LEFT-RIGHT SPLITTING FOR ELECTROMAGNETIC SCATTERING IN 3D

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INTRODUCTION

This paper describes the left-right (L-R) splitting algorithm and its application to electromagnetic scattering by 3D scatterers. Surface currents are as usual expressed as the solution of a boundary integral equation, which can become prohibitively expensive for large or complex problems. L-R operator splitting, used for 2D (scalar) problems in [Spivack 1994 and 1995], and related methods [Kapp & Brown 1996; Tran 1997, Pino it et al 1999], expand the solution of this equation as an operator series, in increasing orders of multiple scattering. Each term can be evaluated rapidly, and the approximation is provided by truncating the series.

The approach has proved versatile and has been applied to extended rough surfaces as well as finite scatterers, converging for a wide range of incident angles. The approximation can provide considerable physical insight, and truncation errors can be examined theoretically, in some sense independently of discretisation.

In many cases only a single iteration is needed to provide accurate results. At its simplest the evaluation time scales with $O(f^4)$, where f is frequency. This is sufficient to treat electrically large scatterers, but could be accelerated to $O(f^3)$ by use of fast matrix mutiply methods (eg fast multipole).

METHOD AND APPLICABILITY

The method arose originally in the treatment of low grazing angle scatter by rough surfaces. Suppose Cartesian coordinates (x, y, z) are specified where z is the vertical, and x is in the plane of incidence and can be thought of as a 'range' direction. If a known radar field is incident on, say, an extended rough surface, then the induced surface current can be obtained by solution of the magnetic field integral equation

$$J_{inc}(\mathbf{r}_s) = \frac{1}{2}\mathbf{J} \; - \; \mathbf{n} \times \int_{\text{surface}} \mathbf{J} \times \nabla G dS$$

or more conveniently in terms of operator notation $\mathbf{J}_{inc} = \mathbf{AJ}$ where $\mathbf{r}_s = (x, y, z)$, all terms except $\mathbf{J}$ are known, $\mathbf{J}_{inc} = \mathbf{n} \times \mathbf{H}_{inc}$ and $\mathbf{n}$ is the surface normal. By splitting the integral into two halves with respect to x to the left and right of the observation point $\mathbf{r}_s$, this can be written $\mathbf{J}_{inc} = (\mathbf{L}+\mathbf{R})\mathbf{J}$, where the Cauchy principal value is assumed to be in $\mathbf{L}$. Formally the solution $\mathbf{J} = \mathbf{A}^{-1}\mathbf{J}_{inc}$ can be expanded and written as a series:

$$\mathbf{J} = (\mathbf{L}^{-1} - \mathbf{L}^{-1}\mathbf{R}\mathbf{L}^{-1} + ...)\mathbf{J}_{inc}.$$

The key step is that under the assumption that the effect of $\mathbf{R}$ is in some sense small, this series converges quickly and can be truncated. Physically this corresponds roughly to the assumption that surface interactions are primarily 'from the left', as expected in this scattering regime.

Once the surface currents are obtained, the scattered electromagnetic fields at all angles are found as an integral of these currents over the surface.

Considerable success has been obtained in applying this approach for perfectly conducting rough surfaces and waveguides, and radar interaction with closed scatterers. For a wide range of incident angles accurate results are obtained using only one or two iterations.

NUMERICAL EVALUATION AND COMPUTATIONAL COST

Discretisation of the above system is straightforward and gives rise to a matrix equation. We can therefore replace $\mathbf{A}$, $\mathbf{L}$, and $\mathbf{R}$ by matrices. The total number of surface points N required in the discretisation scales

with the square of the frequency, f^2. The main computational advantage then arises from the treatment of the inverse of operator $\mathbf{L}$ compared with that of the full form $\mathbf{A}$. In effect $\mathbf{A}$ is a full matrix so that the time needed for 'exact' inversion scales with f^6, whereas $\mathbf{L}$ (which in the 2D case is lower triangular) can be inverted exactly by a marching method of $O(f^4)$. Further terms in the series involve multiplication by $\mathbf{R}$ but inversion only of $\mathbf{L}$, so computation time is proportional to the number of terms used.

The system is well-suited to parallelisation, and such calculations have been carried out on the Cranfield-Cambridge High Performance Computer Facility (HPCF). The marching technique for evaluation of $\mathbf{L}^{-1}$ falls into two main parts at each step: a matrix multiplication and an inversion of a 'small' matrix (whose dimension is given by the number of surface points in the plane transverse to the x axis.) For example, without further approximation, evaluation for a scatterer of around 200 by 60 wavelengths can be carried out in around 100 minutes using 32 processors on the SunFire machine.

For a typical scatterer, in the above steps the matrix inversions form a small part of the overall time, compared with the matrix multiplications. Evaluation time for these scales with $O(f^4)$ because the Green function must be evaluated for each pair of surface points. The L-R series, before futher approximation, allows solution for electrically large scatterers in reasonable computational times. If necessary accelerated methods could be applied, including those based on fast multipole or wavelets, by which this can be reduced to $O(f^3)$.

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