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Analysis of key parameters in a diffusion type beach profile evolution model

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ABSTRACT

Diffusion type formulations are commonly used in beach profile evolution models. The practical idea behind that is to map the behaviour of the beach profile onto a simple mathematical model that exhibits the same behaviour under defined operating conditions. The success of this approach is based on the accurate determination of key parameters in the diffusion model that govern its behaviour, using observed beach behaviour in the field. In order to determine these parameters, i.e. diffusion coefficient and a time and space varying source function, we used observations of historic beach profiles at Milford-on-Sea beach in Christchurch Bay, Dorset, United Kingdom. The relationship between the diffusion coefficient and Dean's equilibrium profile was investigated, leading to a new interpretation of the diffusion coefficient in terms of the sediment characteristics. The analysis also shows the significance of the diffusion process in the medium to long term evolution of the beach profile. A canonical correlation analysis (CCA) was undertaken in order to identify patterns of behaviour between wave conditions and source terms, and the possible correlations between them. The analysis provides strong evidence of a useful link between the source term in the simple dynamical equation and the distribution of wave steepness.

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1. Introduction

Morphodynamic processes in the medium- to long-term time scale entail large scale coastal topographical changes that are important for long term planning and management of the coastal zone. Therefore, understanding large scale beach behaviour and the ability to predict beach changes over timescales of engineering significance is the key to successful coastal management. Predicting beach changes over these time and space scales is a non-trivial task due to the existing gaps in our knowledge of process dynamics and interactions. To overcome this practical difficulty, behaviour-oriented modelling approaches have been proposed during the past decade as a means of mapping the qualitative behaviour of beach changes (Hanson et al., 2003; Cowell et al., 1992,1994).

Diffusion type formulations are widely used in behaviour-oriented models of beach changes (e.g. Pelnard-Considere, 1956; Stive and De Vriend, 1995; Reeve and Fleming, 1997; Hanson et al., 2003; Karunarathna et al., 2008, 2009). In the diffusion formulations applied to cross shore profile changes, the profile depth is described as a function of cross-shore position, with appropriate initial and boundary conditions. This type of formulation is used to reproduce beach profile morphology on the basis that the solutions map the behaviour of the beach profile in a qualitative manner. Diffusion has the effect of smoothing irregularities in the profile. However, smoothing is not the only morphological response of a beach profile. Therefore, other morphological changes to the beach profile such as steepening of the profile and evolution of near-shore bars are included as a source function in the equation. This is an aggregation of changes driven by physical processes.

In a typical diffusion-type model, the variable diffusion coefficient is assumed to hold all the information related to the site climate and sediment characteristics. The source function is considered as representing all other inputs related to climate drivers, human intervention and incidental events.

Determination of the appropriate diffusion coefficient and the source function is the key to the success of this type of model. In order to derive these quantities, field measurements or simulated results from process-based models can be used (Hanson et al., 2003). Using an inverse technique, Karunarathna et al. (2008) derived the source function in a 2D diffusion type model that applied to estuary morphology, from a set of historic bathymetry charts of the Humber Estuary in the UK. In a subsequent paper, Karunarathna et al. (2009) developed a methodology to derive the diffusion coefficient as a space varying function and the source function as a space and time varying function of a 1D diffusion model for beach profile evolution. Avdeev et al. (2009) reproduced beach profile variation in Duck, North Carolina (USA) beach and

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Delfland beach in Holland, using reconstructed parameters in a diffusion model.

The aim of this paper is to investigate the nature of the diffusion coefficient and the source function, which are the key parameters in a diffusion type beach profile evolution model. In particular, we investigate whether these quantities, that may be construed as mathematical artifices, can be related to physical processes. In Section 2 of the paper, a brief description of the model and the numerical techniques used to derive unknown model parameters are presented. A description of the field site from which historic measurements are taken for deriving model parameters and an analysis of the measured profiles are given in Section 3. In Section 4, the reconstructed diffusion coefficient and the source function are analysed and the results are presented and discussed. Section 5 concludes the paper.

2. Diffusion type beach profile model and recovery of model parameters

The model equation we use to describe beach profile evolution is a form of 1D diffusion formulation:

$$\frac{\partial h(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(K(x,t)\frac{\partial h(x,t)}{\partial x}\right)+S(x,t) \tag{1}$$

Eq. (1) is a continuous extension of the discrete 'n-line model' (Hanson et al., 2003), which describes time and space variation of profile depth $h(x,t)$ at a cross shore location x, where x is measured offshore from the mean water shoreline (MWL). $K(x,t)$ and $S(x,t)$ are the unknown space- and time-dependent diffusion coefficient and source function, respectively, which are the key parameters that determine beach profile evolution in time. Cross-shore variation of the diffusion coefficient is expected to represent the typical site climate and sediment characteristics. Natural and anthropogenic changes to morphodynamic forcing are assumed to be embedded in the source function.

Following the theoretical treatment of Karunarathna et al. (2009), both the diffusion coefficient and the source function are taken as a sum of time varying and time averaged components as in a Reynolds expansion. Time averaging is considered to be in several years to decadal (medium-term) time scale. Changes in beach profile due to individual storms are thus considered 'turbulence'. Eq. (1) can be rewritten to give

$$\frac{\partial h(x,t)}{\partial t}=\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial h(x,t)}{\partial x}\right)+G(x,t) \tag{2}$$

where

$$G(x,t)=\frac{\partial}{\partial x}\left(K'(x,t)\frac{\partial[h(x,t)]}{\partial x}\right)+\overline{S}(x)+S'(x,t) \tag{3}$$

where $K'(x,t)$ and $\overline{K}$ are the time varying and time averaged components of $K(x,t)$, respectively. Note that the term involving $K'(x,t)$ is now embedded in the new source function $G(x,t)$ as in Eq. (3). $G(x,t)$ is assumed to contain all information related to changes in natural environmental forcing and human induced inputs, and the time-varying diffusive effects.

Note first that the decomposition of K and S into time-varying and time-averaged components is exact, so that no approximation is made in transforming Eq. (1) to Eq. (2). Consider the problem of reconstructing quantities $\overline{K}$ and G from Eq. (2). Direct solution of an inverse problem to find $\overline{K}$ and G is not feasible as there are key mathematical problems to overcome. In particular, $h(x,t)$ depends highly non-linearly on $\overline{K}$ and G, so that direct reconstruction is extremely sensitive to data resolution and measurement errors. As in common with other ill-posed problems, approximations taking into account all available information must be sought. Here, we propose an approximate two-step procedure to retrieve these quantities, to avoid unwanted instabilities.

First, assume that the time average is taken over a sufficiently long period that for any variable x, $\overline{x'}=0$, $\partial\overline{x}/\partial t\approx 0$. Taking the time average of Eq. (2) gives

$$0=\frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial\overline{h}(x)}{\partial x}\right)+\overline{G}(x) \tag{4}$$

In an analogy to the Reynolds' stresses of turbulent fluid flow, $\overline{G}(x)$ may be considered to be a turbulent morphodynamic stress. As a first order approximation these stresses are taken to be zero.

Eq. (4) is then solved for time averaged component of the cross-shore beach profile, the solution of which gives

$$\overline{K}(x)=\frac{\alpha}{\left(\partial\overline{h}(x)/\partial x\right)} \tag{5}$$

where α is a constant of integration. $(\partial\overline{h}(x)/\partial x)$ is the gradient of the mean cross-shore beach profile, which can be calculated from the measurements of beach profiles. Note that should $(\partial\overline{h}(x)/\partial x)=0$ at any finite number of points, all terms in Eq. (4) are identically zero and so $\overline{K}$ is indeterminate. Using arguments of physical continuity it may be set equal to the average of its values at immediately neighbouring points in cases where the mean profile is not monotonic. However, it should be noted that this may not be a fully generic approach to deal situations with zero gradients.

The time averaged diffusion coefficient $\overline{K}(x)$ is then used in Eq. (2) to derive the source function $G(x,t)$ as an inverse problem as follows.

Re-writing Eq. (2) in operator notation gives

$$h_t=Dh+G \tag{6}$$

where $D=(\partial/\partial x)\left(\overline{K}(x)(\partial/\partial x)\right)$

Spivack and Reeve (2000) showed that if the time variation of $G(x,t)$ is weak enough to be neglected over one time step, then the formal solution of Eq. (6) can be written as

$$h(x_i,t_{j+1})\cong(\exp(D\tau)-1)D^{-1}G+\exp(D\tau)h(x_i,t_j) \tag{7}$$

where τ is the time interval between two time steps t_j and t_{j+1}.

The exponential terms are differential operators acting on the functions $G(x,t)$ and $h(x,t)$. It has been assumed that, for simplicity, the values of $h(x,t)$ are given at uniform intervals at a series of time steps t_j where x_i is evenly spaced; although this is not a necessary condition.

Using first order approximation of exponential terms, an expression for the source function $G(x,t)$ is found as (Spivack and Reeve, 2000)

$$G(x,t)=\frac{1}{\tau}\left[h(t+\tau)-\exp(\tau D)h(t)\right] \tag{8}$$

Eq. (8) gives an explicit expression for the unknown source function. Given the data for the function $h(x,t)$ and the time mean diffusion coefficient, the source function $G(x,t)$ can be recovered using Eq. (8).

For more discussion of the governing equation and details of the recovery of the model parameters, the reader is referred to Karunarathna et al. (2009). In order to recover $\overline{K}(x)$ and $G(x,t)$ using the methodology described above, a set of historic cross-shore beach profile bathymetry data over a sufficient time duration is required. The quality and the accuracy of the recovered parameters depend on the length and precision of cross-shore profile data.

3. Milford-on-Sea beach and historic morphological data

Historic cross-shore beach profile surveys carried out at Milford-on-Sea beach, Christchurch Bay, Dorset, UK, are used to demonstrate the methodology and analyse the model parameters. The morphological and hydrodynamic environment of the beach at Milford-on-Sea and the historic data used in the analysis are described in this section.

Milford-on-Sea beach is located in the eastern part of Christchurch Bay. The location map and a view of the beach are given in Fig. 1. The beach is designated as a site of special scientific interest. It is an extensively surveyed beach due to its strategic and national significance. The beach profiles along the bay have been regularly surveyed since 1986 and incident waves were measured and modelled at a number of locations inshore and offshore of the bay area.

3.1. Morphology

Christchurch Bay is a shallow embayment bounded by Hengistbury Head to the West and Hurst Spit to the east. Milford-on-Sea beach extends several kilometres in the NW–SE direction within Christchurch Bay. It has a landward margin of receding cliffs at the western part of the beach and a shallow, wide beach at the eastern side. The beach consists of composite mixed sand-gravel that has a characteristic low tide terrace and time and space varying inter-tidal bars.

Christchurch Bay is regarded as a self contained sediment system (Halcrow Group, 1999). Historically, Milford-on-Sea beach has shown a general trend of retreat. Shoreline retreat rate of 0.65 m/yr at mean sea level (MSL) has been reported between 1867 and 1969. Part of the beach has been restrained since 1970, with the construction of a seawall and a series of groynes (SCOPAC, 2003).

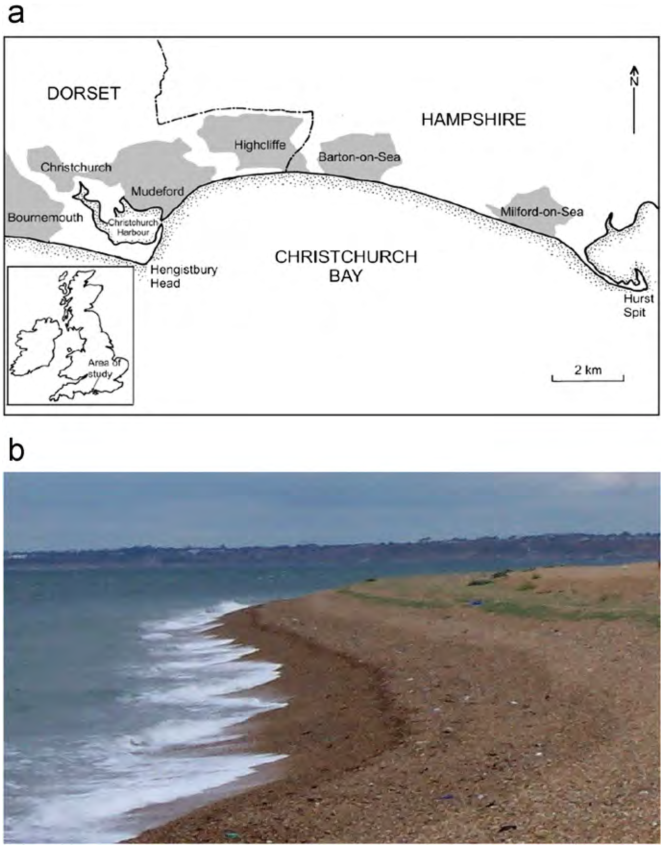

Fig. 1. (a) Location map and the view of the beach at Milford-on-Sea at Christchurch Bay, UK. (b) A view of the study area.

The upper beach face is highly reflective with average gradient varying between 1:5 and 1:7. Inter-tidal beach gradient varies between 1:10 and 1:20. The gentler sub-tidal beach is characterised by highly mobile and segmented sand bars. The number of bars varies between 1 and 2 and their location and shape are highly variable. Cross-shore profiles along the western part of Milford-on-Sea beach are significantly steeper than those in the eastern part. Typical seasonal changes such as flattening and steepening of the profile, appearance and disappearance of beach berms and longshore bars and lowering and lifting of the profile are visible.

The sediment grain size at Milford-on-Sea beach varies significantly along the cross shore profile. Coarse shingles and pebbles with a median grain diameter (D_{50}) around 14 mm dominate the upper beach. A sand-gravel mix with a bimodal distribution, which has D_{50}-gravel = 10 mm and D_{50}-sand = 1 mm with 12% sand fraction, dominates inter-tidal areas (Martìn-Grandes—unpublished data). Sediment grain size at the west side of the beach is slightly coarser than that at the east side, which explains the alongshore variation of the beach slope.

3.2. Hydrodynamic conditions

Tides: Tides in Milford-on-Sea are semi-diurnal and have a modest tidal range of 2.0 m at spring tide. Tidal currents as high as 3.0 m/s have been observed at the close proximity to the beach (SCOPAC, 2003).

Waves: Waves that reach Milford-on-Sea beach are predominantly from the SSW direction. Following a study on wave climate in Christchurch Bay, Bradbury and Kidd (1998) suggested that the 1:100 year wave at Milford-on-Sea has a mean offshore significant wave height of 4.14 m, approaching from 240°. Applying a wave model to determine near-shore waves in Christchurch Bay, Halcrow Group (1999) found that the most frequently occurring wave heights at Milford-on-Sea are between 0.1 and 1.0 m.

3.3. Morphological data

The beach profile measurements were obtained from a series of field surveys conducted between 1987 and 2006. New Forest District Council has operated a data collection programme since 1986, recording cross-shore beach profile surveys and wave and tide measurements. Beach profiles have been surveyed at 45 cross-shore beach transects along Christchurch Bay. Inter-tidal beach was measured using RTK-GPS, using the UK South-East Regional Coastal Monitoring Programme's ground control network. This is tied into Ordnance Survey (OS) Active Network in the UK. Measurements along the profile are accurate to ± 30 mm (vertical and horizontal). GPS was used for all profiles from 1994. Prior to that, profiles were measured by line and level from a fixed marker at the back of the beach (the markers were tied into OS by theodolite height transfer). All height data are to OD (Newlyn). The zero chainage position is a nominal marker at some distance from the back of the beach beyond the area that might erode in the next 100 years. All surveys use this chainage as zero, so the profiles can be overlain for comparison. Earlier line and level survey data was reworked to this start of line position. The topographic and bathymetric profiles are not necessarily done together, but they overlap as seaward limit of topohraphy measurements was MLWS and landward limit of bathymetry measurements was MLWN.

Considering cross-shore location, measurement frequency and cross-shore coverage, two cross-shore transects at the eastern side of the beach were selected for the present analysis. Fig. 2 shows locations of the selected transects 5f00070 and 5f00076.

The reference number and the chainage of the selected transects are given in Table 1.

Fig. 3 shows cross-shore beach profiles measured at transects 5f00070 and 5f00076 during 18 years from November 1987 to January 2006. Each consecutive profile in the figure is shifted by 0.5 m for clarity. There are 51 profile surveys during this period. Survey frequency had been highly irregular. All profile survey dates are given in Appendix A.

Before using cross-shore beach profile survey data in recovering model parameters as described in Section 2, all data are re-arranged to make them amenable to numerical analysis. The offshore limit of the profiles was defined by the spatial coverage of the majority of profiles at a given transect. Incomplete profiles were interpolated up to the chosen offshore cut-off point from adjacent profiles. Following spatial extrapolation, the data are interpolated to 0.5 m intervals along the profile using kriging. The interpolated profiles were compared with their original counterpart to make sure that the interpolations are valid and acceptable. Not more than 6% root mean square error was observed in all interpolated profiles. Finally, as the numerical procedure uses a uniform time step for computation, profiles were interpolated to regular intervals in time, using the nearest available profiles. This time interval for interpolation was chosen on the basis that there were three or more profile surveys in the majority of years; the minimum time interval between surveys was 3 months. Given the irregular nature of the sampling and the focus in this study on medium term variability a simple linear interpolation was used to interpolate the observations to 90-day intervals. More complex interpolation schemes are possible but the nature of the data and the analysis do not warrant it. It might be argued that such a procedure leads to a greater number of new data than in the original set and we are thus ‘creating’ extra data. Some inflation of the data does occur (from 51 original profiles to 76 interpolated profiles). This is justified on the basis that (a) we are concerned with medium term changes that are relatively slowly varying over the sampling interval; and (b) any artefact of the interpolation will be straightforward to identify due to the simple nature of the interpolation scheme.

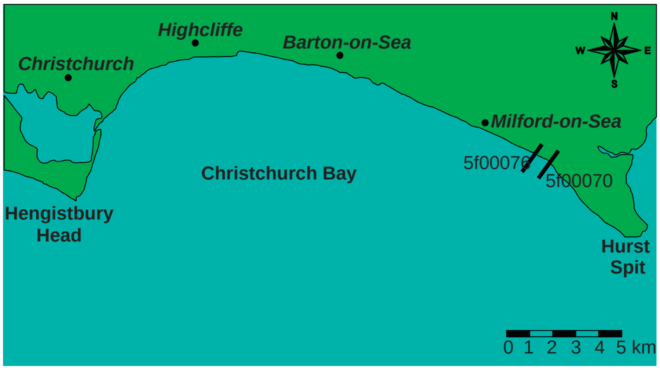

Fig. 2. Locations of beach transects where cross-shore profiles are measured.

Table 1
Beach profile transect reference guide.

Profile transect reference number	Easting (m)	Northing (m)	Coastal chainage (m) (measured from Hengistbury Head)
5f00070	429,409.900	91,161.560	4258.30
5f00076	429,050.000	91,312.000	3868.45

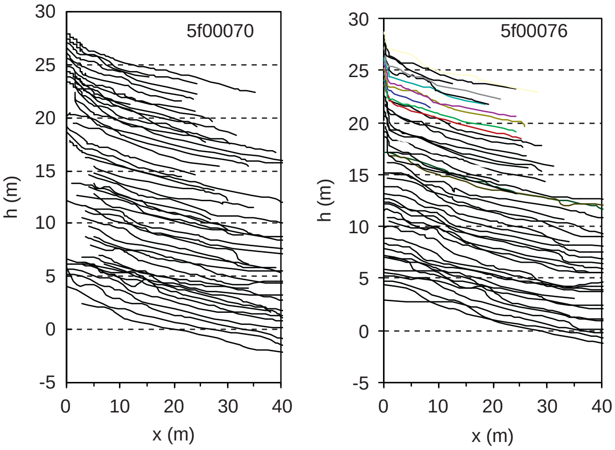

Fig. 3. Measured cross-shore profiles at transects 5f00070 and 5f0076 at Milford-on-Sea beach.

It should be noted that the accuracy of the results may have affected by the interpolation of beach profile data. Root mean square error between time mean beach profiles computed using original beach profile surveys and interpolated profile data was 1.2%.

The re-arranged profiles were then used to determine bulk statistics in order to confirm the suitability of the data for recovering model parameters. Fig. 4 shows time-mean cross-shore

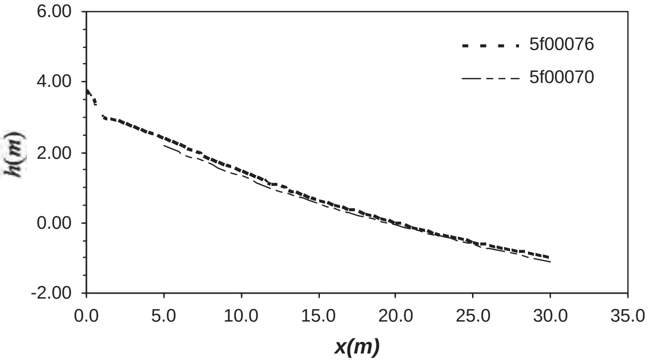

Fig. 4. Time averaged beach profiles at transects 5f00070 and 5f00076 at Milford-on-Sea.

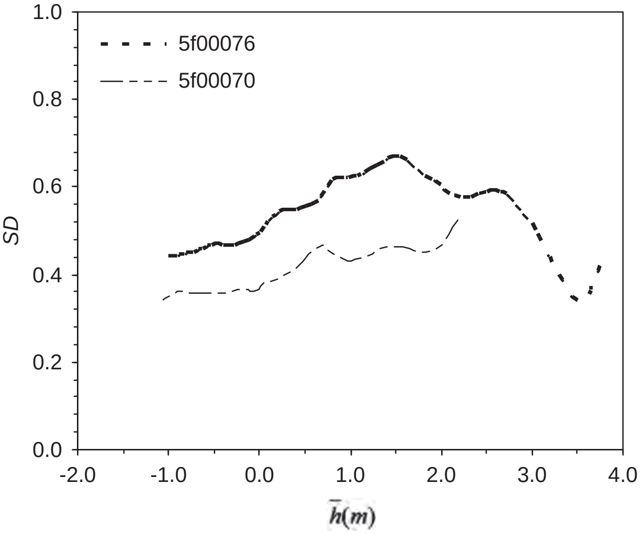

Fig. 5. Standard deviation of mean profile depths at 5f00070 and 5f00076.

beach profiles ($\overline{h}$) at transects 5f00070 and 5f00076. Time averaging was carried out over the total duration of the dataset. Standard deviations (SD) of time variation of beach profile elevation were then derived in order to make sure that the cross-shore extent of the profile measurements is adequate for using them in the recovery of model parameters (Larson et al., 2003). Fig. 5 shows the results. In both transects the SD is at its highest in the supra-tidal beach. Even though SD does not reach zero at the seaward limit of the profile, as our focus here is on inter-tidal and supra-tidal regions where beach variability has the highest engineering significance in terms of beach stability and stability of coastal defence structures, the seaward extent of the profiles was taken as satisfactory.

4. Recovery and analysis of model parameters

The key parameters of the diffusion model, diffusion coefficient and the source function were recovered from the cross-shore profile measurements at Milford-on-Sea beach, following the method described in Section 3. The results and analysis of these parameters are presented and discussed in this section. The objective here is to investigate the nature of the diffusion coefficient and the source function and thereby the potential for using Eq. (2) in a predictive capacity.

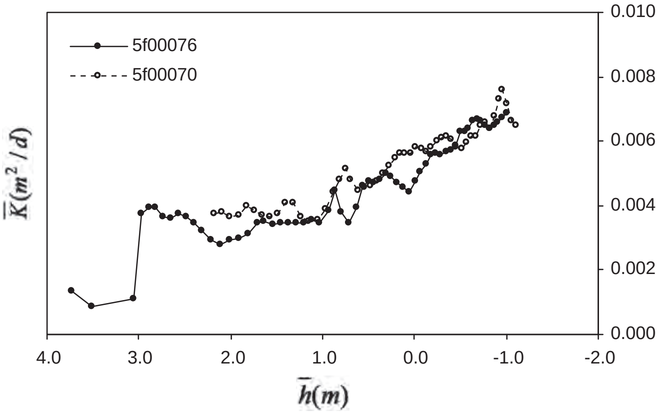

Fig. 6. Mean diffusion coefficient recovered from cross-shore beach profile measurements at 5f00070 and 5f00076.

The time-mean diffusion coefficient $\overline{K}$ is determined following the procedure described in Section 2 (Eq. (5)). Even if it was not necessary to use interpolated profile data to determine $\overline{h}$ (and then $\overline{K}$), we used them here in order to maintain consistency among the data used to determine $\overline{K}$ and G. The variation of $\overline{K}$ with x is expected to reflect changes in beach sediment across the profile and the site climate. Fig. 6 shows variation of $\overline{K}$ against mean profile depth for the two selected cross-shore beach transects. The general trend here is that $\overline{K}$ gradually increases with profile depth. This reflects the variation of beach sediment from coarse to fine material from the upper beach to sub-tidal areas.

Despite the composite nature of the beach and the ephemeral appearance of berms, bars and troughs in the individual beach profiles, the time mean profile is remarkably monotonic. In fact, it is reminiscent of the canonical equilibrium profile described by Dean (1991), in which $h(x)=Ax^{2/3}$ and A is a constant related to grain size by $A=0.21D^{0.48}$ with D in millimetres. Dean's equilibrium profile shape is directly linked to sediment characteristics whereas the diffusion coefficient in the profile evolution model is assumed to characterise sediment properties of the cross-shore profile together with the time-integrated effects of non-diffusive processes. The top and bottom panels in Fig. 7 correspond to the cross-shore transects 5f00070 and 5f00076, respectively. The left hand panels plot the time mean profile together with Dean's profile corresponding to a grain size of 15 mm (SCOPAC, 2003). Deviations of the mean profile shape from the Dean's profile can be attributed to complex cross-shore sediment variability, long-shore non-uniformity of the beach and other site-specific effects such as sediment transport control by coastal defence structures. The right hand panels show values of $\overline{K}$ against $x^{1/3}$. From Eq. (4), if the beach followed Dean's equilibrium form then, $\overline{K}$ and $x^{1/3}$ should have a linear relationship. This is indeed very nearly the case, except on the upper beach face. We conclude that the fact that the mean beach profile closely follows Dean's equilibrium profile and that the mean diffusion coefficient has a natural interpretation as representing the sediment characteristics of the beach. That is, for a beach following Dean's equilibrium form we have $\overline{K}=(3\alpha/2A)x^{1/3}$,

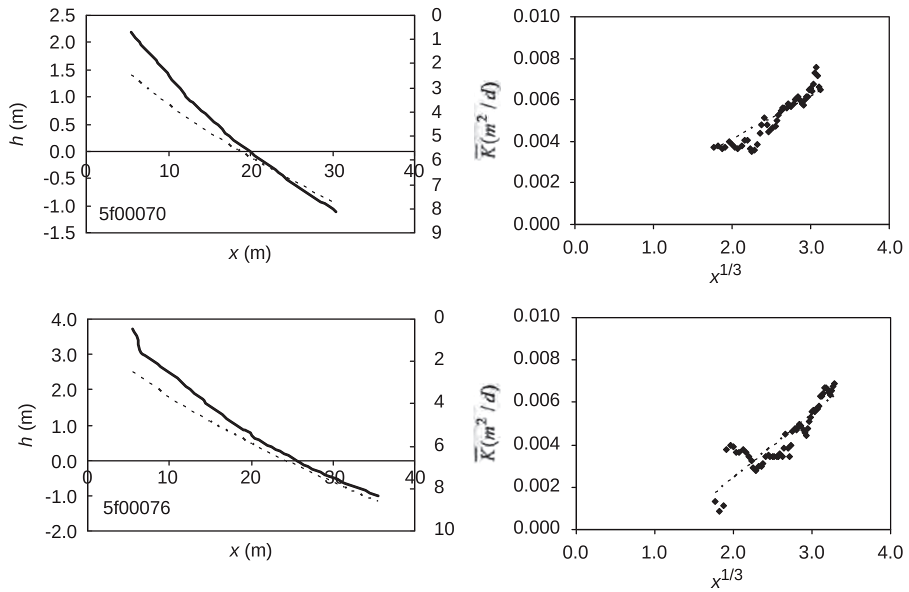

Fig. 7. Comparison of measured mean cross-shore profile and $\overline{K}$ at 5f00070 and 5f00076 with Dean's equilibrium profile.

which is clearly linked to the sediment characteristics. The link between diffusion coefficient and sediment characteristics has been made on the basis of the assumptions inherent in Dean's equilibrium profile model and also the invertibility of Eq. (5). The question arises whether the procedure could be applied to beaches with stable bars (non-monotonic mean profiles) to give a similar relationship between diffusion coefficient and sediment size. As mentioned earlier, at points where the denominator in Eq. (5) vanishes, the coefficient is indeterminate and may be set to ensure continuity. However, to derive a functional relationship would require the equivalent of Dean's model for barred beaches, which currently does not exist.

Detailed investigation of time variation of $G(x,t)$ across the profiles for both beach transects (not shown) shows slow variation of $G(x,t)$ between successive time intervals as assumed in the development of the inverse methodology. It was also observed that a source function derived from two consecutive cross-shore profile surveys is substantially different from the beach profile change between the two corresponding profiles. That is, the source function is not simply the arithmetic difference of the two profiles and that the sediment transport as described by the diffusion process plays a vital role in beach profile evolution. In Fig. 8, the time variations of the spatially integrated (integrated across the cross-shore profile) source function $\int_x G(x,t)dx$, for 5f00070 and 5f00076, are shown. Non-zero values of $\int_x G(x,t)dx$ indicate net removal or accumulation of sediment across the profile as a result of changes to external sediment transport drivers. The traces show strong fluctuations about relatively stable negative mean values of

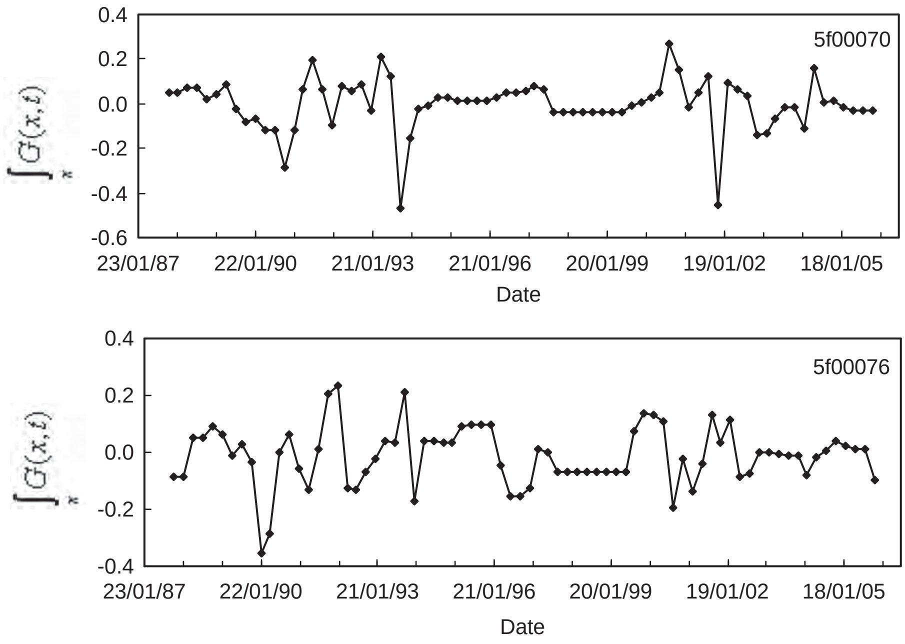

Fig. 8. Time variation of source function integrated cross the cross-shore beach profile at 5f00070 and 5f00076.

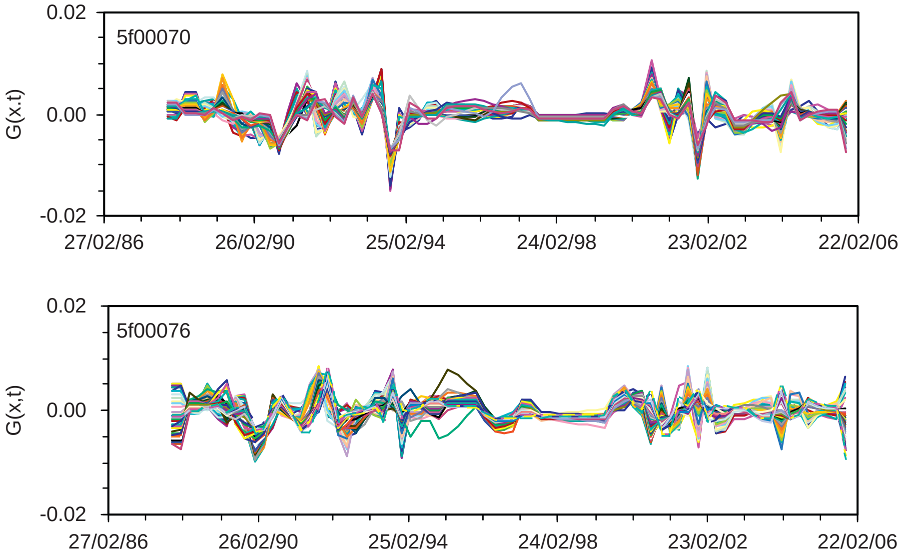

Fig. 9. Time histories of the source function recovered at 5f00070 and 5f00076.

approximately 0.009 m^2/day. If spread uniformly over the profile this equates to a lowering of approximately 3×10^{-3} m/day. On a beach with an average slope of 1:7, this corresponds to a retreat rate of 0.0021 m/day (or 0.77 m/yr), which is in close agreement with the observed rate of retreat (0.65 m/yr) of Milford-on-Sea beach (SCOPAC, 2003).

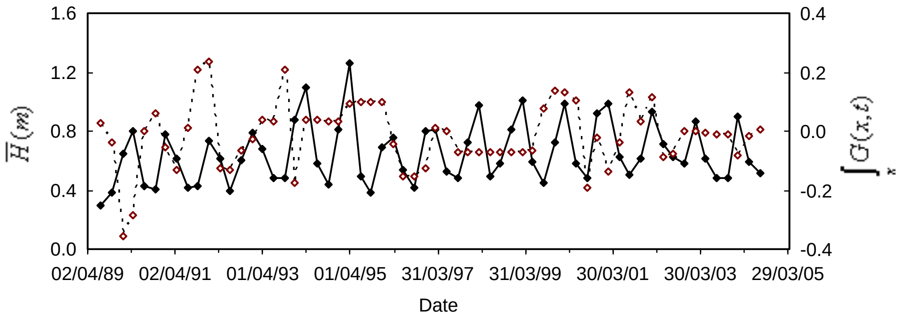

Fig. 10. Comparison of time averaged wave heights (averaged over 90 day period) at Christchurch Bay with cross-shore integrated source function.

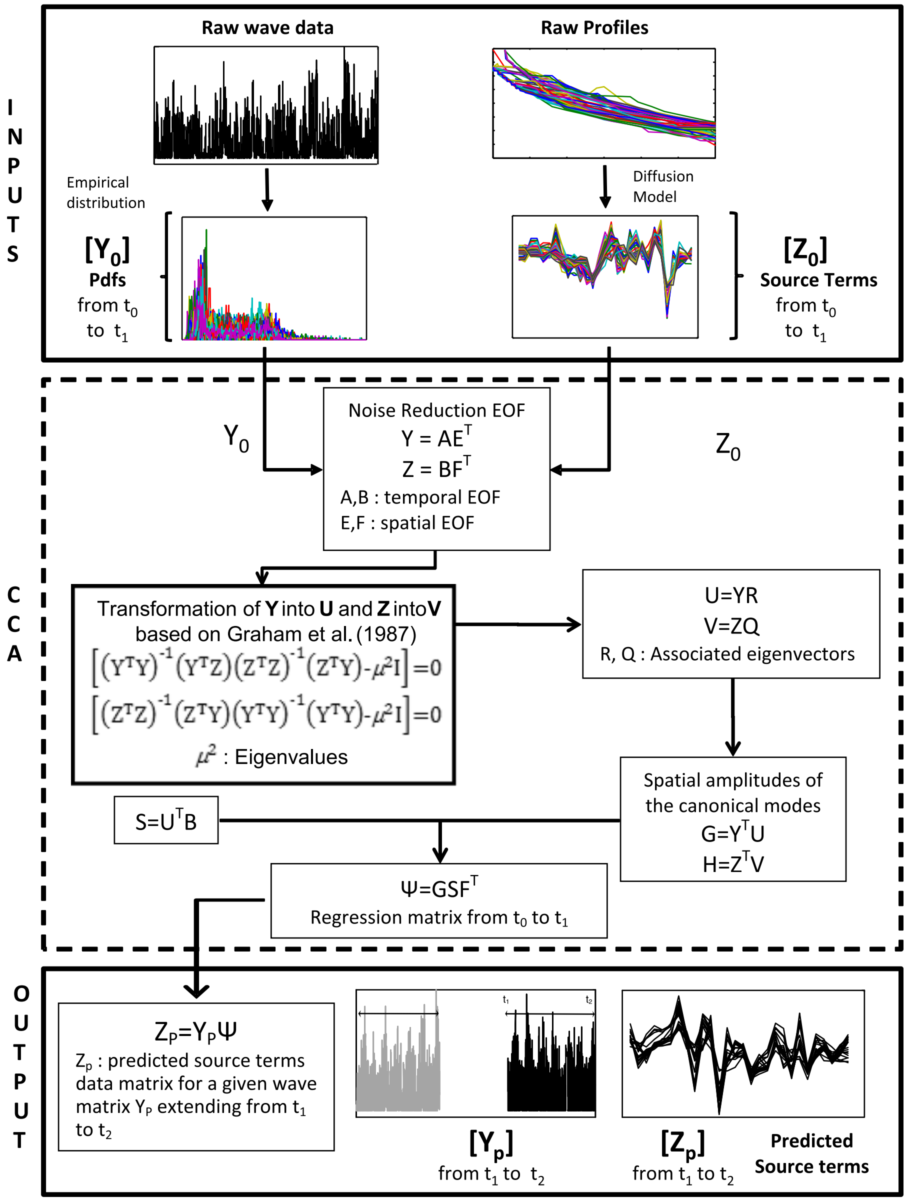

Fig. 11. Scheme of the derivation of the source terms using the CCA method (adapted from Ruiz de Alegria-Arzaburu et al. (2010)).

To investigate the temporal variation of $G(x, t)$ in greater detail, time histories of it are drawn in Fig. 9. Positive values correspond to beach accretion while negative values correspond to erosion. According to Fig. 9, the source function captures alternate erosion and accretion of the cross-shore profile at seasonal/yearly timescales. In the period between 1996 and 1999, the source function shows considerably low variability compared to the rest of the time, indicating a significant deviation from the general trend. It has been reported that beach refilling of Milford-on-Sea had been taken place during this period where the beach was continuously monitored, and if beach levels fall below a certain critical level then there was beach refilling (SCOPAC, 2003). Beach refilling, or reprofiling, is a relatively continuous process, at least in comparison to the profile measurement intervals. Thus between two consecutive profile measurements there may be several refilling exercises. Further, the incoming waves will be reworking the beach material throughout the proceedings. The impact of beach refilling as seen in the measured profiles is thus not a sudden jump in the time series of beach levels but a smoothing of the variation.

To explore the physical meaning of the source function its relationship to morphodynamic forcing associated with incident waves has been investigated. As an initial hypothesis we consider that the bulk properties of the key quantities may be directly linked. That is, some integrated measure of the source function is strongly dependent upon an integrated measure of wave intensity. We thus compare the temporal variation of the cross-shore averaged source function $\int_x G(x,t)dx$ with the time averaged incident wave height $\overline{H}$. Wave heights were averaged over 90-day periods to match with the time interval between two successive source functions. The incident waves have been measured at Christchurch Bay, at a location 1.5 km offshore at 10 m (CD) depth and the measured wave data were provided by New Forest District Council. The results for 5f00070 are shown in Fig. 10. The correlation coefficient between $\int_x G(x,t)dx$ and $\overline{H}$ is 0.2, which does not suggest a strong link between the profile-integrated source term and the time averaged wave height.

To investigate the potential relationship between computed source function and waves further, a more sophisticated testing involving a canonical correlation analysis was performed. This allows us to discern joint patterns of behaviour in the evolution of the source function and the full distributions of wave conditions.

CCA is used to investigate the presence of any patterns that tend to occur simultaneously in two different data sets and the correlation that exist between the associated patterns. A brief outline of the technique is provided here but further details may be found in Clark (1975) and Różyński (2003). If the two original data sets are denoted by Y (wave data matrix with size $nt \cdot ny$) and Z (source terms matrix of size $nt \cdot nz$), then, a linear combination of Y and Z is sought to obtain the new variables U and V (Fig. 11) that are maximally correlated for the same index and zero correlation for differing indices.

A regression matrix (ψ) is obtained through the CCA analysis, which relates the source terms to the wave properties based on the correlation between the dominant patterns of these two variables, source function and wave data. With this regression matrix, predicted source terms represented by the matrix Z_p can be obtained by multiplying the regression matrix ψ by the matrix Y_p corresponding to known or forecast wave conditions over the period between time t_1 and time t_2 (see Fig. 11). As CCA requires two time series sampled at the same rate it is not possible to use source functions at 90-day intervals and 3 h wave time series directly. Thus, in order to generate two time series at equal length, the waves between the dates of each consecutive pair of source functions were used to compile probability density functions (pdf) of significant wave heights and also wave steepness. The empirical pdf suggested by Horrillo-Caraballo and Reeve (2008, 2010) was used. The resulting pdf was then assigned to the latter of the source function. The empirical distribution is a cumulative probability distribution function that concentrates probability $1/n$ at each of the n numbers of a sample. A combined pdf (p_n) may then be

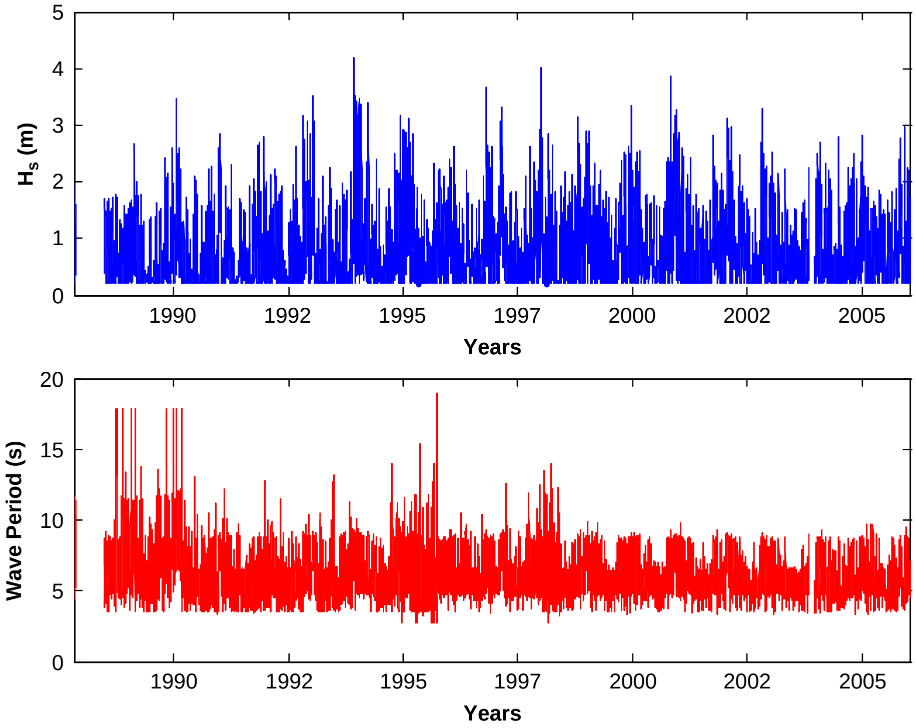

Fig. 12. Time series of significant wave height (H_s) (top panel) and wave period (bottom panel) measured at Milford-on-Sea.

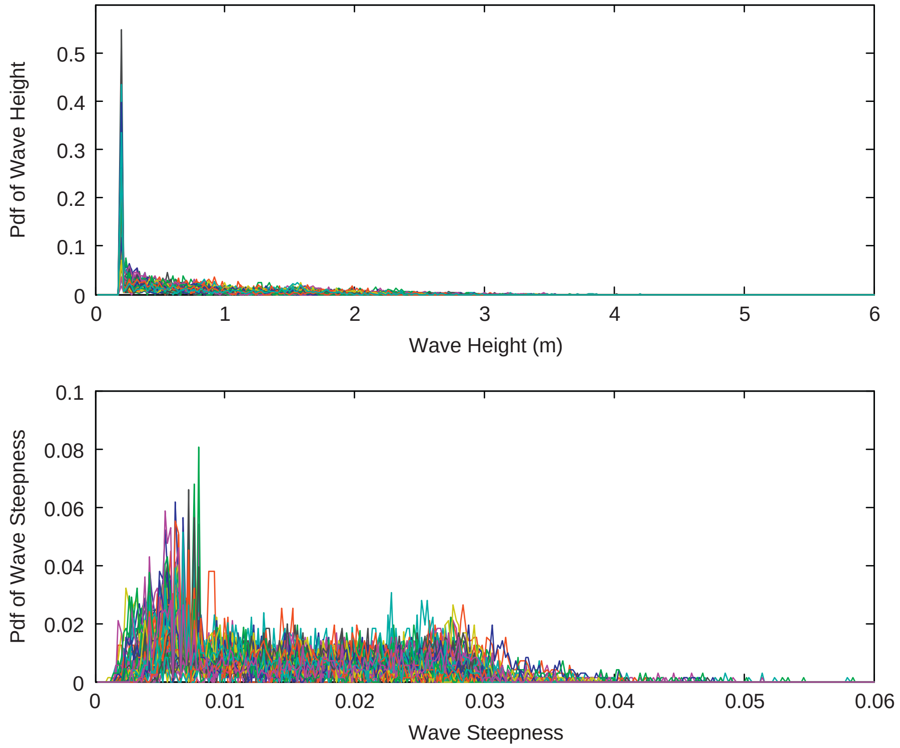

Fig. 13. Composite plot of empirical probability density functions (pdfs) for significant wave height (top panel) and wave steepness (bottom panel).

derived by superimposing the individual pdfs available for the period between two consecutive source functions:

$$p_n(a) = \frac{1}{n}\sum_{i=1}^{n} I(a_i \le a) \tag{9}$$

where a is the wave height or steepness, n is the number of individual wave measurements between two consecutive source functions and i is an index.

The performance of the CCA method can usually be improved by expanding the input time series as empirical orthogonal functions. A filtered sequence of input is obtained by reconstructing the data with a truncated set of empirical orthogonal functions. The 'rule of thumb' (North et al., 1982) was used to establish the appropriate number of CCA modes to obtain the best possible correlation.

Fig. 12 shows the wave height (top panel) and period time (bottom panel) series and Fig. 13a and b shows a composite of the pdfs of wave height and wave steepness. It is evident that the structure of Fig. 13b differs from the structure of Fig. 13a; the inclusion of the wave length (referred to the wave steepness) is related to the suggestion by Pedrozo-Acuña et al. (2008); that higher the wave steepness, the higher the impact of the wave over the beach profile. This means that the wave steepness will be more related to changes on the profile than the wave height. While a statistical technique like CCA cannot provide proof of a direct causal link it can identify strong correlative links that suggest an underlying dynamical link. Here we use the CCA to construct a regression matrix using information in the early part of the records. This matrix is then used, in conjunction with the wave conditions in the latter part of the record to 'predict' the corresponding source functions. These predictions are then compared against the computed ones to establish the strength of the link between the source function and the wave heights or wave steepness.

The first 27 source terms and corresponding wave conditions covering the period from 01/11/1987 to 29/03/1994 are used to create regression matrices. These regression matrices were then used with the wave height and wave steepness pdfs for the period between 03/03/1999 and 25/01/2006 (~9 years) to calculate the corresponding source terms over the same period (see Fig. 11).

Table 2
Skill of the CCA method for transects 5f00070 and 5f00076 at Milford-on-Sea (H_s and WSTP refer to significant wave height and wave steepness, respectively).

Profile	Skill	
	$\boldsymbol{H_s}$	**WSTP**
5f00070	0.72	0.96
5f00076	0.69	0.96

Table 2 shows the 'skill' (prediction ability of the CCA) of the method. The 'skill' is calculated using the matrix Q (see Fig. 11), and the percentage of total variance in the profiles and the percentage of variance of input predictand EOFs (Różyński, 2003). The value of the 'skill' is analogous to the correlation coefficient with a value of 0 corresponding to no skill and a value of 1 being a perfect correlation. The results show that more than 96% of overall variability can be explained by variations of the wave steepness at both profiles and 72% and 69% of the overall variability to variations of the wave conditions at profiles 5f00070 and 5f00076, respectively. Wave steepness appears to be a better indicator of beach profile changes than wave height. Further, a skill of 0.96 suggests a very strong link between the computed source functions and the incident wave steepness.

5. Conclusions

In this paper, the key parameters in a diffusion-type formulation for beach profile evolution are analysed. In the diffusion formulation, the space varying diffusion coefficient and a source function

Table A1
Dates of beach profile surveys at 5f00070 and 5f00076, Milford-on-Sea.

Year	Survey dates
1987	01/11
1988	01/05, 01/11
1989	01/03, 01/07, 01/09
1990	01/01, 01/04, 01/07,30/09
1991	01/03, 01/07, 01/10
1992	01/01, 01/04, 01/09, 01/12
1993	01/03, 01/08, 31/10
1994	01/03, 01/06, 01/09
1995	01/04, 01/04
1996	01/05
1997	01/03, 01/09
1998	01/01, 01/09
1999	01/01
2000	01/03, 01/08, 01/11
2001	01/02, 01/05, 01/08, 01/12
2002	01/02, 01/05, 01/11
2003	20/06
2004	01/02, 17/02, 17/06, 25/06,12/10
2005	09/03, 26/08, 29/11
2006	31/01

that describe the aggregation of all non-diffusive morphodynamic processes govern the success of its application to predict beach profile evolution in future.

The success of recovering the diffusion coefficient and the source function using the inverse methodology described here largely depends on the availability and accuracy of the measured cross-shore profile survey data. In the present study, we have described a two-stage process whereby the diffusion coefficient and the source function can be retrieved from observations of the beach profiles alone. Historic beach profiles measured at the mixed beach at Milford-on-Sea in Christchurch bay, UK, have been used to test the methodology. Comparing results with approximate equilibrium beach profile models a new interpretation of the diffusion coefficient has been proposed, which links it directly to the sediment properties of the beach. The physical nature of the beach concerned is captured well by the diffusion coefficient. The sediment composition, the transition region of the composite beach profile and the gentler sub-tidal beach correlated well with the variability of the diffusion coefficient in the cross-shore direction.

Comparing the source function with incident waves, it is found that the profile-integrated source function had little correlation with the time mean wave height. A more sophisticated analysis using CCA demonstrated a very strong link between the wave steepness and the source function. This supports the postulates of Pedrozo-Acuña et al. (2008) who argued that gravel-sand beach response is conditioned by wave steepness.

The outcome of the analysis indicates that historic data of sufficient length and accuracy can be successfully used to determine the nature of the key parameters in a diffusion type beach profile evolution model. For the specific site used for our analysis the diffusion coefficient can be related directly to the sediment characteristics. Further, the seasonal to yearly time scale response of the beach is well captured by the source function, making it a useful tool in predicting future beach profile behaviour in that time scale. The strength of the link between source function and wave steepness suggests that this could form the basis of a useful predictive tool, whereby forecast wave conditions could be used in conjunction with the regression matrix from the CCA to forecast a future sequence of source functions from which forecasts of beach profile evolution can be made. However, it has to be noted that in the event of significant human intervention, behaviour of the source function deviates from its general trends.

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Appendix A

See Table A1.

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