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Evolution of shoreline position moments

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Abstract

This paper considers the problem of forecasting coastline position in response to a fluctuating wave climate. Statistical moments are introduced as a method for describing shoreline evolution and variability. Equations are derived and solved for the statistical moments of shoreline position. These equations describe analytically the time-dependent ensemble averaged solution, and its dependence on wave climate without the need for computationally intensive Monte-Carlo simulations. The average is understood to be taken over the ensemble of possible wave sequences. An example application, based on a time series of nearshore synthetic wave conditions, is used to illustrate how the technique might be used in practice to account for both short- and long-term variability in wave climate.

Keywords: Shoreline position moments; Statistical moments; Shoreline evolution

1. Introduction

Changes in shoreline morphology are the result of physical processes with a range of temporal and spatial scales. From a practical perspective, there is interest in beach response to distinct storm events (*short-term*) and in coastal evolution over some years and decades (*long-term*). Long-term changes trace the general trend in morphological variations over a time span approximately coinciding with the period of design life for coastal structures (De Vriend et al., 1993). Storm events can lead to large but often transient deformations of the beach, with the effects being smoothed over a period of months. From a long-term perspective, the morphological response to storms is analogous to 'noise' about the long-term trend, caused by the fluctuating morphological forcing of the waves, tides and surges. Estimation of long-term shoreline movement and its variability remains a difficult open problem.

The ability to predict changes in coastal morphology is hampered by a lack of observational data, and by the prohibitive computational complexity of applying deterministic dynamical equations for fluid flow and sediment transport over even relatively short periods of a single storm (De Vriend et al., 1993). Therefore, much research has focused on simplified models for longshore or cross-shore transport. An equation governing the longshore transport of sand on

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a beach was derived by Pelnard-Considere (1956). This has been subsequently extended to account for variations in wave direction and sediment transport along the shore (Larson et al., 1997). At its simplest, the equation for the position of a single chosen depth contour, $y(x,t)$, from a fixed datum line takes the form of a linear diffusion equation with constant coefficient, and has gained the epithet of the 'one-line equation'. This model has proved remarkably robust, and over the past decade has been used widely, typically to examine shoreline changes over periods of months or years (see Hanson and Kraus, 1989; Kamphuis, 1991; Larson et al., 1997).

In practical applications, the one-line model has been used with sequences of forcing conditions that are considered representative of the conditions likely to occur in the future. Wave conditions are required to drive a one-line model and these are often available only as summary statistics. These conditions might be monthly average conditions determined from several years of data or, synthetic time sequences composed by combining fragments of records together. LeMehaute and Soldate (1979) addressed the problem of using wave statistics to drive the one-line model and proposed a procedure for constructing representative wave conditions. A more rigorous approach is to use Monte Carlo simulations. Vrijling and Meyer (1992) applied the one-line model to perform Monte Carlo simulations of the shoreline position near a port, while Dong and Chen (1999) included random temporal variability in a Monte Carlo study based on a one-line model modified to account for some cross-shore sediment exchanges. In both cases, assumptions were made about the statistics of the forcing conditions that restrict the application of the techniques to more general situations. An alternative is to use a formal averaging procedure and solve the resulting equation for the mean shoreline position. Reeve and Fleming (1997) used a time-averaged form of the one-line model and historical shoreline positions to infer the distribution of time-averaged sediment sources and thence to estimate likely future mean position of the shoreline. While providing an indication of the typical position of a beach and its sensitivity to variations in the boundary conditions, none of these approaches provides a direct method for determining the mean and variance of the shoreline in the presence of fluctuating wave conditions.

In this paper, we derive solutions to the equations for the first and second moments of the shoreline position (i.e., the coastal plan shape), as governed by the one-line equation. Specifically, we formulate equations for the ensemble average shoreline position (or coastal plan shape) and its variance. General solutions are obtained for forcing conditions with arbitrary probability density functions and temporal autocorrelation function. The solution expresses the spatial variability of the coastal plan shape explicitly with time in terms of the statistics of the underlying wave climate. An illustration of how the methods can be applied in practice is given, together with some results for specific cases.

The derivation of the equations is given in Section 2, together with their general solutions for the case of the evolution of an arbitrary initial coastline. Solutions for a specific case of beach nourishment are presented in Section 2.4. In Section 3, the steps needed to apply the method to practical situations are illustrated using hindcast wave data covering a period of almost 30 years. A discussion of the results and conclusions are contained in Section 4.

2. Governing equations and moments

A longshore current is generated by oblique breaking waves; this current can generate longshore transport of sediment along a beach. However, quantitative estimation of sediment transport rates is extremely difficult. Transport rates may be inferred from changes in beach volumes derived from ground or aerial surveys. Direct measurement of longshore transport has been attempted using a variety of techniques, such as deposition of a tracer material or installation of traps. Equations proposed for calculating longshore transport have been based on a number of different concepts, including: the energy flux approach, the stream power approach, dimensional analysis, and force-balance methods.

The energy flux approach is based on the principal that the longshore immersed weight sediment transport rate, I_{ls}, is proportional to longshore wave power per unit length of beach, P_{ls}. The most widely used formula in this category is commonly known as the CERC equation (U.S. Army Corps of Engineers, 1984). The equation was derived for sand beaches and has been developed over a number of years. The formula is intended to include both bedload and suspended load and is usually given in the form of:

$$I_{ls} = \varepsilon' P_{ls} \quad (1)$$

where P_{ls} is the longshore component of wave energy flux (sometimes referred to as wave power) per unit length of beach, given by:

$$P_{ls} = (EC_g)_b \sin\theta_b \cos\theta_b \quad (2)$$

and where ε' is a dimensionless empirically derived coefficient. The volumetric transport rate, Q_{ls}, is related to I_{ls} by:

$$Q_{ls} = \frac{I_{ls}}{\Gamma} \quad (3)$$

where

$$\Gamma = (\rho_s - \rho_w) g \lambda \quad (4)$$

and λ is the porosity. A suggested value for ε' using H_s is 0.39 for sand sized sediments (U.S. Army Corps of Engineers, 1984). More recently, Schoonees and Theron (1994) suggested a value of ε' =0.41 for sand with D_{50}<1 mm based on fitting an energy flux expression to observational data. Bailard (1984) used the concept of stream power to derive an equation for ε' (depending on sediment fall velocity, bed velocity and breaking wave angle), which could be used in the CERC formula. He concluded that this modification of the ε' coefficient extended the range of application of the CERC equation, which can also be applied to a range of sediment sizes due to the dependence on fall velocity.

Kamphuis (1991) developed a formula for longshore transport for use on sand beaches, derived from mathematical relationships between groups of dimensionless variables, rather than from physical principles. For a typical sand, the formula may be written as an annual transport rate (Q_{ls}) with units of m^3/year, given by:

$$Q_{ls} = 6.4 \times 10^4 H_{sb}^2 T_p^{1.5} (\tan\beta)^{0.75} D_{50}^{-0.25} (\sin 2\theta_b)^{0.6} \quad (5)$$

Eq. (5) was found to be valid for data obtained in both laboratory and field conditions.

Equations such as Eq. (3) or (5) may be used in the one-line model of beach plan shape evolution. The starting point for this study is the extended one-line equation for the evolution of the beach plan shape presented in Larson et al. (1997, Eq. (9)):

$$\frac{\partial}{\partial x}(\alpha K) + \frac{\partial y}{\partial t} = K \frac{\partial^2 y}{\partial x^2} + \frac{\partial K}{\partial x}\frac{\partial y}{\partial t} \quad (6)$$

where $y(x, t)$ denotes the beach position in relation to a fixed datum line set parallel to the trend of the shoreline, K is a measure of the sand transport rate along the beach (Q_{ls} divided by the depth of closure) and α is the angle of breaking wave crests relative to the datum line. It is noted that this equation is applicable strictly only for cases where the wave crests make a small angle with the shoreline contour. Fig. 1 illustrates the definition of terms.

In general, K and α will be functions of longshore distance and time (x and t, respectively). For given values of K and α Eq. (6) may be solved numerically to find y. In practice, it can be more usual to have K and α as functions of time at a point taken to be

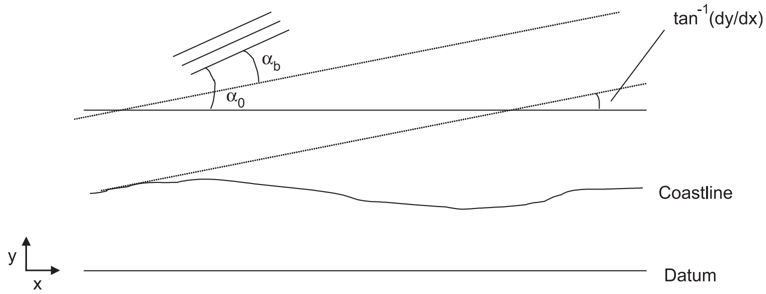

Fig. 1. Definition of terms for the one-line model.

representative of the region of beach under consideration. This analysis, therefore, is restricted to the case where K and α vary in time only, but in a random fashion. In this case Eq. (6) becomes:

$$\frac{\partial y}{\partial t} = K(t)\frac{\partial^2 y}{\partial x^2} \tag{7}$$

If we treat the shoreline position as a randomly varying function (or stochastic variable) then we may characterize it by, for example, its mean and variance over time at a fixed point along the shore. We may also be interested to find out how the mean and variance varies with position along the shoreline. The mean and variance are the first and second central moments of a distribution. More generally, the variance may be determined from the first and second moments. In this paper, we consider the first and second moments only. In order to derive expressions for the moments of shoreline position it is necessary to assume knowledge of the statistical variation of K; this is of course also required in approaches using Monte Carlo simulations. We consider K to be a realization of an ensemble and so may be written as the sum of its ensemble average and a deviation from this average:

$$K = \langle K \rangle + \delta(t) \tag{8}$$

where the fluctuating component, $\delta(t)$, has mean zero. In what follows we also assume that $\delta(t)$ is statistically stationary and has known statistics. The assumption of stationarity is not crucial but simplifies the analysis slightly. K is thus treated as a stochastic variable. In turn, this means that through Eq. (7) the shoreline position is also a stochastic variable. The chosen model (Eq. (8)) allows us to describe cases where K is not statistically stationary, specifically, where the mean value of K is a function of time. That is we write $\langle K \rangle = K_0 + \kappa(t)$, where K_0 is a reference value that could be taken as the value used in a deterministic application of Eq. (7) and $\kappa(t)$ accounts for deterministic seasonal or annual cycles, or longer term departures from the reference value. For practical applications K_0 would be determined as the 'best estimate' of the value of K for a coastal region, while $\kappa(t)$ would describe, for example, changes in this best estimate arising from models of climate change impacts on wave heights.

In what follows we use the angled brackets to denote an ensemble average; the average taken over all possible realizations of the random function K. The uncertainties in wave climate, sediment distribution and other key variables are then implicitly represented in the stochastic diffusion coefficient. The relationship may be made explicit by adopting a specific alongshore sediment transport equation. An example of how such an approach can be implemented is given in Section 3.

As an illustrative example we consider the case in which δ is well represented by a Gaussian distribution. Other distributions such as exponential or uniform can be treated similarly. However, on purely physical considerations K must remain bounded and, because of the small angle assumption, K must be positive. Thus, a cut-off is introduced and the corresponding truncated Gaussian distribution can be written

$$\frac{C_0}{\sigma_0\sqrt{2\pi}} \int_{-\varepsilon}^{\varepsilon} \exp\left(-\delta^2/2\sigma_0^2\right)\mathrm{d}\delta \tag{9}$$

where the cut-off ε is a positive number smaller than K_0, and σ_0 is the standard deviation of δ. Here C_0 is a normalization constant given by

$$C_0 = 2 \Big/ \left[\mathrm{erf}\left(\frac{\varepsilon}{\sigma_0\sqrt{2}}\right) - \mathrm{sgn}(-\varepsilon)\mathrm{erf}\left(\frac{-\varepsilon}{\sigma_0\sqrt{2}}\right)\right] \tag{10}$$

where erf(x) is the error function (see Abramowitz and Stegun, 1964).

In what follows we will require the autocorrelation function of δ, denoted by ρ, and defined by

$$\rho(t' - t'') = \langle \delta(t')\delta(t')\rangle \tag{11}$$

where t' and t'' vary independently over the values of t being considered. The variance of $\delta(t)$ is, therefore, given by $\rho(0)$ and will be denoted by ρ_0. We will also need the time-integral of δ

$$f(t) = \int_0^t \delta(t')\mathrm{d}t \tag{12}$$

It may also be shown (see Papoulis, 1987) that $f(t)$ also obeys a Gaussian distribution with cut-off, which can be written:

$$f(t) = \frac{c}{\sigma\sqrt{2\pi}} \int_{-t\varepsilon}^{t\varepsilon} \exp\left(-F^2/2\sigma^2\right)\mathrm{d}f \tag{13}$$

where $\sigma(t)$ is the standard deviation of f, which varies with time so that f is not stationary, and c is again a normalization constant, and that

$$\sigma^2(t) = \left\langle \int_0^t \int_0^t \delta(t')\delta(t'')\mathrm{d}t'\,\mathrm{d}t'' \right\rangle = \int_0^t \int_0^t \rho(t' - t'')\mathrm{d}t'\,\mathrm{d}t'' \tag{14}$$

where the last expression follows because the order of integrating and averaging may be reversed. For many cases of practical interest the integral in Eq. (14) can be evaluated explicitly as shown in the following sections.

2.1. First moment

In this section, we derive and solve an equation for the first moment, which describes the mean shoreline position as a function of time. The first moment of the shoreline position is a function of time and longshore position, and is defined as

$$m_1(x,t) = \langle y(x,t) \rangle \tag{15}$$

It is convenient to work with the spatial Fourier transform of the shoreline position. Accordingly, we denote the Fourier transform of the first moment by M_1 and write

$$M_1(v,t) = \langle \hat{y}(v,t) \rangle \tag{16}$$

where $\hat{y}(v,t)$ is the Fourier transform of y with respect to x and v is the transform variable. Given the statistics of K we will obtain an expression for m_1 as a function of time. Taking Fourier transforms of each side of Eq. (7) we get

$$\frac{\partial \hat{y}}{\partial t} = -v^2 K \hat{y} \tag{17}$$

This has the following solution over any time step $[t_0,\ t_1]$:

$$\hat{y}(v,t) = \exp\left(-v^2 \int_{t_0}^{t} K(t')\mathrm{d}t'\right)\hat{y}(v,t_0) \tag{18}$$

Without loss of generality we set $t_0=0$. Taking the ensemble average of Eq. (18) and substituting Eq. (8) into the result yields

$$M_1(v,y) = \exp\left(-v^2\left(tK_0 + \int_0^t \kappa(t')\mathrm{d}t'\right)\right) \times \left\langle \exp[-v^2 f(t)] \right\rangle \hat{y}(v,0) \tag{19}$$

In this equation, the only random part on the right-hand side is the term $\langle \exp[-v^2 f(t)]\rangle$. The ensemble average is found by integrating the product of the quantity in the angle brackets and the density function of the stochastic variable $f(t)$ over all possible values. From Eq. (13) this is given by

$$\left\langle \exp[-v^2 F(t)] \right\rangle = \frac{c}{\sigma\sqrt{\pi}} \int_{-t\varepsilon}^{t\varepsilon} \exp\left(-v^2 F - \frac{F^2}{\sigma^2}\right)\mathrm{d}F \tag{20}$$

Eq. (20) can be written

$$\left\langle \exp[-v^2 F(t)] \right\rangle = \frac{c}{\sigma\sqrt{\pi}} \int_{-t\varepsilon}^{t\varepsilon} \exp\left[-\left(\frac{F}{\sigma} + \frac{v^2\sigma}{2}\right)^2 + \frac{v^2\sigma^2}{4}\right]\mathrm{d}F \tag{21}$$

The integral can be performed analytically to give,

$$\left\langle \exp[-v^2 F(t)] \right\rangle = \frac{c}{2} \mathrm{e}^{v^4\sigma^2/4}[\mathrm{erf}(A+B) - \mathrm{erf}(A-B)] \tag{22}$$

where $A=v^2\sigma/2$ and $B=t\varepsilon/\sigma$. Substitution of Eq. (22) into Eq. (19) yields the transform of the first moment as required:

$$M_1(v,t) = \frac{c}{2}\mathrm{e}^{-v^2\left(tK_0 + \int_0^t \kappa(t')\mathrm{d}t'\right) + v^4\sigma^2/4} \times [\mathrm{erf}(A+B) - \mathrm{erf}(A-B)]\hat{y}(v,0) \tag{23}$$

The first moment m_1 can be obtained from this by performing the inverse Fourier transform. The coefficients A, B, and therefore m_1, depend on the statistics of the coefficient K via the quantity $\sigma(t)$. However, the behavior at large v must be considered to ensure that M_1 is bounded so that its inverse transform exists and can be computed. For further details, including the demonstration that the expression for M_1 is bounded for large v, the reader is referred to Spivack and Reeve (2003).

While the underlying equations such as Eq. (7) are useful in simulating changes in beach plan shapes and examining qualitatively the dependence on K, the first moment allows us to look at the evolution of the mean plan shape, examine underlying persistent effects and quantify their dependence on the statistics of K.

Equally important, however, is the spatial variability, and the variation about this mean. This is given by the second moment, and is important in "worst case" estimation.

2.2. Second moment

The second moment m_2 is defined to be the spatial autocorrelation function of

$$m_2(x_1, x_2, t) = \langle y(x_1, t) y(x_2, t) \rangle \tag{24}$$

where x_1 and x_2 are any two points. Note that the second moment is a function of three variables. From the equations above, we can form and solve an exact equation for the second moment as follows.

Define the two-dimensional Fourier transform M_2 of m_2, with respect to x_1, x_2, so that say

$$M_2(v, \omega, t) = \int \int \langle y(x_1, t) y(x_2, t) \rangle e^{i(vx_1 + \omega x_2)} dx_1 dx_2 \tag{25}$$

The angled brackets can be interchanged with the integrations, so that

$$\begin{aligned} M_2(v, \omega, t) &= \left\langle \int \int y(x_1 t) y(x_2 t) e^{i(vx_1 + \omega x_2)} dx_1 dx_2 \right\rangle \\ &= \left\langle \int y(x_1 t) e^{ivx_1} dx_1 \int y(x_2, t) e^{i\omega x_2} dx_2 \right\rangle \\ &= \langle \hat{y}(v, t) \hat{y}(\omega, t) \rangle \end{aligned} \tag{26}$$

This average can be evaluated explicitly in the following way. Substituting Eqs. (18) and (8) into Eq. (26) we obtain

$$\begin{aligned} M_2(v, \omega, t) &= \left\langle e^{-\left(v^2+\omega^2\right)\left\{tK_0 + \int_0^t \kappa(t')dt'\right\}} \right. \\ &\quad \left. \times e^{-\left(v^2+\omega^2\right) \int_0^t \delta(t')dt'} \hat{y}(v, 0) \hat{y}(\omega, 0) \right\rangle \\ &= \hat{y}(v, 0) \hat{y}(\omega, 0) e^{-\left(v^2+\omega^2\right)\left\{tK_0 + \int_0^t \kappa(t')dt'\right\}} \\ &\quad \left\langle e^{-\left[v^2+\omega^2\right] f(t)} \right\rangle \end{aligned} \tag{27}$$

The last term on the right is similar to the expression arising for the first moment (Eq. (20)), and the analysis carries through with v^2 replaced by $(v^2+\omega^2)$ in Eqs. (20) to (25). For brevity, we set $\kappa(t)=0$ and therefore obtain the solution, written in terms of error functions, as

$$\begin{aligned} M_2(v, \omega, t) &= \frac{c}{2} e^{-\left(v^2+\omega^2\right)K_0 t} e^{\left(v^2+\omega^2\right)^2 \sigma^2/4} [\text{erf}(C + B) \\ &\quad - \text{erf}(C - B)] \hat{y}(v, 0) \hat{y}(\omega, 0) \end{aligned} \tag{28}$$

where $C=(v^2+\omega^2)\sigma/2$ and again $B=t\varepsilon/\sigma$. As in the case of M_1 the dependence on the statistics of K is contained in the quantity $\sigma(t)$. The second moment can then be found for any time t as the two-dimensional inverse Fourier transform of M_2.

2.3. Variance of shoreline position

Our aim is to develop a set of equations to examine the mean spatial pattern and to quantify the departure of the typical beach plan shape from this mean. This departure is expressed as the variance of $y-m_1$, which can be expanded and expressed in terms of the second moment as follows:

$$\begin{aligned} \langle (y - m_1)^2 \rangle &= \langle y^2 \rangle - 2\langle y(x, t) \rangle m_1 + m_1^2 \\ &= \langle y^2 \rangle - 2m_1^2 + m_1^2 \end{aligned} \tag{29}$$

so that

$$\langle (y(x, t) - m_1(x, t))^2 \rangle = m_2(x, x, t) - m_1^2(x, t) \tag{30}$$

Note that the evolution of the variance cannot be determined directly but must be evaluated from the second and first moments by setting $x_1=x_2\equiv x$.

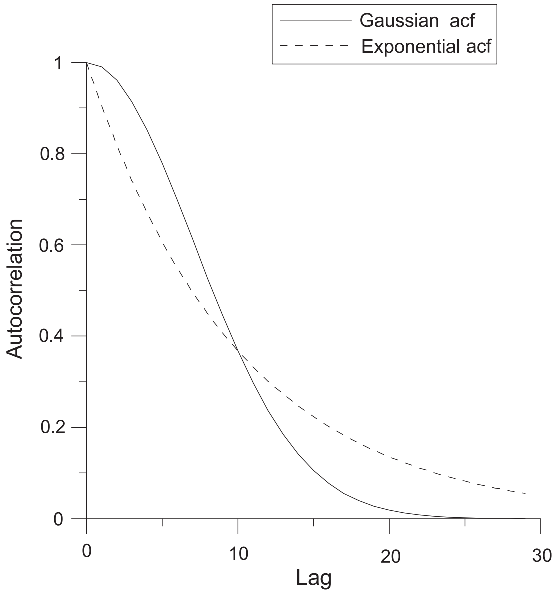

Fig. 2. Gaussian and exponential autocorrelation functions, both with a correlation time of 10.

2.4. Variance of morphological forcing

The random component in the morphological forcing is $\delta(t)$. The statistical properties of the evolving shoreline are determined by the statistical characteristics of the forcing. In particular, both the distribution function and the temporal autocorrelation function appear explicitly (namely, Eqs. (14) and (20)). In principle, the distribution and autocorrelation functions can be determined empirically and the expressions for the shoreline moments evaluated numerically. Alternately, measurements may be used to determine the parameters that give a best fit to an analytical function. Two such analytical forms are the Gaussian and exponential autocorrelation functions, given respectively by

$$\rho(\eta) = \rho_0 \mathrm{e}^{-\left(\frac{\eta}{T}\right)^2} \tag{31}$$

and

$$\rho(\eta) = \rho_0 \mathrm{e}^{-\left|\frac{\eta}{T}\right|} \tag{32}$$

where $\eta = t' - t''$ and ρ_0 is the mean square of the fluctuating component of δ. The typical rate of variation of the coefficient K is characterized by the correlation time scale, T. Fig. 2 illustrates the two autocorrelation functions.

A diffusion coefficient with variations obeying a Gaussian correlation function will exhibit irregularities that are very closely grouped about the single scale, T. In contrast, the exponential correlation function falls off to zero more slowly and so considerable contributions to the fluctuations in the diffusion coefficient come from a wide range of temporal scales. Fig. 3a and b shows realizations of a stochastic variable that have a Gaussian and exponential autocorrelation function, respectively. In both cases, the variables are Gaussian distributed.

As mentioned above, if the initial coastline configuration depends on longshore position then the statistics of y (x, t) will not be stationary with respect to either x or t. The first and second moment solutions are valid for an arbitrary initial coastline configuration. In what follows, we will use the autocorrelation function (32). In this case the variance in Eq. (14) may be found in closed analytical form to be:

$$\sigma^2(t) = 2\rho_0\Big(T(t-2T) + T\mathrm{e}^{-t/T}(t+2T)\Big) \tag{33}$$

For any given initial coastline position, inserting Eq. (33) into Eq. (22) or (23), and Eq. (28) yields

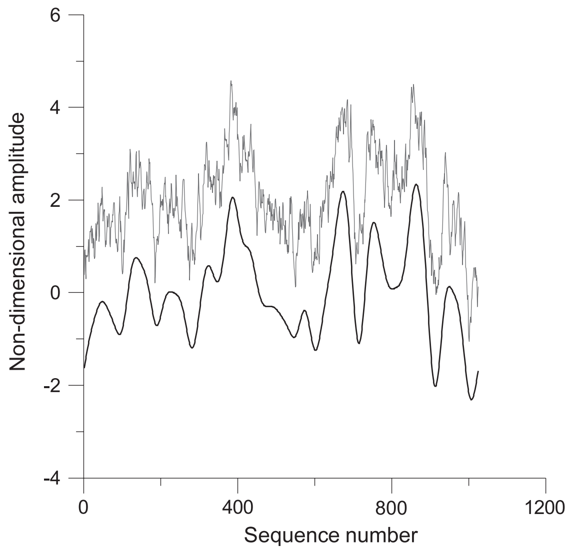

Fig. 3. Realizations of a stochastic process with Gaussian statistics and Gaussian autocorrelation function (lower curve), and exponential autocorrelation function (upper curve).

expressions for the transforms of the first and second moments. The moments themselves or the variance can then be obtained numerically using inverse fast Fourier transforms.

Examples for an idealized situation with a Gaussian autocorrelation function have been presented in Spivack and Reeve (2003). Here, we consider the following situation. The initial condition is a Gaussian-shaped nourishment on an otherwise plane beach line. The nourishment has a half-width of 500 m and a peak extent of 50 m at x=1000. We take K_0=267 m^2/year, $\kappa(t)$=0, $\delta(t)$ to be Gaussian distributed with cut-off at 15 m^2/year, with mean=0 and standard deviation=457 m^2/year. Fig. 4 shows the computed beach plan shape at a time t=0.1 T after nourishment, where T is the 'correlation time' of the fluctuations. For comparison, the solution for the case where no fluctuations in forcing are present is also shown (see Carslaw and Jaeger, 1959).

2.5. Interpretation

The form of the analytical solutions for the first and second moments of beach position (Eqs. (23) and (28)) demonstrates that one is unable to obtain the ensemble average results by simply putting a representative value of K into Eq. (7). Fluctuations in the forcing have a cumulative effect over time that must be taken into account in order to get accurate results.

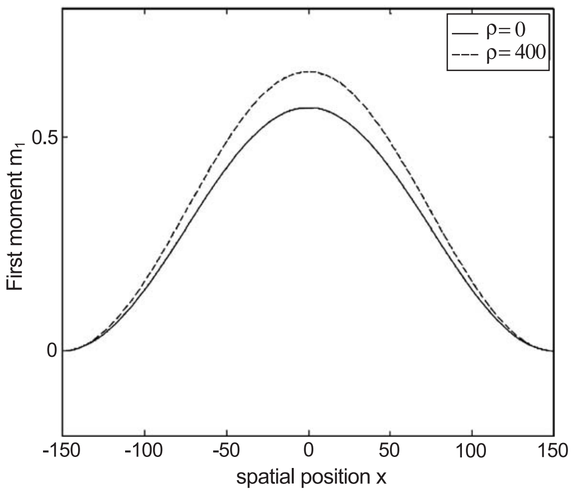

Fig. 4. Simple line graph for Gaussian nourishment example with and without fluctuations with an exponential ACF.

The cumulative effect depends on the probability density function and autocorrelation properties of the fluctuations. The probability density function of K reflects the distribution of storm intensity, and the shape of the autocorrelation function will be determined by the duration of individual storms and the groupiness of sequences of storms. The mean effect of fluctuations in forcing is to accelerate the movement of beach material along the shore relative to the case of no fluctuations. Why? The effect of storms (during which $K>K_0$) shortly after nourishment has a proportionately greater effect than later on as the transport is driven by the second derivative of the plan shape. The transport will be greater immediately after nourishment when the shoreline curvature is larger than in the later stages of evolution. On average, this leads to accelerated diffusion of the nourishment material, in comparison to the case where no variations in wave climate are present.

3. Application and results

In this section, we demonstrate the application of these concepts to a situation that is closer to what might be found in practice. Hourly hindcast wave conditions for a location on the southeast coast of the United Kingdom (UK) have been obtained for the period February 1, 1971 to May 31, 1998. Waves were hindcast in deep water using the surface winds output from a global meteorological model, and transformed inshore to a fixed depth contour using a spectral refraction model. The transformations were done using a fixed water level corresponding to Mean High Water Spring tide. The resulting time series of wave conditions provided estimates of mean H_s, T_z and wave direction at hourly intervals, a total of just under 250,000 data values.

Time series of the diffusion coefficient K were then calculated assuming a water depth of 6 m, a beach slope of 1/50, a depth of closure of 10 m with the CERC formula. This formula was chosen so that the transport is a function of wave angle and height only. This simplifies the statistical dependency but eases the interpretation of results. The resulting time series was analyzed to determine the interannual behavior of some basic statistics, an empirical probability density function and the autocorrelation function.

Fig. 5 shows the annual average values of K over the period 1971 to 1998, calculated on the basis of calendar years. The dashed line shows the best-fit linear trend line, which is $-12{,}980+6.67t$, where t is in years and runs from 1971 to 1998. A small but well-defined annual signature is also evident in the autocorrelation function calculated for the whole series from the detrended time series of K (see Fig. 6).

The amplitude and phase of regular variations may be found by Fourier analysis. Here, given its relatively small amplitude we do not remove the annual cycle from the series. Referring to Eq. (8) we can see that in this case we will write $\kappa(t)=-12{,}980+6.67t$. Subtracting this from the raw time series leaves the residual, random component. This has a mean value of K_0=267 m^2/year. Removing this mean value yields the residual, random component $\delta(t)$. Fig. 7 shows the empirical density function of the random component, $\delta(t)$. This has a mean of zero and a standard deviation of 456 m^2/year.

The shape of this density function is similar to a truncated Gaussian, although the empirical distribution has a large 'tail'. For the purpose of our illustration we take the density function to have the form $0.6e^{-(K+300)/300}$. This function is plotted in Fig. 7 as the dashed line. A comparison with the mean and standard deviation of the empirical distribution shows that this choice is approximate, and further refinement of the statistical distribution model is required to capture the long tail in the empirical distribution.

Finally, the autocorrelation function of $\delta(t)$ was calculated and is shown in Fig. 8a, b, c and d at scales

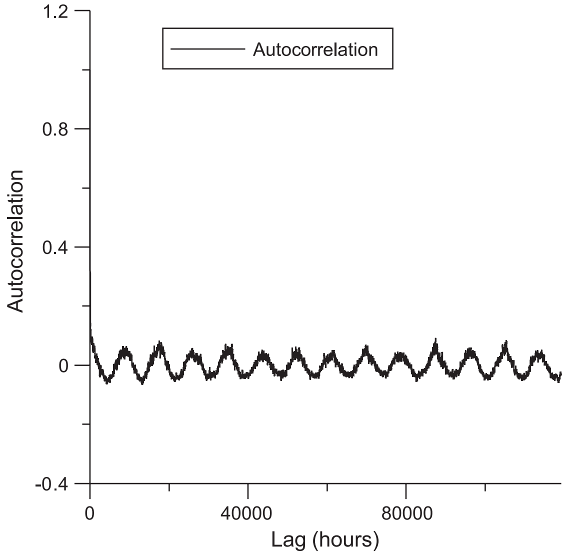

Fig. 6. Autocorrelation function of the detrended series of K.

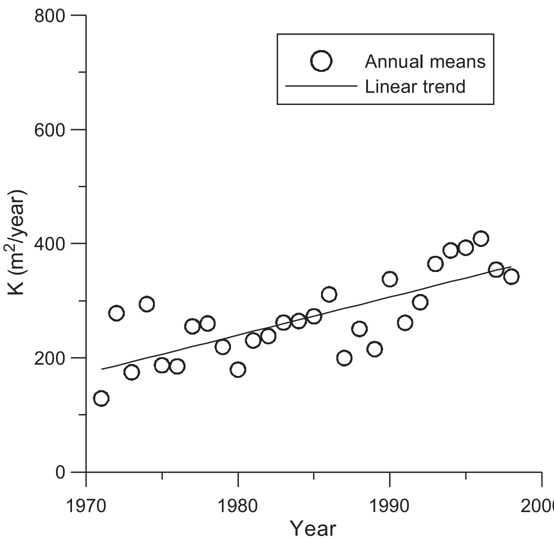

Fig. 5. Annual mean values of K computed over calendar years for the period 1972 to 1998. K is quoted in units of m^2/year. The linear trend has a slope of 6.6 m^2/year and has been determined by least squares fitting to the annual means.

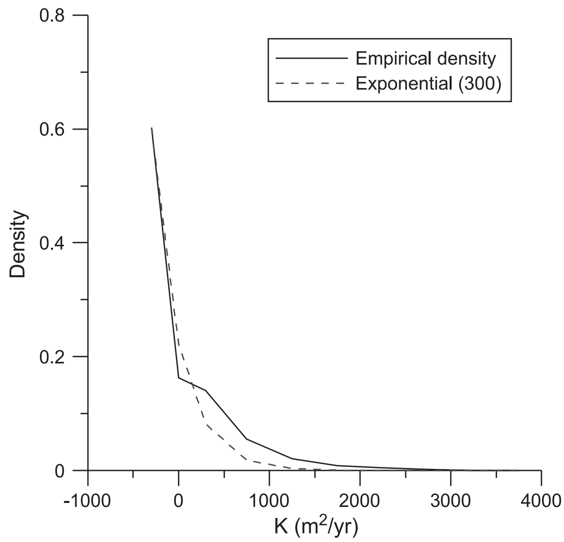

Fig. 7. Empirical probability density function of the detrended time series of K. Also shown is the fitted exponential curve which has an e-folding scale of 180 m^2/year.

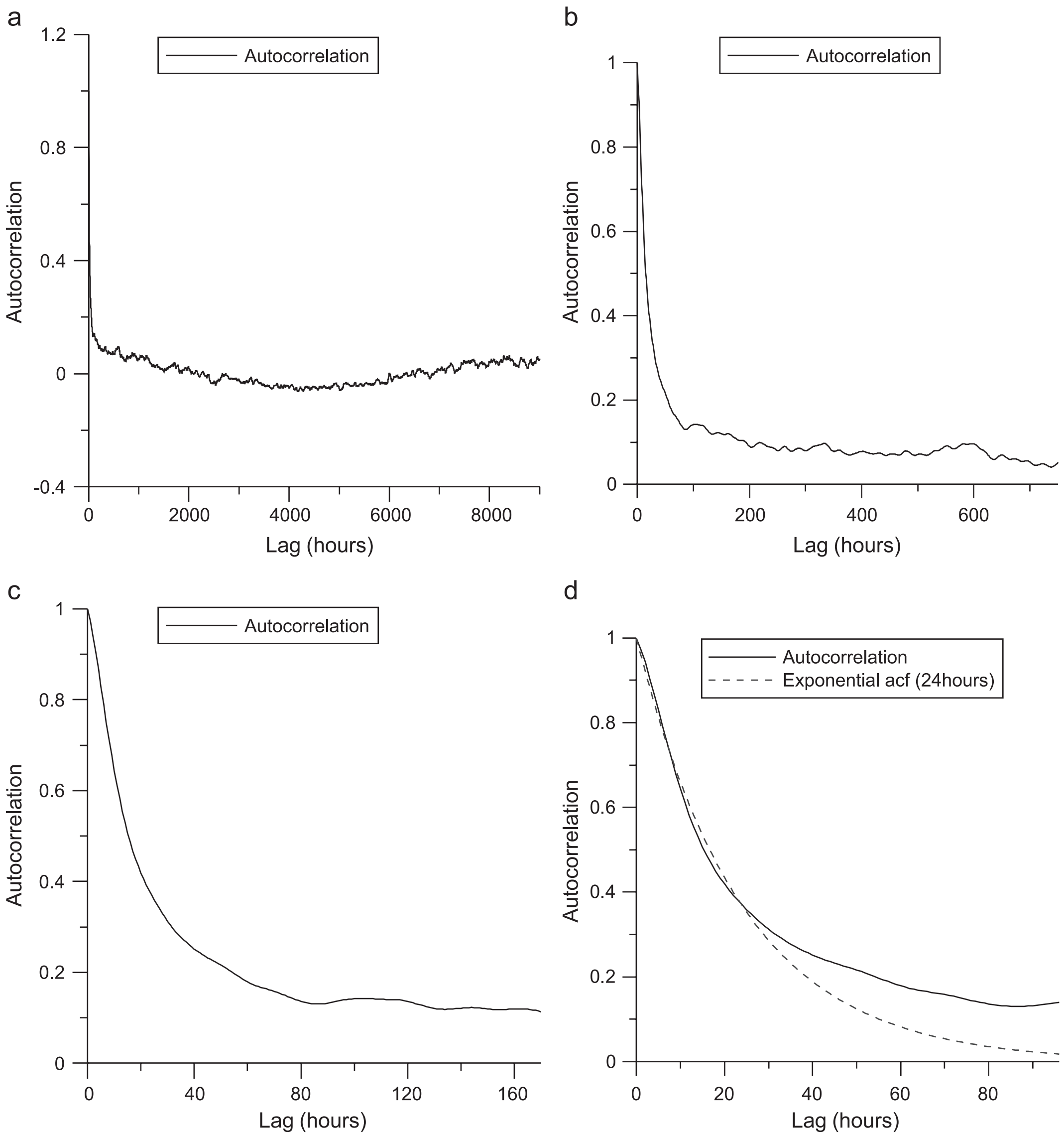

Fig. 8. Autocorrelation function of the detrended time series of K. (a) Over ~1 year, (b) ~1 month, and (c) ~1 week, and (d) 4 days. In (d) the fitted exponential curve, with an e-folding time of 1 day, is shown as the dashed line.

varying from a year to days. The plots illustrate the rich nature of the sediment transport rate behavior over time.

Here, we approximate the autocorrelation function by an exponential function:

$$\rho(\eta) = \mathrm{e}^{-\frac{\eta}{24}} \tag{34}$$

which is shown by the dashed line in Fig. 8d. Note that η and the time scale (Eq. (20)) are in units of hours.

Results were computed for the same beach nourishment situation as described in Section 2.4 but with K_0, $\kappa(t)$ and $\delta(t)$ as described above and with $\delta(t)$ having a standard deviation equal to 456 m/year$^{1/2}$. Results are shown in Fig. 9 for the fluctuating non-stationary wave climate and for the case of perpetual wave conditions giving K equal to its average value over the period 1971 to 1998.

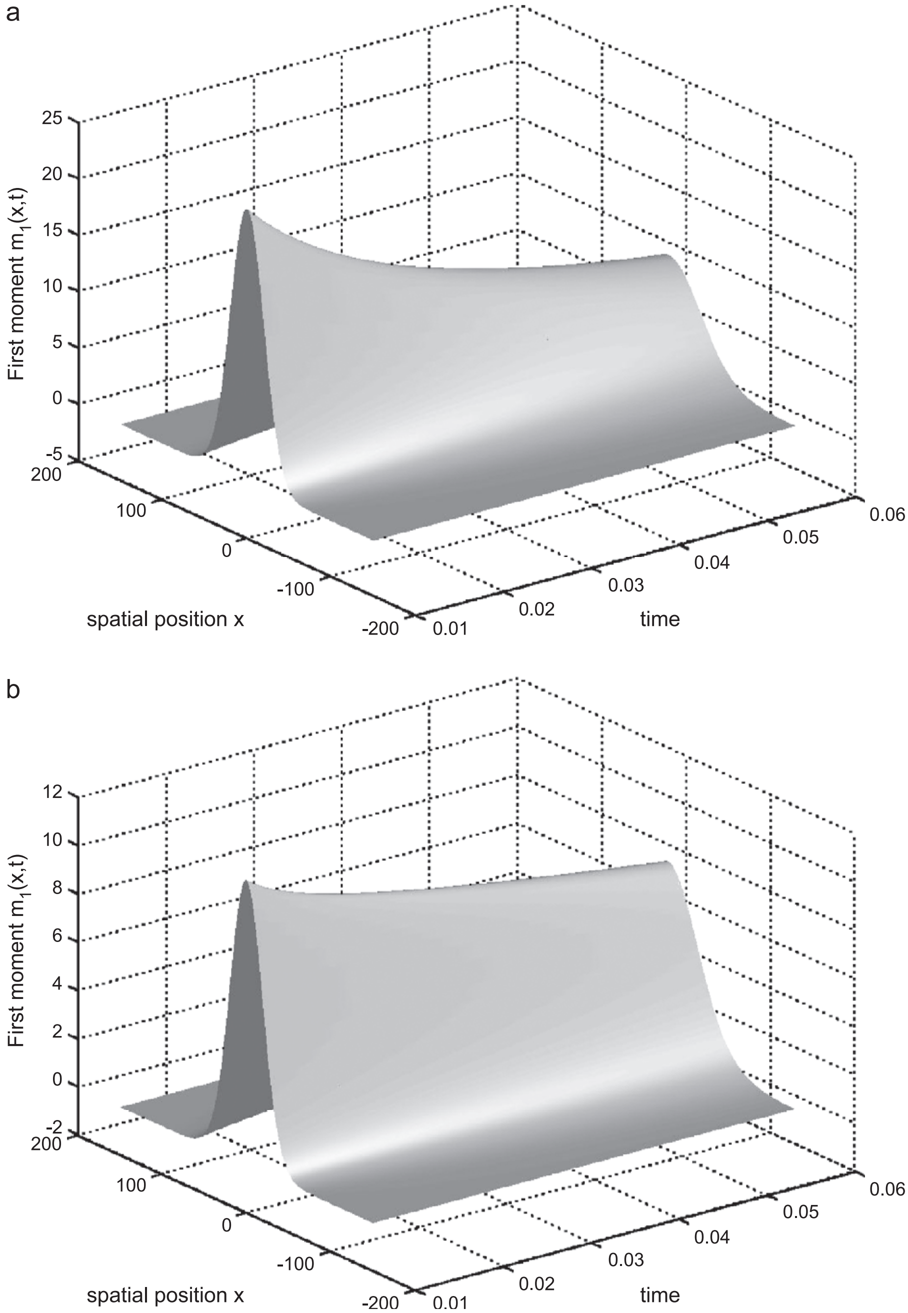

Fig. 9. Results using exponential fit to pdf and acf of 'real' data.

Fig. 9a shows the evolution of a Gaussian-shaped nourishment over time in the case where the coastal coefficient is constant in time and set equal to the long-term average (267 m^2/year), and no fluctuations are present. Fig. 9b shows the evolution of the same initial beach configuration but in the presence of fluctuating conditions and long-term trend in the longshore transport.

4. Conclusions

In this paper, we have considered the problem of forecasting coastline position in response to a fluctuating wave climate. Statistical moments have been introduced as a method for describing shoreline evolution and variability. The basis for the theoretical development has been the small wave angle form of the one-line equation. Equations governing the evolution of the first and second moments of beach position in time and alongshore directions have been derived. Solutions of these equations have been presented for some specific cases. For more general cases, solutions can be obtained using numerical integration techniques. The solutions describe the time-dependent ensemble average beach position, and its dependence on wave climate without the need for computationally intensive Monte-Carlo simulations. An example application, based on a time series of nearshore synthetic wave conditions, has been presented to illustrate how the technique might be used in practice to account for both short- and long-term variability in wave climate. The solutions for the moments of shoreline position provide a rigorous basis on which to examine long-term shoreline evolution from a stochastic perspective.

Several important points arise from the theoretical development and numerical solutions. These include:

- Solutions for the mean and variance of shoreline position have been found for idealized cases, and a more general approach has been illustrated by application to hindcast wave data.
- The solutions demonstrate that the mean shoreline position is not obtained by inserting mean forcing conditions into the governing equation. Fluctuations in the forcing conditions have a cumulative effect on the evolution of the shoreline; the nature of this effect is determined by the distribution function and the autocorrelation properties of the fluctuations.
- The solutions also provide an explanation of why beach nourishment is often observed to spread at a faster rate than predicted using representative values of coastal coefficient, K (or wave height and period) in a deterministic one-line model.
- Cases where conditions are not statistically stationary, such as in climate change scenarios, are amenable to analysis by the methods described, as demonstrated by an example application.
- The methods described can be used to provide a rigorous and independent check on shoreline position statistics computed by Monte Carlo models.
- The moment equation solutions complement Monte Carlo simulation because they allow quantitative analysis of the dependence of shoreline position on wave climate.

Research on developing the moment equation approach to the problem of predicting long-term coastal morphological changes is ongoing. The solution of the equations for more complicated situations, such as shoreline change near groynes, is the subject of current work by the authors.

List of symbols

α	angle of breaking wave crests relative to datum line
β	angle between horizontal and beach
δ	randomly fluctuating component of the coastal diffusion coefficient (m^2/year)
ε	cut-off value of coastal diffusion coefficient
ε'	coefficient relating sediment transport to the longshore component of wave energy flux
η	time lag ($=t'-t''$)
$\kappa(t)$	deterministic time varying component of K
λ	Porosity
θ	angle between the wave crest and the shoreline
ν, ω	spatial frequencies (Fourier transform variables)
$\rho(t'-t'')$	temporal autocorrelation function of δ
ρ_0	value of ρ at zero time lag ($t'-t''=0$)
ρ_s	density of beach sediment
ρ_w	density of sea water
$\sigma(t)$	standard deviation of $f(t)$, the integral of $\delta(t')$ over the interval $t'=0$ to t
σ_0	standard deviation of δ
C_g	wave group velocity
E	wave energy
erf	error function
g	acceleration due to gravity
H_s	significant wave height
H_{sb}	significant wave height at breaking
I_{ls}	longshore immersed weight sediment transport rate

K beach diffusion parameter
K_0 best estimate or reference value of K
M_1 Fourier transform of m_1
M_2 Fourier transform of m_2
m_1 first moment of shoreline position
m_2 second moment of shoreline position
P_{ls} longshore component of wave energy flux per unit length of beach
Q_{ls} volumetric transport rate
T_p peak wave period
T_z wave mean zero up-crossing period
T temporal correlation scale

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