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Long-term morphodynamic evolution of estuaries: An inverse problem

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Abstract

A new technique for predicting long-term variations in estuary morphology is developed using a morphological evolution equation that isolates diffusive and non-diffusive processes in estuaries. The contribution from non-diffusive processes to the morphological changes of the estuary is incorporated in the governing equation by a source function. The source function is derived by solving an inverse problem using historic survey data of the Humber estuary, UK covering a period of 150 years.

Source functions derived for the consecutive pairs of bathymetry surveys show a significant structure persistent throughout the entire data set. Large scale features such as tidal channels, tidal flats and linear banks in the outer estuary are persistently visible in the source function. Empirical Orthogonal Function (EOF) analysis is used to analyse the spatial and temporal variation in the source function. The first spatial eigenfunction which corresponds to the time-mean value of the source function captures over 92% of the mean square of the data. Over 65% of the data variance is captured by the first six eigenfunctions. The first temporal eigenfunction which corresponds to the mean value of the source function is almost constant as expected.

Keywords: estuaries; long-term morphodynamics; inverse method; source function

1. Introduction

Estuaries occur in many coastal areas worldwide. They are important to mankind as places of navigation, recreation and commerce as well as being extensive and diverse habitats for wildlife. Estuarine environments constitute complex physical systems subject to the influences of waves, tides, river flow and construction. In the context of estuarine morphology, these influences are often considered to 'force' a 'response' from the shore and sea bed. The exact form of the response will depend on the particular mixture of the waves, tides and so on at a particular estuary, together with other factors such as sediment characteristics and local geology.

The forcing acts at many scales of time and space often through complicated interactions. Time scales of estuary morphology evolution phenomena may vary from hours to days (short term), months to few years (medium term), decades to few hundred years (long-term) and several millennia (geological scale). In terms of spatial scales, the smallest morphological phenomena are ripples and dunes formed on the bed, and these are categorised as micro-scale features. Alternating and interacting ebb and flood channels and shoals are categorised as meso-scale elements. Features such as ebb tidal deltas, tidal flats and inlet channel belong to the macro-scale. The entire estuary and the adjacent coastal areas are classified as mega-scale (De Vriend, 1996; Hibma et al., 2004).

Prediction of the morphological behaviour of estuaries generally follows two approaches. The first is the use of geological and geomorphological evolution models which are designed to simulate morphological evolution over very long periods, some times referred to as top-down models (Di Silvio, 1989; Stive et al., 1998; Dennis et al., 2000; Karunarathna and Reeve, in press). These models, also known as behaviour oriented models, are based on empirical rules or expert analysis

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of long-term morphology data. The majority of top-down models are designed to predict the long-term physical response of an estuary to natural changes in forcing (e.g. sea level rise) and to changes in morphology following human interference. This category of models, in general, predicts large scale and long-term changes. The second approach is the use of process models based on two- or three-dimensional hydrodynamic models combined with sediment transport and morphodynamic modules known as bottom-up models (De Vriend and Ribberink, 1996; Friedrichs and Aubrey, 1996; Dronkers, 1998). These models are designed to simulate physical properties of estuaries in the short term.

However, numerical prediction of long-term estuary morphology evolution is in its infancy. Even though top-down models are relatively effective in predicting morphological changes on a comparatively long-term estuary-wide basis, they have inherent limitations due to lack of detailed physics. Process-based prediction models on the other hand are a valuable tool for assessing local, short term morphodynamic changes in an estuary, but have limitations due to insufficient knowledge of sediment transport processes and their linkage to hydrodynamics. Uncertainties in the predictions are amplified by treating sediment with a range of grain sizes and cohesiveness. Further, numerical predictions can exhibit great sensitivity to the initial conditions. In a more basic context there are limits to the predictability of morphological variables due to non-linearity of many coastal systems that may induce chaotic behaviour.

This has encouraged the development of hybrid models with simplified dynamics that are designed to predict qualitative behaviour by including only predominant processes. Some progress has been made by inferring potential sediment transport from tidal flow statistics. For example, Zimmerman (1981) argued that the dynamical characteristics of the tidal residual eddies could assist our understanding of the geomorphology of the sea bed. Spearman et al. (1995) and Van De Kreeke (1996) combined hydrodynamic models with regime relationships for estuaries. Wang et al. (1998) developed a dynamical–empirical model combining the advection–diffusion equation(s) with empirical equilibrium relationship of estuaries.

This paper presents a technique for analysing long-term evolution of estuary morphology formulated as an inverse problem. The governing equation used in the model is the two-dimensional diffusion equation with a source function. The equation does not explicitly describe the full spectrum of underlying physical processes other than diffusive processes but the source function is considered to represent the morphological response of the estuary to the tidal and wave forcing and increase in mean water level. The nature of the source function is derived as an inverse problem using the measurements on historic morphological evolution of the estuary concerned. If the source function exhibits a level of predictability, then there would be the potential for using the governing equation in a predictive mode with the estimates of the source function being used to do the prediction. For these reasons, the approach is to be considered as a 'hybrid' technique combining physical processes and the behavioural aspects of the estuary.

In Section 2 of the paper, re-formulation of a morphological evolution equation that isolates diffusive and non-diffusive processes in estuaries is presented. Non-diffusive processes are incorporated through a source function, which was derived solving an inverse problem. Section 3 describes the numerical inversion procedure. The field site and historical data are presented in Section 4. Results are presented in Section 5, and the paper finishes with a discussion and conclusions in Section 6.

2. Formulation of the model

The strategy for developing the model is to use elements of what has proven to work well in the past. The approach is pragmatic and behaviour oriented, being based on physical intuition, rather than being derived from first principles as a 'bottom-up' model would be.

We begin by considering evolution to occur due to cross-axis and along-axis transport. Models for each direction of transport have been reported separately.

Pelnard-Considére (1956) derived a diffusion equation governing the long-shore transport of sand on the basis of theoretical considerations and physical model experiments. At its simplest, the equation for the position of a depth contour, $h(x,t)$ from a fixed datum line takes the form:

$$\frac{\partial h(x,t)}{\partial t} = K\frac{\partial^2 h}{\partial x^2} \tag{1}$$

K is treated as a constant and determines the rate at which the beach responds. The value of K depends on incoming wave energy, the characteristics of beach material and the depth of closure, that is the depth beyond which there is no significant change in mean water depth with time. The equation predicts the movement of a depth contour arising from wave driven transport of material along the shoreline.

If the long-shore transport and the incoming wave angle vary as a function of x, then Eq. (1) takes the form (Larson et al., 1997):

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K(x)\frac{\partial h}{\partial x}\right) - \frac{\partial(\alpha K)}{\partial x} \tag{2}$$

Fig. 1 shows schematics of Eq. (2).

The second term in the right hand side represents contributions due to spatial variations in wave angle, α and diffusion coefficient K. This equation has since been extended to its use to describe the evolution of shorelines with coarser and finer sediment than originally used by Pelnard-Considére (1956).

An equation of similar form has been proposed for predicting long-term cross-shore changes in beach profiles by Stive et al. (1991) and Niedorora et al. (1995):

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial y}\left(K(y)\frac{\partial h}{\partial y}\right) + S(y) \tag{3}$$

where $S(y)$ is a source function and $K(y)$ is a diffusion coefficient.

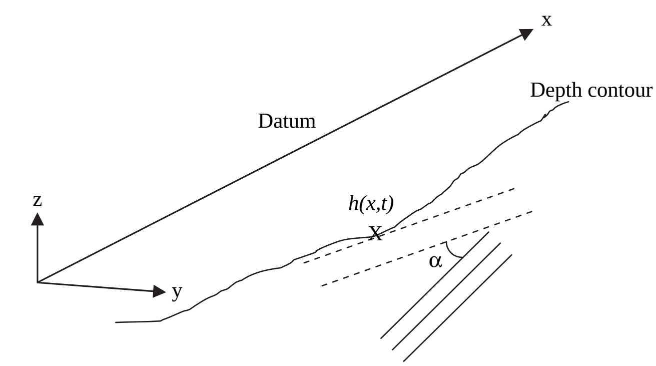

Fig. 1. Schematic diagram for Eq. (2).

Eqs. (3) and (1) or (2) provide the means of predicting the morphological changes in the long-axis (longitudinal) and cross-axis (transverse) directions, respectively. If we extend either of these in the natural way to two horizontal dimensions, a single form of equation is formed, of the form:

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_x(x)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y(y)\frac{\partial h}{\partial y}\right) + S'(x,y,t) \quad (4)$$

where $S'(x,y,t)$ is a new source function.

This equation is not derived from first principles, but is proposed in the context of 'behaviour oriented' model in a manner analogous to the justification of Eq. (3) and to a lesser extent of Eq. (1).

If $K_x(x)$ and $K_y(y)$ are written as the sum of a reference value and a spatially varying component (i.e. $K_x(x) = K_x' + k_x$; $K_y(y) = K_y' + k_y$) then, Eq. (4) can be written in the form of a diffusion equation with a constant diffusion coefficient and a source function:

$$\frac{\partial h}{\partial t} = K_x'\frac{\partial^2 h}{\partial x^2} + K_y'\frac{\partial^2 h}{\partial y^2} + G(x,y,t) \quad (5)$$

Sharp features of the bathymetry will be smoothed by a diffusion equation. A continual smoothing of sea bed features is not always what is observed in practice. Hence the additional source function G is required to represent an aggregate of the effects of all the processes other than those described by diffusion.

The source function G in Eq. (5) is given by:

$$G(x,y,t) = S'(x,y,t) + \frac{\partial}{\partial x}\left(k_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\frac{\partial h}{\partial y}\right) \quad (6)$$

K_x' and K_y' are the reference values of the diffusion coefficient in the long-shore and cross-shore directions, respectively. The additional terms involving the spatially varying components of the diffusion equation are incorporated in the source function G. Eq. (5) represents a drastic simplification of all the processes involved in shaping and transforming estuary morphology. However, it should be noted that the aim here is to analyse and predict changes in shape rather than detailed processes. It is thus anticipated that Eq. (5) provides a useful predictive model if the aggregated effects of the physical processes in the medium to long-term can be described as the linear combination of diffusive processes and a source function.

While the formulation of Eq. (5) may seem somewhat arbitrary, it is worth noting that a similar form of equation can be obtained by taking the continuity equation of sediment in two dimensions and setting the sediment transport rates in the cross-shore and long-shore directions proportional to the gradients of the bathymetry. This is physically reasonable in that the wave and tidal action is effective at smoothing or eroding high relief features. However, there are some features in the estuarine bathymetries which are maintained by waves and tides. The inclusion of a general source function provides a mechanism in the equation to maintain sea bed gradients without including the processes in the equation explicitly.

Huthnance (1981, 1982) formulated a similar model for the formation of offshore sand banks. His model was based on an assumption of enhanced down-slope transport derived from the consideration of the work of Bagnold (1956). This lead to an evolution equation for the time-averaged sea bed level in terms of its gradient and the residual currents:

$$\frac{\partial \bar{h}}{\partial t} = \left(\frac{C}{1-p}\right)\nabla\cdot\left[\mu\overline{|\underline{u}|^n\underline{u}} + \lambda\overline{|\underline{u}|^{n+1}}\nabla h\right] \quad (7)$$

The over-bars denote a time-average, $\underline{u}$ is the depth averaged flow velocity, p is porosity of sediment, μ and λ are coefficients and C is a scaling coefficient. The local change in bathymetry is governed by the residual tidal current and the Laplacian of the bathymetry (diffusive term). In this interpretation, the reference diffusion coefficient is a function of both x and y, and is given by the product of $(n+1)$th power of the magnitude of a characteristic residual velocity, λ, scaling coefficient S and the porosity p. This model has been later used subsequently by Hulscher et al. (1993) and Van Leeuwen (2002) to study offshore tidal sand banks and tidal inlet systems, respectively. One interpretation of the equation used in this paper (Eq. (4)) is an analogy of Eq. (7) with the first term of Eq. (7) replaced with a source function and the second term modified such that the residual velocities are replaced with actual velocities.

3. The inversion

The key concept behind the inverse method is that of using available data to determine unknown parameters or functions in the governing equation. The aim of the approach is to examine the historical trends of morphological evolution in estuaries and to find out the extent to which the source function is able to capture the trends associated with non-diffusive processes at longer time scales. If the source function reflects the long-term morphological changes due to non-diffusive processes, then it could be used in the evolution equation to predict changes in estuary morphology.

In the present study, a two-dimensional diffusion equation with a source function (Eq. (5)) is taken as the governing equation for long-term changes in the bottom topography of the estuary. The diffusion coefficients in x and y directions, K_x' and

K_y' are constants and $G(x,y,t)$ is a continuous bounded source function which is assumed vary slowly in time. The constants and the function are taken to be real valued. For convenience, it is also assumed that h and G have well defined spatial Fourier transforms at each time t, and that $G = Df$ for some function f, where D is the Laplacian operator. That is:

$$D(f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{8}$$

x and y are rescaled so that the coefficients of the spatial derivatives are equal and rescaled variables are given as:

$$\widehat{x} = \frac{x}{\sqrt{K_x}}$$
$$\widehat{y} = \frac{y}{\sqrt{K_y}}$$

and

$$\widehat{h}(\widehat{x}, \widehat{y}, t) = h(x, y, t)$$
$$\widehat{G}(\widehat{x}, \widehat{y}, t) = G(x, y, t)$$

For convenience, in what follows we drop $\widehat{}$, and the governing equation may then be written as:

$$\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + G(x, y, t) \tag{9}$$

or in operator notation:

$$h_t = Dh + G \tag{10}$$

Spivack and Reeve (2000) showed that if the time variation of G is weak enough to be neglected over one time step τ then the solution of Eq. (10) can be written as:

$$h(t + \tau) = \tau \exp\left(\frac{\tau D}{2}\right) G + h(\tau) \exp(\tau D) \tag{11}$$

The exponential terms are differential operators acting on the functions G and h. It has been assumed that, for simplicity, the values of $h(x_i, y_j, t)$ are given on a rectangular grid (x_i, y_j) at a series of time steps t_m where x_i and y_j are evenly spaced. The time step $\tau = t_{m+1} - t_m$. It is also assumed that the spatial resolution of h is sufficient to ensure that the Fourier transform of h is well represented by its Fast Fourier transform and that the time steps are sufficiently close for Eq. (11) to be valid.

The inverse of the diffusion term $\exp(\tau D/2)$ is given exactly by $\exp(-\tau D/2)$. Re-arranging Eq. (11) and multiplying by this term to bring the exponentials over to one side of the equation gives:

$$G(x, y) = \frac{1}{\tau}\left[\exp\left(-\frac{\tau D}{2}\right) h(t_m + \tau) - \exp\left(\frac{\tau D}{2}\right) h(t_m)\right] \tag{12}$$

Eq. (12) gives the source function in the two-dimensional diffusion equation given in Eq. (5). The exponential terms on the right hand side of Eq. (12) can be evaluated using the two-dimensional FFT of h where:

$$\exp\left(\frac{\tau D}{2}\right) h(x, y, t) = \Im^{-1}\left[\exp\left\{\tau\left(\frac{\upsilon^2 + \sigma^2}{2}\right)\right\} \Im\{h(x, y, t)\}\right] \tag{13}$$

where $\Im$ is the two-dimensional Fourier transform of h with respect to x and y, and $\Im^{-1}$ is the inverse Fourier transform. υ and σ are the corresponding transform variables. The other term may be evaluated in a similar manner. In Eq. (11) the effects of diffusion and the source function are combined and occur continuously over each time interval. It is approximated that the effects of G can be integrated and replaced by an instantaneous sediment transfer of strength τG. In this way, G becomes partially decoupled from D. The evolution of bottom bathymetry (h) over one time step can be represented by pure diffusion up to the next time step and then interaction with instantaneous sediment transfer due to G.

The derivation of the source function using Eq. (12) requires the selection of diffusion coefficients in the x and y directions, K_x and K_y. This is discussed further in Section 4.

4. Field site and bathymetry data

The data used to construct the inverse solution to derive the source function in Eq. (9) are a set of detailed bathymetric surveys covering the Humber estuary of UK. The gridded bathymetry data were provided by ABP Marine Environmental Research, UK. Fig. 2 shows a map of Humber estuary and its location in the UK. The estuary is located in the east coast of UK. The River Ouse and River Trent drain into the estuary and then into the North Sea. The estuary receives water from a catchment covering an area of 24,000 km^2 and is the largest freshwater supplier to the North Sea. It is a macro-tidal, "tide dominated" estuary subjected to semi-diurnal tides. The estuary is about 60-km long and its width varies from 13 km at the mouth to 1.5 km inland.

Morphological studies (Eurosion Case Study – Humber Estuary, UK, 2003) have suggested that the estuary can be divided into four main units as follows.

1. The outer estuary extending from Spurn head to a line across the estuary between Grimsby and Hawking Point.
2. The middle estuary extending from Grimsby to the Humber bridge.
3. Inner estuary extending from the Humber bridge to Trent Falls.
4. Rivers Trent and Ouse.

The outer estuary behaves as a coastal inlet or bay. More typical estuarine behaviour begins in the middle region of the estuary. Approximately one-third of the mud and sand flat are exposed at low water. The inter-tidal area is about 12,000 ha. Of this area, more than 90% is mud and sand flats and the remainder is largely salt marsh.

The main physical features of the estuary are:

- Spurn Head, which is connected to shore by a sand and shingle bank and fed by sediment moving south along the Holderness coastline;

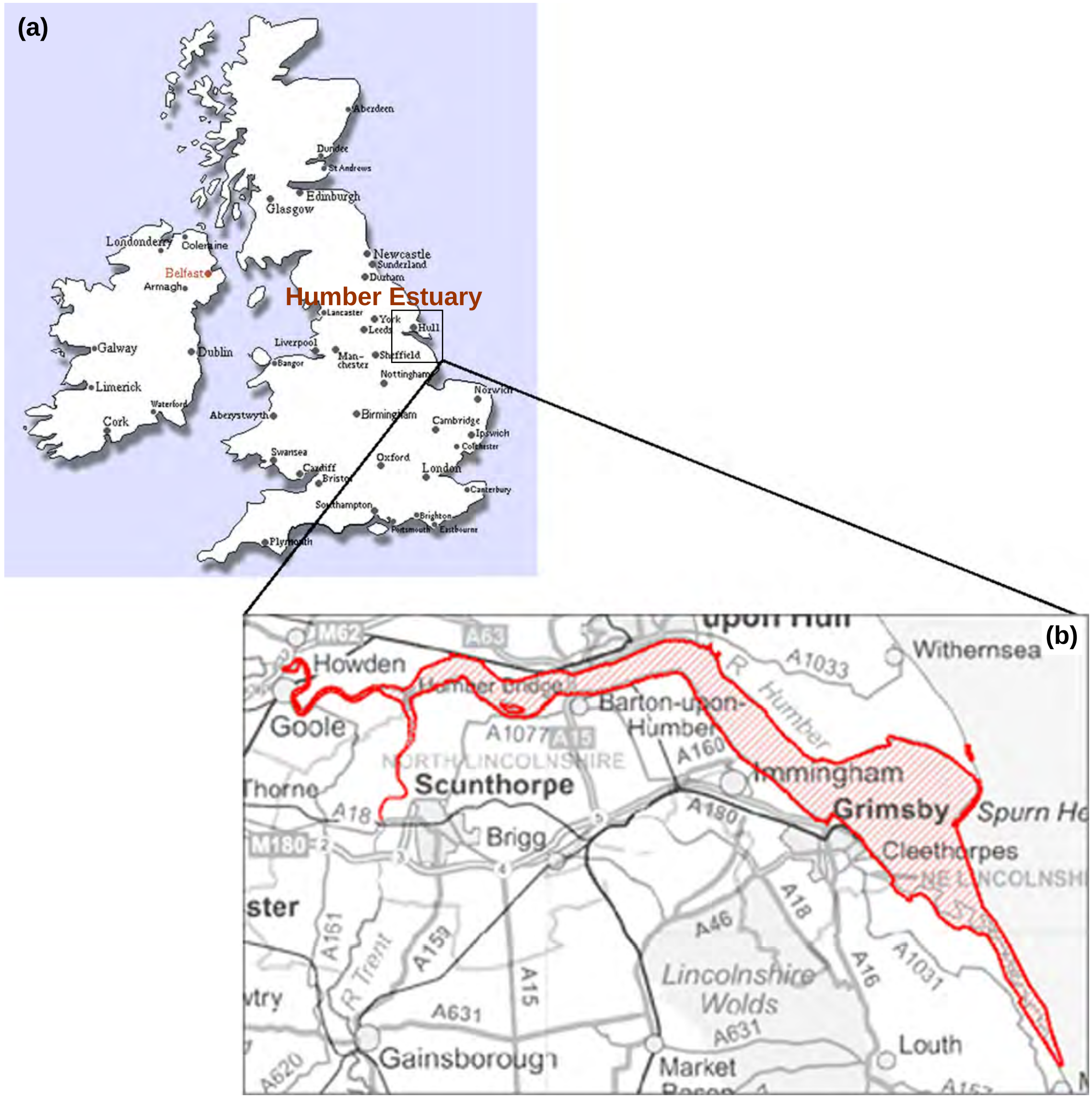

Fig. 2. (a) Map of United Kingdom. (b) Map of the Humber estuary.

- the narrows at Grimsby, which were formed by the development of Sunk Island (originally as a natural source and then by reclamation);
- the Wolds, which provide the foundation for the Humber Bridge and prevents the estuary from widening at this point; and
- Trent Falls, where the estuary divides in to the rivers Ouse and Trent.

Fig. 3 shows the main physical features of Humber estuary. The tidal range in the Humber estuary varies from 3 m at neap to 6 m at spring. The estuary has a tidal prism varying between 1 and 2.5×10^9 m^3 over a spring–neap cycle and the long-term averaged total river flow is 240 m^3. Waves up to 4 m can occur in the outer estuary but reduce to little more than 1 m upstream of Hull (Eurosion Case Study – Humber Estuary, UK, 2003).

The data used in the present research were obtained from bathymetric surveys of the Humber estuary between 1851 and 2000. The surveys covered the entire length and width of estuary from Spurn head to Trent Falls. The data were provided by ABP Marine Environmental Research Limited, UK. The entire data set contains 20 bathymetric surveys collected in: 1851, 1875, 1900, 1910, 1925, 1936, 1940, 1946, 1950, 1956, 1960, 1966, 1970, 1976, 1980, 1986, 1990, 1997, 1998 and 2000. Data from the maps were interpolated, (using kriging), onto a 15 m $\times$ 15 m square grid. The bathymetry data values refer to levels relative to Ordnance Datum Newlyn.

All data were first transformed to a common grid with the same origin. Then grid points outside the +2 m contour line were blanked and assigned zero value. Considering the shape of the estuary, all maps were divided in to two separate sections (grids A and B) and each section was subjected to coordinate transformation. The new Cartesian coordinate system and the grid size are selected such a way that the x and y coordinate directions represent cross-shore and long-shore transport directions of the estuary, respectively. The bathymetry data from the original grid were interpolated to the new grid

Fig. 3. Physical features of Humber estuary. (a) Sand and shingle bank at Spurn head. (b) Mud flats and main channel.

using a method based on area-coordinates. The grid spacing of the new grids in x and y directions was selected as 30 m. The bathymetry data were then smoothed using a weak low pass filter to eliminate high frequency noise.

Fig. 4 shows some historic bathymetry maps of the Humber estuary covering the entire outer and middle estuary with respect to the new coordinate system. It can be seen that the outer estuary has two main natural channels, Bull and Haile, and one regularly dredged Sunk Dredged Channel, separated by shoals and mud flats. The depths and extent of the shoals and the mud flats vary from time to time but the changes are relatively gradual. Over the course of years, the shallow shoal at Grimsby Middle (Middle Shoal) recedes in both depth and extent while accumulation can be seen in the elongated tidal flat dividing the two main channels. The tidal flat accumulates and extends towards the mouth of the estuary. The two channels merge in the middle estuary to form one main channel. But, there is a shallower secondary channel in the middle estuary. This secondary channel began to appear in 1936 and developed into a fully fledged channel in 1960 and then began to disappear in 1986. The Sunk dredged Channel which is 7-km long, 213-m wide and 8.8-m deep has been dredged in 1969 (Townend and Whitehead, 2003). The depth of the main channel is as deep as 20–25 m at certain locations but about 10–15 m deep on average. The main channel is surrounded by extensive shallow tidal flats in both outer and middle estuary.

It has been found that during the period of 85 years from 1851 to 1936, there was a general trend of accretion within the estuary. The main areas of accretion were Grimsby Middle/Middle Shoal, Foul Holme Sand and Read's Island. Following 1936, many areas appeared to have eroded (ABP MER, 2004).

4.1. Sediment diffusion coefficient

Selection of a suitable sediment diffusion coefficient for sediment in an estuarine environment is extremely difficult. In this study we have used a value of 10^6 m^2/yr for both longitudinal and transverse directions, which is within the bounds found by other investigators. For example, Masselink and Pattiarachchi (1998) found that large scale sediment diffusion coefficientsfor micro-tidal sandy beach in Australia is in the order of 10^5–10^6 m^2/yr. Baugh (2004) and Burgh and Manning (2007) used a horizontal diffusion coefficient of the order 10^7 m^2/yr for morphodynamic modelling of the Thames estuary, UK. The work of Huthnance (1982), Flather (1984) and White (1995), on offshore sand banks suggests a value in the order of 10^5 m^2/yr. The value of the horizontal diffusion coefficient of 10^6 m^2/yr is used in the present study is in the middle of the range of these values. The longitudinal and transverse diffusion coefficients are taken to be equal.

5. Results and discussion

This section of the paper presents results derived from the inversion procedure. First, bathymetry changes between each successive chart are shown. These results show the morphological changes of the estuary due to all external forcing. The reconstructed source functions, which describe the morphological response of the estuary to non-diffusive processes, are shown for each survey interval. Finally the results of the Empirical Orthogonal Functional analysis of the source function are presented.

Fig. 5 shows bathymetry changes of the outer and middle regions of the estuary between 1925/1936, 1960/1966 and 1998/2000. Evaluation of the entire set of bathymetry change maps shows a pattern of alternative erosion and accretion in the periphery of the outer and middle estuary. Prior to 1925, these areas are mostly accretive, but from 1925 to 1966 erosion and accretion have taken place in approximately 10-year cycles. After 1966, these areas show alternate accretion and erosion in the following 20 years. During the period from 1986 to 1998, changes in these peripheral areas were almost negligible. A small amount of accretion has taken place within the period between 1998 and 2000.

Accumulation in the main channels and erosion of shallow flats are eminent throughout apart from a few exceptions. This could well be due to dredging operations to maintain the navigation channel. However, morphological changes in the outer and middle estuary are comparatively small in the recent years

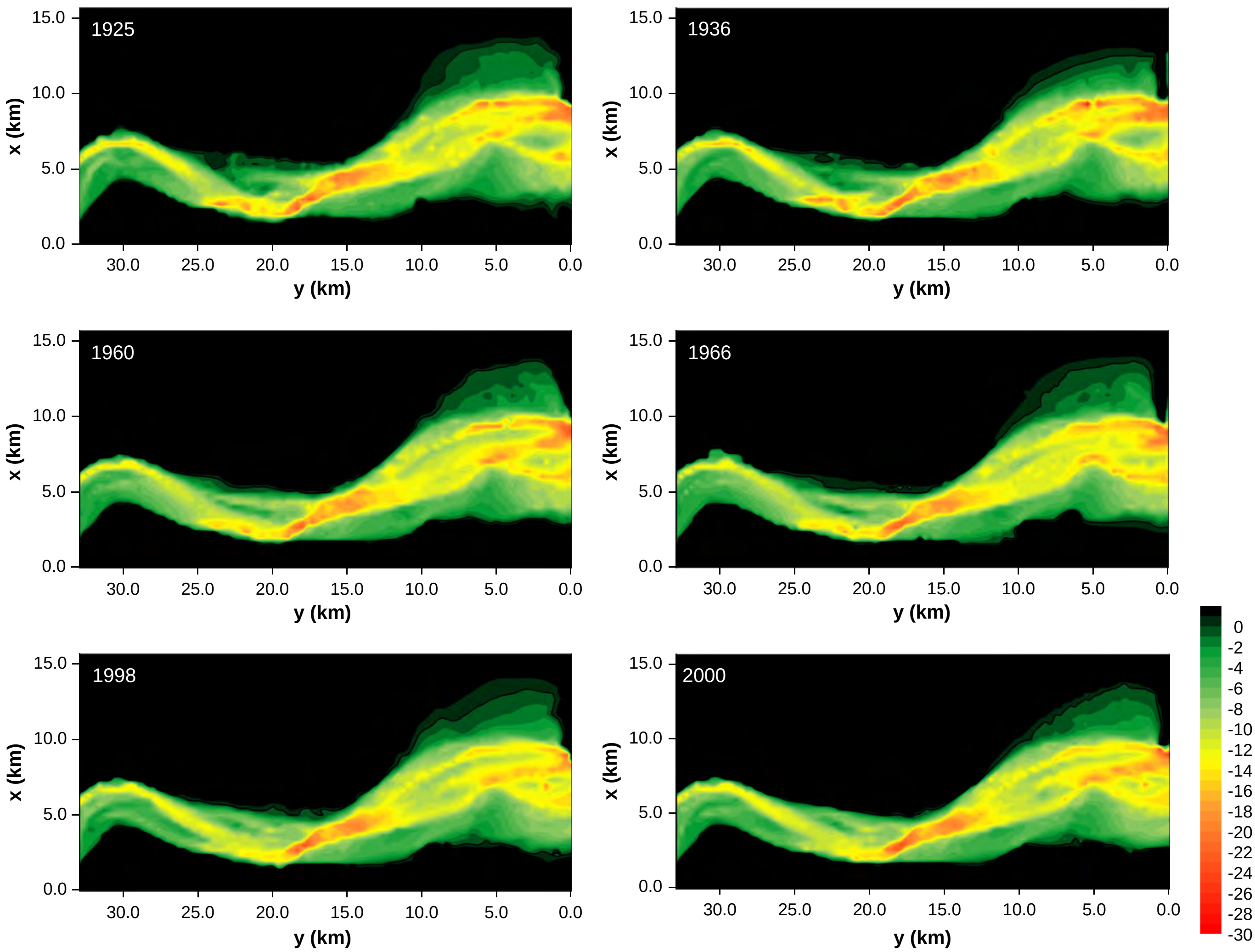

Fig. 4. Historical bathymetry of the Humber estuary, UK.

after 1986 except near the mouth of the estuary and at Hull. Apart from these changes, other localised changes in the estuary morphology show erosion/accretion behaviour with little apparent structure. It may also be seen that the estuary has been more morphologically active prior to 1960. Morphological changes show a more gradual and streamlined nature after 1960.

Inverse solutions were obtained using the method described in Section 3 for each pair of consecutive bathymetric surveys to construct the source function corresponding to each survey interval.

Fig. 6 show some selected reconstructed source functions. Several broad features are apparent in the source functions collectively. A significant structure is persistent throughout the entire set of results. Overall, there is no rapid variation of source function from one interval to the other. Large scale features such as tidal channels, tidal flats and linear banks in the estuary are persistently visible in the source function and also smaller scale structures are apparent than in bathymetric data itself. Other large scale elongated features, possibly mud banks are also visible in the middle estuary.

Large positive source functions in the tidal channels during the entire period of consideration indicates consistent accretion, faster than predicted by the process of large scale diffusion. In other words, tidal channels in the outer and middle estuary draw sediment from surrounding mud flats and external sources and are subjected to accretion. This is in-line with the findings of ABP MER (2004) where infilling of the estuary was observed during the last 150 years. Localised negative source functions in the south and north banks of the outer and the middle estuary indicate erosion or sediment removal from those areas either by wave and tidal forcing or by dredging. Localised alternate erosion and accretion in certain areas of mud flats in the outer estuary between main channel and north bank is indicated by the negative and positive source functions, respectively.

It may be observed that the source functions derived from the inversion of the governing equation have significant differences from the corresponding bathymetry changes. These differences demonstrate that the large scale sediment diffusive process plays a significant role in the long-term evolution of estuary morphology.

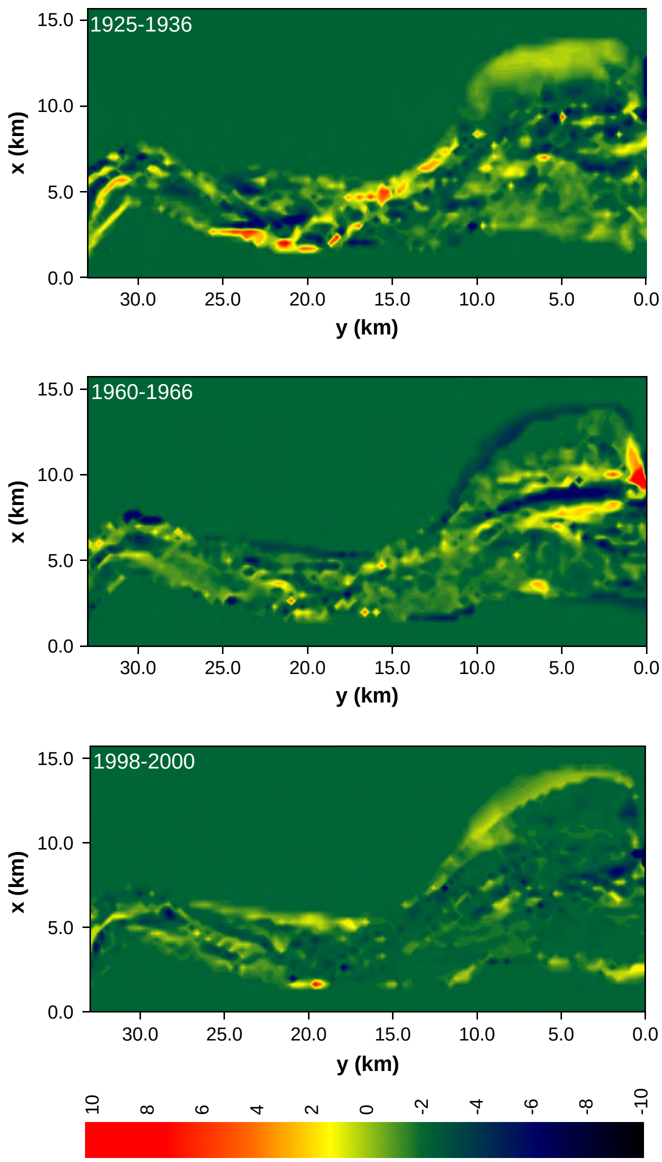

Fig. 5. Bathymetry change of Humber estuary, UK between 1925/1936, 1960/1966 and 1998/2000.

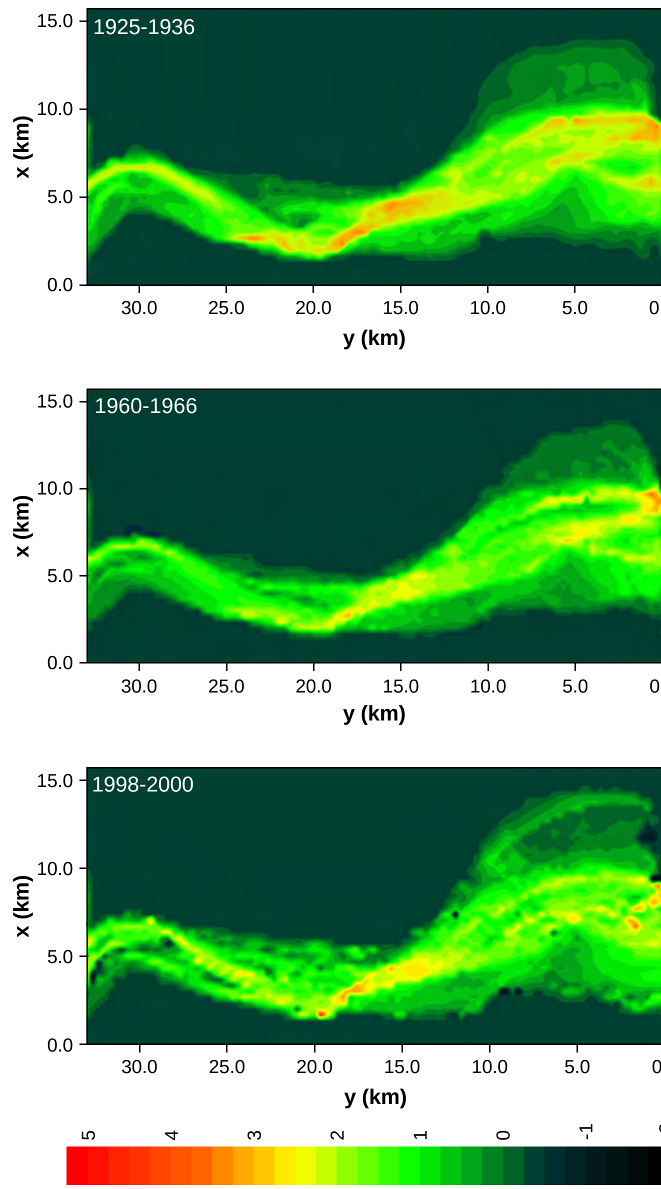

Fig. 6. Reconstructed source function.

The sensitivity of the source function to the selected diffusion coefficient was investigated by reconstructing the source function with ±50% of the selected value for several cases. It was found that there is no apparent difference to the structure of the source function when the diffusion coefficient is varied.

To be of practical use to engineers and planners, predictions of future morphological evolutions of the estuary are required. In principal, Eq. (5) can be used for prediction, although in this paper it has been used as the basis to derive an inverse problem. For prediction, it should be necessary to define the diffusion coefficients and the source function. If one is prepared to accept that past historical behaviour provides a useful basis on which to extrapolate forward, that is that the future will not include some change of forcing that has not occurred in the past. Then extrapolating the sequence of source functions into the future provides one means of estimating the source function in Eq. (5) when used in a predictive mode.

Before embarking on such an exercise it is worth investigating whether the sequence of source functions contains any spatial and in particular temporal structure that would be exploited. To this end, an Empirical Orthogonal Function (EOF) analysis was performed on the sequence of computed source functions. This technique is widely known and further details can be found in Reeve et al. (2001) and Horrilo-Caraballo et al. (2002).

Table 1 summarises the results of the EOF analysis for the first six spatial orthogonal eigenfunctions. It can be seen from these results that 92% of the mean square data are contained in the first eigenfunction. The first eigenfunction corresponds to the mean source function for the entire period considered. The first six functions collectively capture 97% of the mean square of the data. The mean square value of source function is the average of the square of all the source function values of the entire set of results.

Table 1
Eigenvalues and variance of the first six eigenfunctions

Eigenfunction	Normalised eigenvalue	Variance (%)
1	0.92301	–
2	0.01396	18.13693647
3	0.01294	16.81174484
4	0.00948	12.31648694
5	0.00824	10.70546966
6	0.00568	7.379498506

Table 2
Eigenvalues of the first six eigenfunctions for a diffusion coefficient 50% greater than the selected value

Eigenfunction	Normalised eigenvalue
1	0.9789
2	0.00817
3	0.00424
4	0.00149
5	0.00129
6	0.00107

Second and subsequent eigenfunctions represent the variation of source function about the mean. In this case, more than 65% of the data variance about the mean is captured by the 2nd through 6th eigenfunctions, which indicates the fact that the orthogonal eigenfunctions very efficiently describe the variability of the source function.

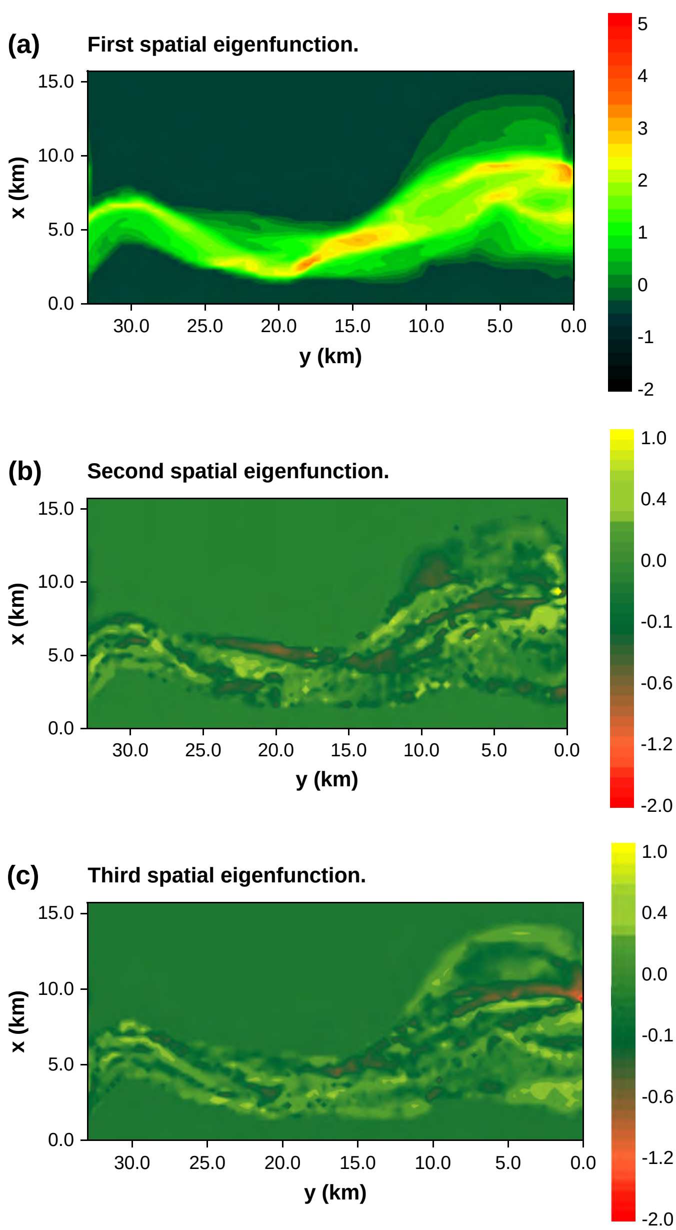

Fig. 7. Spatial Orthogonal eigenfunctions.

Fig. 7a–c show plots of 1st to 3rd spatial eigenfunctions. The first eigenfunction gives the temporal mean value of the source function. The second eigenfunction, which depicts the shape of the strongest variation in the source function, shows a strong spatial structure with areas of maxima and minima. Most of these spatial patterns are a few kilometres long and are elongated along the estuary. The 3rd eigenfunction shows spatial patterns which are of smaller scale to that of the 2nd function. The 4th, 5th and 6th eigenfunctions (not shown) have much less coherent spatial structure.

Table 2 shows eigenfunctions resulting from the source functions calculated with a diffusion coefficient 50% greater than the selected value and the values are very similar to the values given in Table 1. Temporal eigenfunctions describe the temporal variation of the source function. Fig. 8 shows first three temporal eigenfunctions. The first temporal eigenfunction is almost constant as expected because it corresponds to the temporal mean source function. The second eigenfunction, in general shows an upward trend during the period concerned, but show an oscillatory nature between 1960 and 1990. The oscillations may be attributed to the bathymetry changes associated with large scale dredging operations and other development work took place in the estuary from time to time between 1960 and 1994 (Townend and Whitehead, 2003). However, the survey frequency in general is not sufficient to a definite temporal signature of the second eigenfunction.

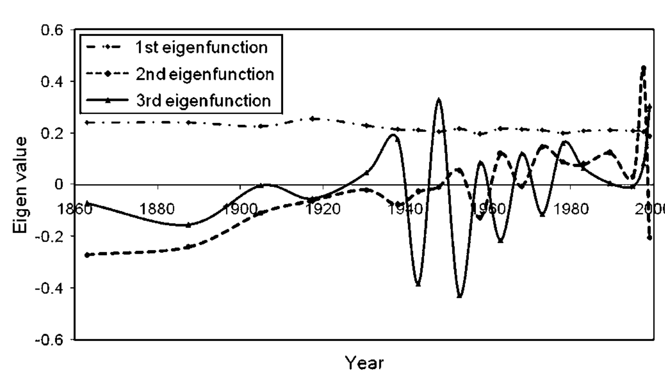

Fig. 8. Temporal eigenfunctions.

6. Conclusions

In this paper, long-term morphodynamic evolution of estuaries is discussed as an inverse problem. The present approach may be considered as an extended behaviour oriented approach for predicting morphodynamic evolution of estuaries. An equation that isolates diffusive and non-diffusive processes in estuaries is used as the governing equation describing long-term morphodynamic evolution of the estuaries. The effects of non-diffusive processes were incorporated in the governing equation through a source function. The source function then represents the aggregation of all non-diffusive phenomena which lead to long-term morphodynamic evolution of an estuary.

The success of the present approach largely depends on the availability and accuracy of the historic bathymetric survey data. A comprehensive data set covering a considerable time period is needed to provide quantitative results.

Inverse solutions for the source function were obtained using historic bathymetric data of the Humber estuary, UK for a period of 150 years. It was found that the source functions derived from the data are significantly different from the change in bathymetry during the corresponding time intervals, which indicates a significant contribution from diffusive processes to the morphodynamic evolution of the estuary. It was also found that there is no rapid variation of the source function from one survey interval to the other, which indicates a steady long-term morphodynamic evolution of the estuary against external (non-diffusive) forcing. Large scale features such as tidal channels, tidal flats and linear banks in the outer estuary are persistently visible in the source function.

The results of the Empirical Orthogonal Function analysis of the source function show a significant spatial and temporal structure of the source function during the period covered by the bathymetry surveys. Ninety two percentage of the mean square of the source function results are contained in the first eigenfunction which corresponds to the temporal mean.

In the light of these findings, the mean source function can be taken as a suitable representative value of the source function to be used in the governing equation considered in this study. Extrapolating the sequence of mean source functions into the future will provide a means of estimating the source function when the governing equation is used in a predictive mode.

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