Computing lower order terms in the full moment conjecture for ζ.

Michael Rubinstein

Amer. Inst. of Math., Palo Alto, Calif.

I will describe an algorithm for obtaining both analytically and numerically the lower order terms in the full moment conjecture for ζ, and will present evidence supporting the conjecture. This work is joint work with Brian Conrey (A.I.M.), David Farmer (A.I.M.), Jon Keating (Bristol, UK) and Nina Snaith (Bristol, UK).

Calculating Maass forms.

David Farmer

Amer. Inst. of Math., Palo Alto, Calif.

Nonholomorphic modular forms, commonly called Maass forms, are fundamental objects in number theory. There do not exist closed form expressions for these functions, so one must perform numerical experiments in order to find them. We will describe methods for locating Maass forms on arbitrary cofinite subgroup of $SL(2, \mathbf{R})$. Examples will illustrate properties of the family of L-functions associated with Maass forms. Joint work with Stefan Johansson.

Some remarks on the distribution of the zeros of the Riemann zeta function.

Akio Fujii

Rikkyo University, Tokyo, Japan

Some problems on the vertical distribution of the zeros of the Riemann zeta function will be presented. Especially, the distribution of $S(t)$ and the problems on the uniform distribution of the imaginary parts of the zeros will be discussed.

Maass forms of CM-type and quantum chaos.

Dennis Hejhal

University of Minnesota and Uppsala University

Maass waveforms of CM-type are a special “almost explicit” type of eigenfunction of the Laplacian that occur on certain congruence subgroups of $SL(2, \mathbf{Z})$. In this talk, we report on some recent numerical work with such forms, focussing in particular on their autocorrelations and mimickry of random waves.

Siegel zeros and a New Question in Harmonic Analysis.

Harold Stark

Univ. of Calif., San Diego, Calif. 92093

We will discuss ongoing work regarding Siegel zeros.

A new database of elliptic curves

Mark Watkins[a] (and William A. Stein[b])

[a]The Pennsylvania State University

[b]Harvard University

Over a decade ago, Brumer and McGuinness formed a (nonexhaustive) database of elliptic curves which had $|\Delta|$ both prime and less than 10^8. We extended their database in two directions, pushing 10^8 up to 10^{10} for curves with prime $|\Delta|$, and relaxing the primality constraint for curves with conductor less than 10^8. For each curve, we computed various invariants, such as the rank. We report on the data obtained, and some conjectures which we have been led to make.

Verifying the jumping champion conjecture

Rahul Athale

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In this paper, we present results concerning the computational verification of the jumping champion conjecture made by Odlyzko et. al. in the paper *Jumping Champions.* They have given heuristics and computational results for two intervals of size 10^7, claiming that 210 takes over as jumping champion from 30 somewhere between 10^{425} and 10^{426}. In one of those two intervals, starting from $10^{450}, 210$ was not a dominant gap. This motivated us to verify the conjecture computationally. We considered intervals of size 10^7 starting from $10^{410}, 10^{420}, \ldots, 10^{550}$ and in many of them 210 did not appear as a dominant gap. Therefore we continued our experiments with intervals of size 10^8. We present the results of these experiments. We used computer algebra system Mathematica (Mathematica is a product of Wolfram Research, Inc.) for the computation.

Verifying the full Birch and Swinnerton-Dyer Conjecture for certain specific abelian varieties

William Stein

Harvard University

I will talk about my attempts to verify every part of the Birch and Swinnerton-Dyer Conjecture for certain specific examples of abelian varieties of analytic rank at most 1. I'm most successful for elliptic curves, and obtain interesting bounds for many modular abelian varieties of rank 0. The methods that I use should be of general interest to anyone who wants to know how to compute with modular abelian varieties. This work is joint with Michael Stoll and Amod Agashe.

Biases in the Shanks-Rényi Prime Number Race

Greg Martin [a] (and Andrey Feuerverger[b])

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As Chebyshev first noted in the middle of the 19th century, if you separate the first however-many primes according to their least residue modulo 4, there always seem to be more primes congruent to 3 (mod 4) than to 1 (mod 4). This tendency persists despite the fact we now know that asymptotically *half* the primes belong in each category! Similar phenomena occur with other moduli as well; in general, the congruence classes that are nonsquares (mod q) more often have a surplus of primes rather than a deficit relative to the congruence classes that are squares (mod q). In 1994, Rubinstein and Sarnak proposed a framework for quantifying these biases, and (subject to conjectures on the distribution of zeros of Dirichlet L-functions) proved that the asymptotic "probability" that there are more primes congruent to a than to b (mod q) exists and is strictly between 0 and 1; moreover, this probability is greater than $\frac{1}{2}$ if and only if a is a nonsquare and b is a square modulo q. They calculated, for example, that there is a 99.59% chance that there are more primes congruent to 3 than to 1 modulo 4. In this talk, we discuss our efforts to extend the computation of these sorts of asymptotic probabilities. We describe what goes into such computations and point out some interesting observations about the results. For example, the probabilities that various nonsquare congruence classes run ahead of various square congruence classes can differ from class to class even when the modulus is fixed. Moreover, there are asymmetries in the analogous "prime number race" among three congruence classes that have nothing to do with squares and nonsquares!

Computing Artin L-functions of Graph Coverings using Methods from Number Theory

Audrey Terras

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The L-functions we investigate are analogues of Artin L-functions of Galois extensions of number fields and Selberg zeta functions of Riemannian manifolds. They are connected to finite unramified normal coverings of a graph X. Here, in joint work with H. Stark, I will consider various methods from number theory which aid in the computation of these L-functions.

Computing special values of motivic L-functions

Tim Dokchitser

University of Durham, UK

We present an algorithm to compute values $L(s)$ and derivatives $L^{(k)}(s)$ of L-functions of motivic origin numerically to required accuracy. Slightly more generally, the method applies to any L-series whose Γ-factor is of the form $A^s \prod_{i=1}^{d} \Gamma(\frac{s+\alpha_j}{2})$ with d arbitrary and complex α_j, not necessarily distinct. The algorithm relies on the known (or conjectural) functional equation for $L(s)$.

Maassforms on the Picard group $SL(2, \mathbf{Z}[i])$ and applications in cosmology

Holger Then

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Eigenfunctions of the Laplacian are of importance in physics. In this talk we will report on our numerical computations of Maassforms (eigenfunctions of the Laplacian) on the Picard group $SL(2, \mathbf{Z}[i])$ acting on hyperbolic 3-space. We will then briefly discuss an application to the microwave background radiation in cosmology. (Joint work with Ralf Aurich and Frank Steiner.)

Numerical computation of modular forms

Andreas Strömbergsson

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A few years ago, Hejhal presented a new, robust algorithm for the computation of both modular forms and Maass waveforms on certain cofinite Fuchsian groups Γ. His elegant algorithm represents a major step forward compared to earlier existing methods, regarding both numerical stability and range of applicability. The essential restriction on Γ in Hejhal's algorithm is that it should have exactly one cusp. In my talk, I will present a generalization of this algorithm to the case of cofinite Fuchsian groups having several cusps. I will mainly focus on the case of modular forms. I will also discuss some applications of the algorithm. (Part of this work is joint with Björn Selander of Uppsala.)

Computers in Undergraduate Education and Zeros of Elliptic Curves

Steve Miller

Princeton University

For the last two years at Princeton, the Junior Research Seminar has involved students numerically in investigating hot conjectures. I will mention some of their results from Ramanujan Graphs and Random Matrix Theory (and if time permits, Prime Spacings).

Based on theory from the function field case, Katz and Sarnak predict every natural family of L-functions has an associated symmetry group controlling the distribution of zeros. Certain statistics (n-level correlations) are the same for all classical compact groups; others (n-level densities) differ. For families of elliptic curves, for small support, the 1-level density unconditionally agrees with the appropriate classical compact group; the 2-level conditionally agrees. I will sketch the proofs, and provide numerical evidence for the needed conjectures (as well as consequences of being able to prove these results for larger supports). These computations were done with last year's Junior Seminar.

ZetaGrid – Computations Connected with the Verification of the Riemann Hypothesis

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Results of recent and ongoing internet computing inside the intranet of IBM Corporation which were obtained during the numerical verification of the Riemann Hypothesis are presented, e.g. the first 75 billion nontrivial zeros of the Riemann zeta function $\zeta(s)$ are simple and lie on the line $\mathrm{Re}(s) = \frac{1}{2}$.

In August 2001, we started the project "ZetaGrid" as internet computing inside the intranet of IBM Corporation, and now there are more than 600 office computers/notebooks involved in this computation. These computers spend all of their idle time on the computation.