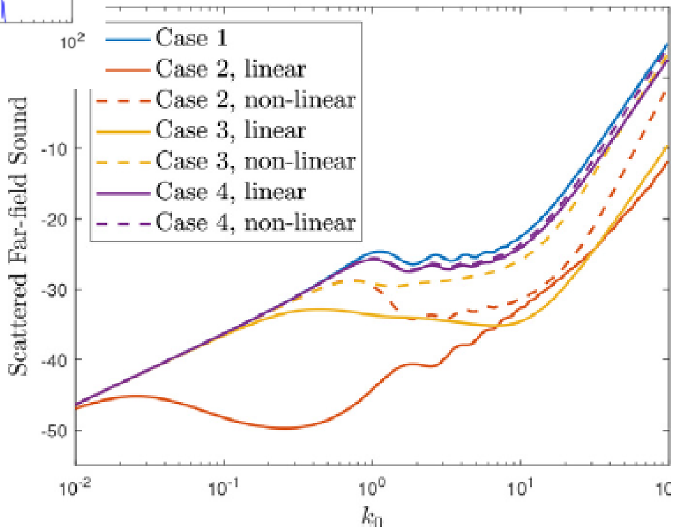


Geyer & Sarradj “Trailing edge noise of partially porous airfoils” 20th AIAA/CEAS Aeroacoustics Conference, 2014.

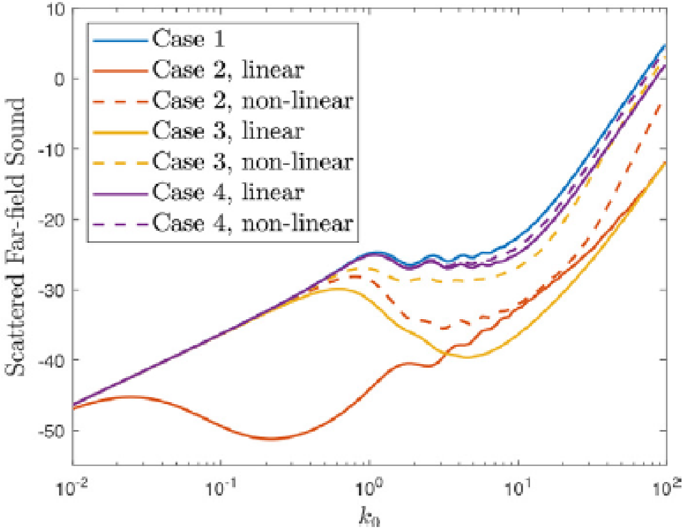
Case	K (m^2)	β	Material
1	0	–	Impermeable
2	2.7×10^{-9}	0.14	Alantum NiCrAl open-cell metal foam
3	5.72×10^{-11}	$0.5^{\dagger}$	Sintered PE granulate (Porex)
4	3.65×10^{-12}	0.613	Sintered SUS316L powder (Group 2, 9 mm)

$$\text{scattered sound} = 10 \log_{10} \int_0^{2\pi} |D(\theta)|^2 d\theta$$

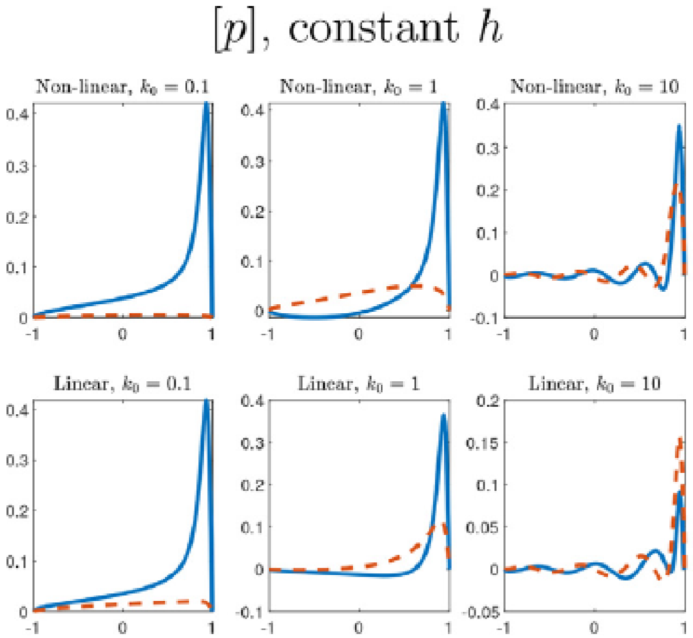
Constant h

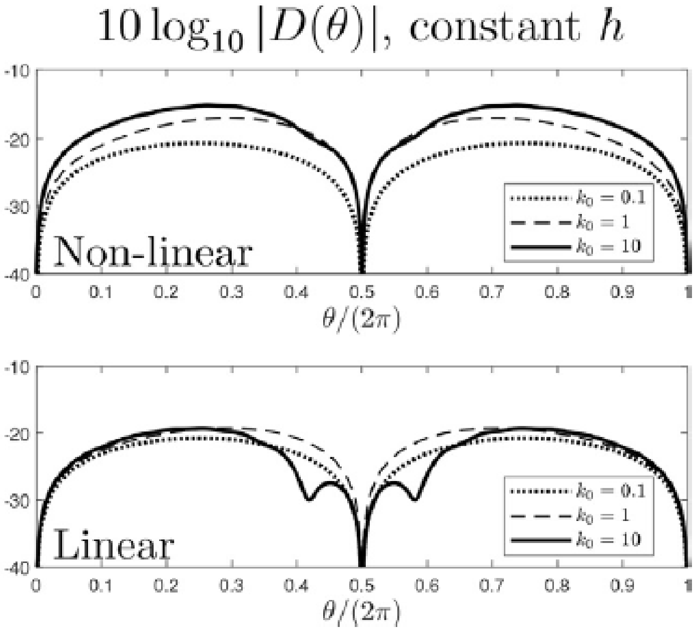


Variable h



Far-field Directivity and Surface Pressure Distribution





Conclusion

- Developed a fast numerical method that can account for variable thickness.
- Linear model at high frequencies can hugely over-predict noise reduction.
- Saw good agreement with experiments when supplemented with a non-linear Forchheimer correction.
- For mid/high frequencies and typical high permeability materials, local inertial effects at trailing edge can dominate overall acoustic scattering behaviour.
- Thus accurate modelling of aft of aerofoil is unnecessary in comparison to the importance of including inertial effects local to the source (boundary layer).
- Future directions: multiple aerofoils, non-constant inertial coefficient, surface roughness.

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