The combinatorial geometry of general flag manifolds

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We give and interesting review about some geometrical properties on general flag manifold $\mathbb{F}$ in connection with some differential structures like almost complex structures and f-structures. A tensor field $\mathcal{F}$ of type (1,1) on a Riemannian manifold is called an *f-structure* if $\mathcal{F}^3 = -\mathcal{F}$, and *almost complex* if $\mathcal{F}^2 = -I$. We call $\mathcal{F}$ (1,1)-symplectic if the $(+,-)$ part of the derivative $d^{\nabla}\mathcal{F}$ vanishes. Although this property is different than the $(1,2)$-symplectic property, the two are identical if $\mathcal{F}$ is almost complex . An f-structure $\mathcal{F}$ on a Riemannian manifold will be called (1,1)-admissible if it is (1,1)-symplectic with respect to some metric on the manifold. Admissibility on a general flag manifold $\mathbb{F}$ is known to play an important role in the analysis of complex harmonic functions with values in $\mathbb{F}$, but its characterization is known when $\mathcal{F}$ is almost complex or when $\mathbb{F}$ is the classical full flag manifold $\mathbb{F}(n)$. We provide a simple characterization for admissibility, valid for any invariant f-structure on any general flag manifolds. Namely, we reduce admissibility to the local transitivity of an associated intersection graph.

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