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Beach profile evolution as an inverse problem

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A B S T R A C T

Beach evolution models are normally applied in a prognostic fashion, with parameters and boundary conditions estimated from previous experience or other forecasts. Here, we use observations of beach profiles to solve a beach profile evolution equation in an inverse manner to determine model parameters and source function. The data used to demonstrate the method are from Christchurch Bay in Dorset, UK. It was found that there is a significant contribution from diffusive processes to the morphodynamic evolution of the beach profiles and that the development and disappearance of near-shore coastal features such as upper beach berms and inter- and sub-tidal bars are well captured by the source function in the governing equation.

1. Introduction

Predicting morphological changes in coastal systems is a non-trivial task due to the complexity of the underlying physical processes involved and the sensitivity of the system behaviour to natural variability. Interaction between system components and dynamic forces behind its evolution spans a wide range of time scales. The uncertainty of deterministic predictions of dynamic forces beyond certain time scales and ambiguity of non-linear interactions between the system and dynamic forces makes medium to long-term morphodynamic predictions of coastal systems extremely difficult.

Morphodynamic predictions of coastal systems are based on two modelling approaches (De Vriend et al, 1993; De Vriend, 2003). The first approach is the use of process models based on two or three dimensional hydrodynamic models combined with sediment transport and morphodynamic modules (van Rijn et al., 2003; Roelvink et al., 2001). These models are a valuable tool for assessing local, short-term morphodynamic changes in a beach, but have inherent limitations due to the lack of knowledge of sediment transport processes and their linkage to hydrodynamics. Uncertainties in the predictions are amplified by treating sediment with a range of grain sizes. Further, numerical predictions can exhibit great sensitivity to the initial conditions. This is due not just to the accumulation of numerical rounding errors in the computations required to solve the equations but also due to nonlinearity of many coastal systems that may induce chaotic behaviour. The second group of models have been termed 'behaviour-oriented models'. These models are designed to overcome the difficulties arising out of application of process based modelling (Cowell et al., 1992, 1994; Dean, 1991; Stive and de Vriend, 1995; Reeve and Fleming, 1997). The aim of behaviour-oriented models is to reproduce the qualitative behaviour of beach morphology using a simplified governing equation, parameterising the key processes. The governing equations are rarely derived from first principles; rather, they are defined along the lines of physical arguments. This and the parameterisation of processes are both the strength and potential weakness of such methods.

Diffusion type formulations have been used in the past to model long-term coastal and estuarine morphodynamic behaviour. It is important to note that this type of equations that have been applied to coastal morphology have not derived rigorously from basic process equations but are selected because their solutions qualitatively exhibit the behaviour of the application (Pelnard-Considere, 1956; Reeve and Spivack, 1994; Stive and De Vriend, 1995; Reeve and Spivack, 2000; Hansen et al., 2003; Karunarathna et al., 2008). The success of these models depends on the identification of fundamental parameters as the space and time varying coefficients of a simplified dynamic equation. In the application of a diffusion type model to beach profile change, collective changes to beach profile morphology including development, disappearance and evolution of near-shore morphological features and flattening and steepening of the profile, which are driven by external forces are all included in a source function, which is reproduced based on field evidence.

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In this paper we present a technique for the determination and recovery of the diffusion coefficient and an unknown source function in an advection-diffusion type governing equation for long-term beach profile evolution. The diffusion coefficient is derived as a problem of error minimisation and the source function is recovered as the solution of an inverse problem, using measurements of historic cross-shore beach profiles.

In Section 2 of the paper, the governing equation of the model and the methodology used to derive the diffusion coefficient and source function are presented and explained. The field site and the historic data used to demonstrate the methodology are presented in Section 3. Results are presented and discussed in Section 4, and the paper finishes with conclusions in Section 5.

2. Formulation of the model

This section of the paper describes the simplified beach profile evolution model and the method of recovery of the diffusion coefficient and the source function.

Following the approach suggested by Stive and de Vriend (1995) we take the governing equation for the evolution of beach profiles, relative to a fixed reference level, as a form of advection-diffusion equation:

$$\frac{\partial h(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K(x)\frac{\partial h(x,t)}{\partial x}\right) + S(x,t) \quad (1)$$

where $h(x,t)$ is the cross-shore beach profile depth measured relative to a fixed reference line, x is the cross-shore position, $K(x,t)$ and $S(x,t)$ are space and time dependent diffusion coefficient and an unknown external source function respectively. Fig. 1 shows the schematics of the model.

There are two unknowns to be resolved in the governing equation, Eq. (1): the diffusion coefficient $K(x,t)$ and the time and space-varying source function $S(x,t)$. Once these unknowns are found (up to a fixed time t_1 say), the governing equation can be used to predict future evolution of beach profiles. Finding suitable values for the diffusion coefficient and the source function is the key element to the success of the model.

The spatial variation of the diffusion coefficient allows us to represent the variation of morphological time scale with cross-shore position. All information about the typical site climate, sediment characteristics and short-term dynamics are assumed to be summarised in $K(x,t)$. All other natural inputs from climate change and human induced inputs are included in the source function $S(x,t)$.

Next we perform a 'Reynold's expansion', writing the profile depth $h(x,t)$, the diffusion coefficient K and the source function S as the sum of their time averaged values and a time varying component as follows:

$$h(x,t) = \overline{h}(x) + h'(x,t) \quad (2)$$

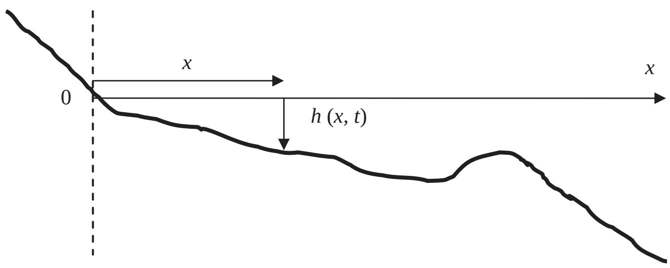

Fig. 1. Schematics of profile evolution model.

$$K(x,t) = \overline{K}(x) + K'(x,t) \quad (3)$$

$$S(x,t) = \overline{S}(x) + S'(x,t) \quad (4)$$

where an over-bar denotes the time averaged components and a prime denotes the time varying residuals.

Then, Eq. (1) can be re-written as

$$\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial t} = \frac{\partial}{\partial x}\left([\overline{K}(x)+K'(x,t)]\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial x}\right) + \overline{S}(x)+S'(x,t) \quad (5)$$

or

$$\frac{\partial[\overline{h}(x)+h'(x,t)]}{\partial t} = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial h(x,t)}{\partial x}\right) + \overline{G}(x) + G'(x,t) \quad (6)$$

where we have written

$$G(x,t) = \frac{\partial}{\partial x}\left(K(x,t)\frac{\partial[h(x,t)]}{\partial x}\right) + S(x,t) \quad (7)$$

We assume that the time average is taken over a sufficiently long period that for any variable x, $\bar{x}'=0$, $\partial\bar{x}/\partial t \approx 0$ and that to a first approximation $\bar{S} \approx 0$.

Then,

$$\overline{G}(x) = \overline{\frac{\partial}{\partial x}\left(K(x,t)\frac{\partial[h(x,t)]}{\partial x}\right)} \quad (8)$$

Taking the time average of Eq. (6) gives:

$$0 = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial\overline{h}(x,t)}{\partial x}\right) + \overline{G}(x,t) \quad (9)$$

In an analogy to the Reynolds' stresses of turbulent fluid flow, $\bar{G}(x)$ may be considered to be a turbulent morphodynamic stress. As a first order approximation we take these stresses to be zero.

Eq. (9) is then solved for time averaged component of the beach profile,

$$\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial\overline{h}(x)}{\partial x}\right) = 0 \quad (10)$$

The solution of which gives

$$\overline{K}(x) = \frac{\alpha}{(\partial\overline{h}(x)/\partial x)} \quad (11)$$

where α is a constant of integration. $(\partial\bar{h}(x)/\partial x)$ is the gradient of the mean cross-shore beach profile, which may be calculated from the measurements of the beach profiles, and must not be equal to zero anywhere in the range of x considered.

The physical interpretation of Eq. (11) is quite straightforward. To maintain a steep beach the mean diffusion coefficient must be small. Conversely, a large value of the diffusion coefficient corresponds to a gently sloping beach. This accords with the observation that gravel beaches are generally steep (with material that has a relatively slow rate of movement) while fine sand beaches, composed of highly mobile sediment, adopt a gentler incline.

2.1. Determination of time averaged diffusion coefficient

In any application it is understood that a time history of beach profile measurements is available. From these, it will be possible to estimate the mean beach profile and hence it's gradient. This can be used in Eq. (11) as a known quantity. However, there are two unknowns: $\bar{K}(x)$ and α. To solve Eq. (11) for $\bar{K}(x)$ a value for α must be specified. Rather than select a value we adopt a procedure similar to that used by Reeve & Fleming (1997). Let x_i $(i=1, 2, \ldots, N)$ denote the x-coordinates for which the average beach profile $\bar{h}(x)$

is known and let the times for which beach profile depths are known be denoted by t_j (j=1, 2, …, M).

A sequence of discrete values of α, α_k (k=1, 2, …, L) are defined and a corresponding sequence of solutions for $\bar{K}(x)$, $\bar{K}(x)_k$ (k=1, 2, …, L), were calculated. Then for each time interval t_j–t_{j+1} with $1<j<M$, predictions of the beach profile for t_{j+1} were made using $\bar{K}(x)_k$ in place of K in Eq. (1) without the source function as follows:

$$h(x,t+\tau)_C = h(x,t) + \frac{\tau}{\Delta x}\left[\overline{K}(x)_{i+1}\left(\frac{\partial h(x,t)}{\partial x}\right)_{i+1} - \overline{K}(x)_{i-1}\left(\frac{\partial h(x,t)}{\partial x}\right)_{i-1}\right] \quad (12)$$

The discrepancy between predictions and known measured beach profiles for each time interval ε_j is calculated as the root mean square difference between the predicted and measured beach profile depth:

$$\varepsilon_j = \sqrt{\frac{\sum_{i=1}^{N}[h(x_i,t_{j+1})_M - h(x_i,t_{j+1})_C]^2}{N}}. \quad (13)$$

where $h(x_i,t_{j+1})_M$ and $h(x_i,t_{j+1})_C$ are the measured and predicted values of profile depth at the next time step, respectively.

The discrete values of ε_j can be considered to define ε_j as a continuous function of α. The value of α which corresponds to the minimum of this function provides the best fit value of $\bar{K}(x)$. Physically, this process corresponds to selecting the mean diffusion coefficient so that it explains as much of the observed change as possible. However, any morphological change not consistent with Eq. (12) will appear as part of the source function.

2.2. Recovery of the source function

The time averaged diffusion coefficient $\bar{K}(x)$ is then used in Eq. (7) to derive the source function $G(x,t)$ as an inverse problem as follows.

Re-writing Eq. (6) in operator notation gives

$$h_t = Dh + G \quad (14)$$

where the operator $D=\partial/\partial x(\bar{K}(x)(\partial h/\partial x))$.

Spivack and Reeve (1999, 2000) showed that if the time variation of $G(x,t)$ is weak enough to be neglected over one time step τ then the formal solution of Eq. (14) can be written as

$$h(x_i,t_{j+1}) \cong (\exp(D\tau)-1)D^{-1}G + \exp(D\tau)h(x_i,t_j) \quad (15)$$

where $\tau=t_{j+1}-t_j$.

The exponential terms are differential operators acting on the functions $G(x,t)$ and $h(x,t)$. It has been assumed that, for simplicity, the values of $h(x,t)$ are given at uniform intervals at a series of time steps t_j where x_i is evenly spaced.

Using first order approximation of exponential terms, an expression for the source function $G(x,t)$ is found as

$$G(x,t) = \frac{1}{\tau}[h(t+\tau) - \exp(D\tau)h(t)] \quad (16)$$

Eq. (16) gives an explicit expression for the unknown source function. Given the data for the function $h(x,t)$ and time-mean diffusion coefficient, source function G can be recovered from Eq. (16).

3. Field site and beach profile data

The data used to derive the diffusion coefficient and source function are a set of historic cross-shore beach profiles at Christchurch Bay, Dorset, UK. The profile data was provided by the Channel Coastal Observatory, UK. Fig. 2 shows a map of Christchurch Bay, its location in the UK and a view of the beach.

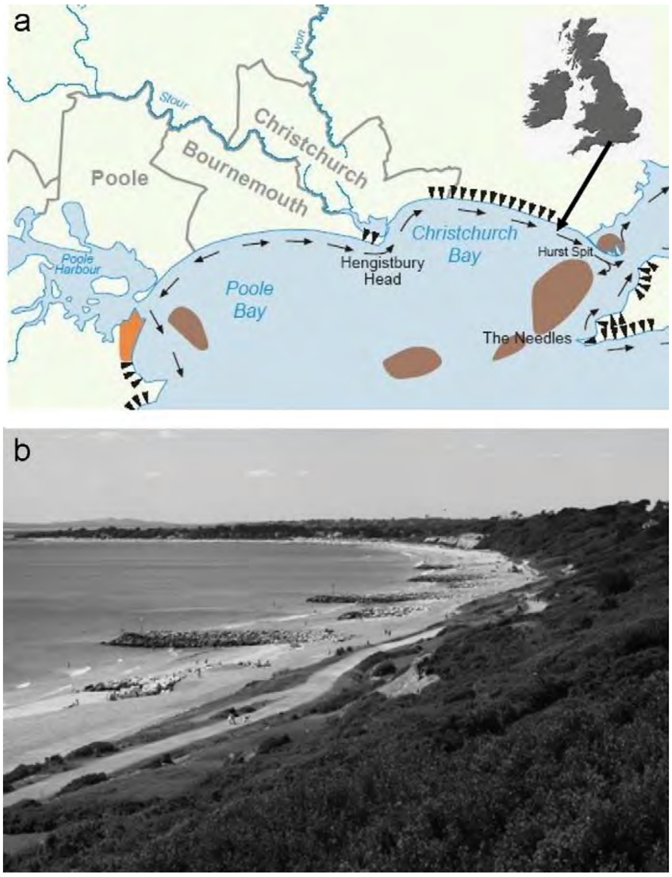

Fig. 2. (a) Christchurch Bay, its location in the United Kingdom; (b) A view of the study area (Few et al., 2007).

The bay, located in the south coast of the UK, is about 4 km long and bounded by Hurst Spit to the east and Hengistbury Head to the west. The beach at Christchurch Bay is a mixed sand–gravel beach with multiple shore parallel bars and is rapidly evolving. The upper and middle beach is mostly shingle while lower inter-tidal and sub-tidal areas including sub-tidal bars are mostly sand. A part of the beach is backed by soft, eroding cliffs and a part is protected by coastal defence structures including seawalls and shore normal groynes.

Cross-shore beach profiles at Christchurch Bay have been surveyed since 1987 at numerous transects along the entire coastline. However, survey interval and frequency are not uniform and also all transects have not been covered at each survey. Considering the availability of survey data, 8 beach transects (5f00070, 5f00076, 5f00091, 5f00099, 5f00107, 5f00121, 5f00135, 5f00140), which had the highest number of surveys data, were selected for the present study. Each selected transect contains at least 38 profile surveys during the period from 1987 to 2005. Fig. 3 shows locations of the selected beach profile transect.

For the demonstration of methodology adopted to determine the diffusion coefficient and recover the source function, measured cross-shore profiles at transect 5f00070 is used. There were 47 surveys available at 5f00070. All measured profiles available at 5f00070 are shown in Fig. 4. It was observed that the overall beach level at this location varied between 1 and 2 m. The still water shoreline moved 10–15 m shoreward or seaward accordingly. An upper beach berm seems to appear and disappear from time to time. A shallow trough and an alongshore bar was observed in some surveys. The sub-tidal area of the beach is much gentler than the steep upper beach face and the inter-tidal beach.

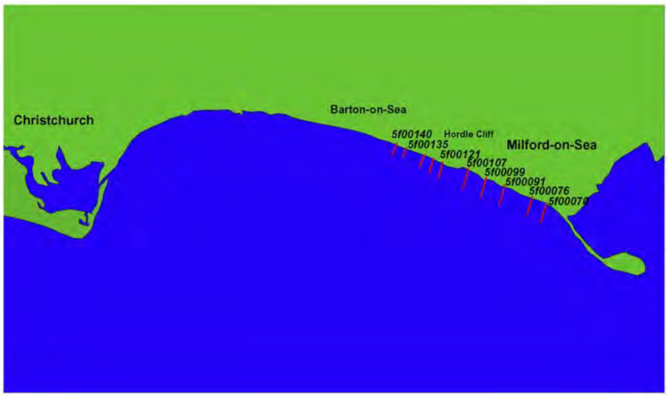

Fig. 3. Locations of beach transects where profiles were surveyed.

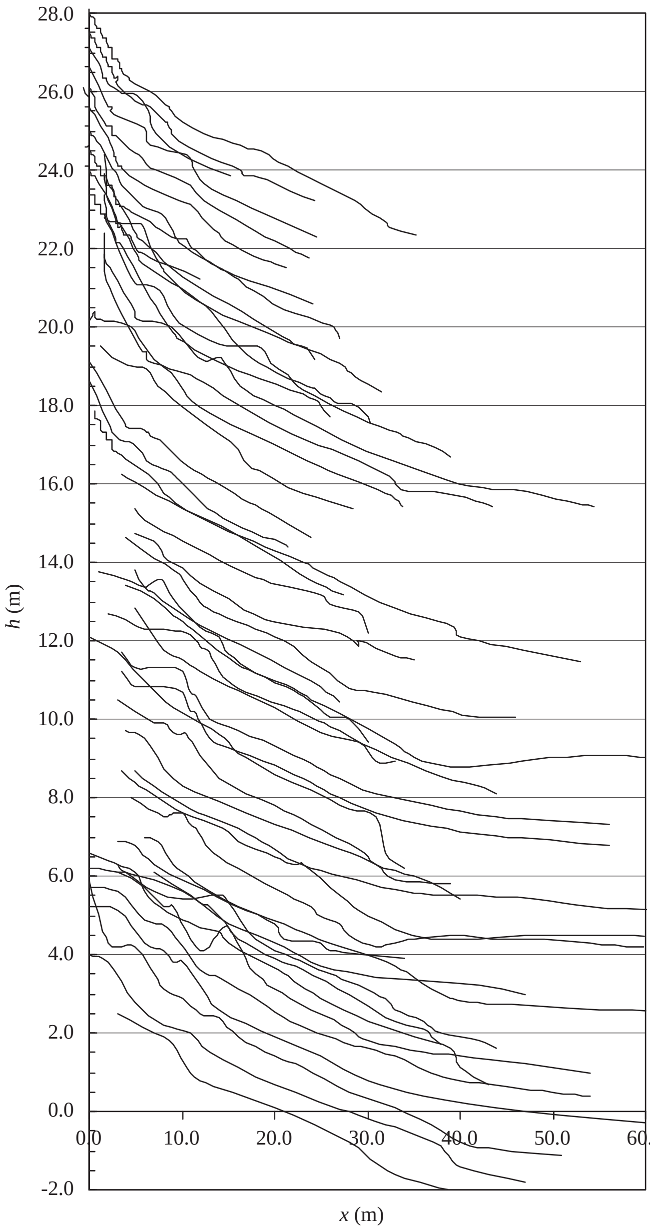

Fig. 4. Cross-shore beach profile surveys at transect 5f00070 from 1987 to 2006.

Some of the beach profile surveys cover the entire profile from the upper beach to the sub-tidal terrace while some surveys are incomplete and cover only the upper and inter-tidal beach. Also, survey intervals were not uniform and profile depths have not been surveyed at uniform spatial intervals. To maximise the number of usable survey data, the beach profiles were first curtailed at 30 m chainage from the cliff foot. Then the time-mean profile was calculated using all measured profile data. The time-mean profile is fairly uniform with a slight reduction in gradient towards the sub-tidal beach. The average profile gradient is around 1:8. The profile data were then interpolated on to uniform spatial intervals (0.5 m) along the profiles and temporal intervals (90 days) using Akima interpolation routine. This is a continuously differentiable sub-spline interpolation built from piece-wise third order polynomials. This was necessary as the method used to recover the source function demands uniform spatial and temporal interval of beach profile measurements.

4. Results

This section of the paper presents and discusses results obtained for the diffusion coefficient and the source function using measured beach profile depths at transect 5f00070 described in Section 3.

Based on sediment diffusion coefficients for different sediment sizes reported in literature (Burgh and Manning, 2007; Huthnance, 1982a, b) first, we estimated an initial value of 10 m^2/yr (0.027 m^2/day) for the mean diffusion coefficient $\bar{K}$. The (spatial) mean of the beach profile gradient ($\partial\bar{h}(x)/\partial x$) was determined from the cross-shore profiles measured at transect 5f00070 as 0.11 leading to an initial estimate of $\alpha \approx 3\times10^{-3}$ m^2/day. Based on this initial estimate for α, ε_j were calculated, following the method described in Section 2, using the measured values of $h(x,t)$ at transect 5f00070, for a range of α values varying from 10^{-4} to 10^{-2} m^2/day. Values of ε_j varied between 0.05 and 0.08; the minimum ε_j was found at $\alpha = 7\times10^{-4}$.

Following the method described in Section 2, the mean diffusion coefficient $\bar{K}$ across the transect 5f00070 was then calculated using the α value given above. Fig. 5 shows cross-shore variation of $\bar{K}$. Mean cross-shore beach profile is also shown in the figure. It can be seen in Fig. 5 that the mean diffusion coefficient gradually increases in the offshore direction. This is as might be expected as the beach sediment varies from coarse shingle in the upper and middle beach to sand in the lower sub-tidal areas of the profile.

The spatially varying mean diffusion coefficient is then used in recovering the source function in the advection-diffusion profile evolution equation. The source function was computed using

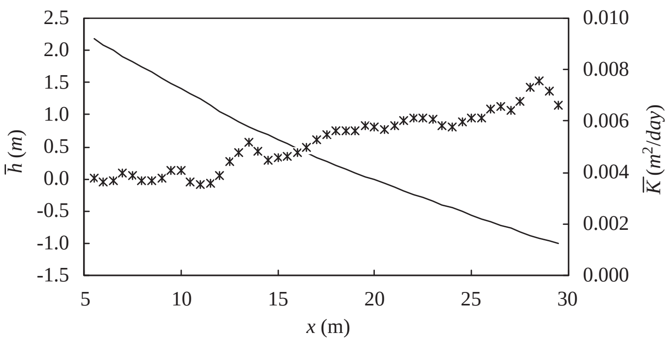

Fig. 5. Mean diffusion coefficient $\bar{K}$ and mean cross-shore beach profile at transect 5f00070.

Eq. (16) as a solution of an inverse problem using measured profile survey data described in Section 3.

Fig. 6 shows few selected source functions and beach profiles used in recovering them. Several broad features are apparent in the source functions. A significant structure is persistent throughout the entire set of results. Evolution of profile features such as upper shore face berms and bars are persistently visible in the source function. Positive source functions indicate net accretion of the profile over the period, faster than predicted by the process of large scale diffusion. Negative source functions indicate net profile erosion. Furthermore, all the source functions tend to zero in the offshore direction, indicating a gradual convergence to a depth of closure, or point at which zero net crosshore transport occurs. It is also apparent from Fig. 7 that the source functions derived from a particular survey interval have significant differences from the corresponding beach profile change during that interval. The source functions are not simply a volume difference between two successive surveys. They represent the changes that have occurred relative to the (time) mean beach profile. The cross-shore shape of the source functions exhibits variation with time, reflecting the seasonal variation in wave energy reaching the beach.

Figs. 7(a) and (b) show composite plots of the entire set of source functions $[G(x,t)]$ recovered from summer and winter beach profiles respectively. During the summer months, the source functions are mainly positive at the upper and middle beaches, correlating well with the observed accretion of the beach profile in this part of the beach. There is a significant spatial variation of source function cross the beach profile at a given time interval. Largest values were observed at the upper and middle beach showing considerable changes between surveys while

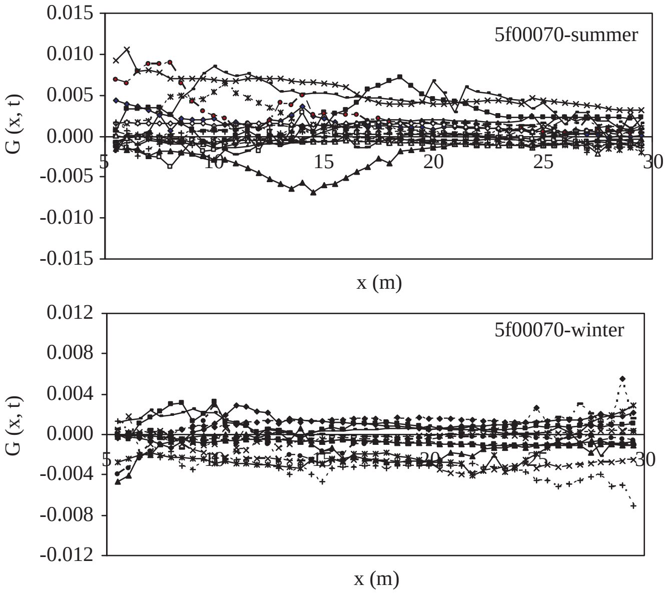

Fig. 7. Source functions recovered from summer and winter beach profiles.

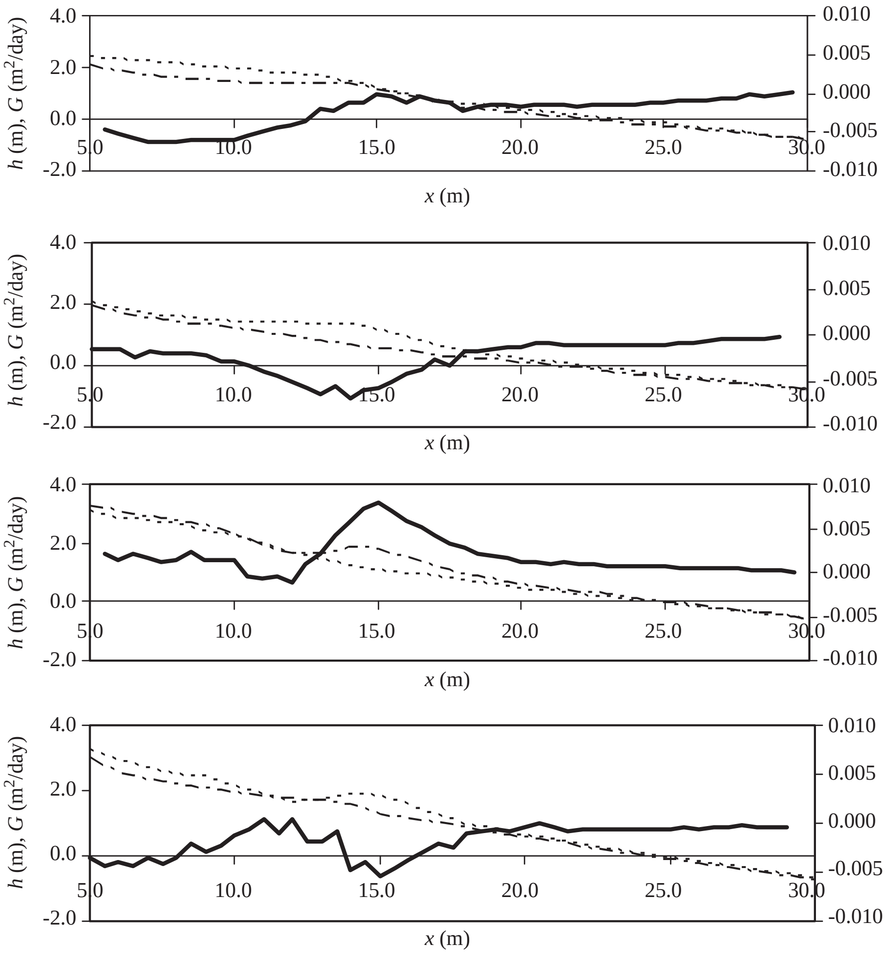

Fig. 6. Source functions and corresponding beach profiles.

smaller values were observed over the lower beach. In contrast, during the winter months the source function varies only slightly along the profile and it is mostly negative, indicating beach erosion. The largest values were observed at the lower middle part of the beach.

5. Conclusions

In this paper, a methodology for the recovery of the diffusion coefficient and the source function in an advection-diffusion type beach profile evolution model is described. This type of model is considered to be an extended behaviour-oriented approach for predicting long-term beach profile evolution. The governing equation isolates non-diffusive processes which drive sediment movement leading to profile evolution from the diffusive processes through an unknown source function. The source function then represents the aggregation of all non-diffusive phenomena which lead to morphodynamic evolution of the beach profile. The modelling approach determines a spatially varying diffusion coefficient and a source function using measured historic data on beach profile evolution.

The success of recovering a suitable diffusion coefficient and a source function using the present approach largely depends on the availability and accuracy of the measured beach profile survey data. A good data set covering a considerable time period is needed to provide quantitative results. The implementation of the method is relatively straightforward and is very efficient computationally.

The methodology has been demonstrated by obtaining solutions for the time-mean, space-varying diffusion coefficient and time- and space-varying source function using historic data of beach profile evolution at the Christchurch Bay, Dorset, UK. It was found that there is a significant variation in the shape of the source function from winter to summer periods, which corresponds to the formation and disappearance of near-shore coastal features such as upper beach berms and inter- and sub-tidal bars.

As a final consideration we mention that the method described here may, possibly, form the basis of a method to forecast the beach shape. Such a method would rely on extrapolation of statistical analysis, and rely on past historical behaviour being a reliable indicator of future behaviour. The raw measurements could be extrapolated directly, of course. However, the level of noise makes this an uncertain procedure at best. Extrapolations of Fourier analysis, Empirical Orthogonal Function analysis, wavelet decomposition and so on succeed or fail through being able to identify strong underlying patterns of behaviour such as trends and cycles. In a similar manner, identifying the diffusion coefficient and source functions in a simple beach model may provide some predictive capacity through extrapolation; and is the subject of ongoing research by the authors.

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