Book of Abstracts

Foundations of Computational Mathematics

Santander, 30 June – 9 July 2005

Welcome Note

The Society for the Foundations of Computational Mathematics supports fundamental research in a wide spectrum of computational mathematics and its application areas. As part of its efforts to promote research in computational mathematics, it regularly organises conferences and workshops which are a prominent meeting point for leading researchers in diverse fields that impinge on various aspects of computation. Major conferences of the Society were previously held in Park City (1995), Rio de Janeiro (1997), Oxford (1999) and Minneapolis (2002).

Similarly to previous FoCM conferences, FoCM'05 will be attended by several hundred scientists. Workshops are to be held in twenty-one fields which include: the foundations of the numerical analysis of partial differential equations, geometric integration and computational mechanics, information-based complexity, learning theory, optimization, special functions and orthogonal polynomials, approximation theory, computational algebraic geometry, computational number theory, multiresolution and adaptivity in the numerical analysis of partial differential equations, numerical linear algebra, relations with computer science: algorithmic game theory and metric embeddings, real-number complexity, computational dynamics, geometric modelling and animation, image and signal processing, stochastic computation, symbolic analysis, computational geometry and topology, mathematical control theory and applications, and random matrices.

In addition to these workshops, eighteen plenary lectures will be presented, covering a broad range of topics connected to computational mathematics.

On behalf of the Board of the Society for the Foundations of Computational Mathematics and the Local Organising Committee of the conference, we would like to use this opportunity to thank the local organisers and administrative staff of our host — the Universidad de Cantabria — for their hospitality. We would also like to express our gratitude for financial support to the International Mathematical Union, the University of Cantabria, the Departamento de Matemáticas, Estadística y Computación, the Escuela Técnica Superior de Ingenieros Industriales y Telecomunicaciones, the Vicerrectorado de Investigación de la Universidad de Cantabria, and the Programa Nacional de Acciones Complementarias (proyecto MTM2004-20180-E). We are deeply indebted to the organisers of the twenty-one workshops for their enthusiasm and tireless efforts. Above all, however, we wish to express our gratitude to all participants of this conference for being here and making this meeting such an exciting and scientifically stimulating event.

Welcome to Santander!

Luis Miguel Pardo
Chair, Local Organising Committee FoCM 2005

Endre Süli
Chair, Society for the Foundations of Computational Mathematics

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Plenary Speakers

Douglas Arnold (IMA & University of Minnesota)

James Demmel (University of California, Berkeley)

Jan Denef (Katholieke Universiteit Leuven)

Michael Griebel (Universität Bonn)

Ernst Hairer (Université de Genève)

Adrian Lewis (Simon Fraser University)

Stephane Mallat (Ecole Polytechnique)

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List of Workshops by Period

PERIOD 1
Thursday 30th June - Saturday 2nd July

WORKSHOP 3
Information-based complexity
ORGANISERS: Leszek Plaskota & Ian Sloan

WORKSHOP 6
Special functions and orthogonal polynomials
ORGANISERS: Mourad Ismail, Guillermo Lopez & Ed Saff

WORKSHOP 8
Computational algebraic geometry
ORGANISERS: Teresa Krick & Andrei Gabrielov

WORKSHOP 10
Multiresolution and adaptivity in numerical PDEs
ORGANISERS: Wolfgang Dahmen, Bob Russell & Endre Süli

WORKSHOP 15
Geometric modelling and animation
ORGANISERS: Tom Lyche & Larry Schumaker

WORKSHOP 19
Computational geometry and topology
ORGANISERS: Rich Schwartz & Abigail Thompson

WORKSHOP 20
Mathematical control theory and applications
ORGANISERS: Eduardo Casas, Uwe Helmke, Jean-Pierre Raymond & Enrique Zuazua

PERIOD 2
Monday 4th July - Wednesday 6th July

WORKSHOP 2
Geometric integration and computational mechanics
ORGANISERS: Begoña Cano, Debra Lewis & Brynjulf Owren

WORKSHOP 4
Learning theory
ORGANISERS: Steve Smale, David McAllester, Tomaso Poggio & Gábor Lugosi

WORKSHOP 5
Optimization
ORGANISERS: Raphael Hauser, Jim Renegar & Philippe Toint

WORKSHOP 12
Relations with computer science: Algorithmic game theory and metric embeddings
ORGANISERS: Avrim Blum & Allan Borodin

WORKSHOP 16
Image and signal processing
ORGANISERS: Albert Cohen & Guillermo Sapiro

WORKSHOP 18
Symbolic analysis
ORGANISERS: Elizabeth Mansfield, Peter Olver & Mike Singer

WORKSHOP 21
Random matrices
ORGANISERS: Ioana Dumitriu, Alan Edelman & Raj Rao

PERIOD 3
Thursday 7th July - Saturday 9th July

WORKSHOP 1
Foundations of numerical PDEs
ORGANISERS: Christoph Schwab & Eitan Tadmor

WORKSHOP 7
Approximation theory
ORGANISERS: Martin Buhmann & Juan Manuel Pena

WORKSHOP 9
Computational number theory
ORGANISERS: Alan Lauder & Jonathan Pila

WORKSHOP 11
Numerical linear algebra
ORGANISERS: Lothar Reichel & Steve Vavasis

WORKSHOP 13
Real-number complexity
ORGANISERS: Peter Buergisser & Gregorio Malajovich

WORKSHOP 14
Computational dynamics
ORGANISERS: Jean-Pierre Ramis, Carles Simo & Warwick Tucker

WORKSHOP 17
Stochastic computation
ORGANISERS: Brad Baxter & Thomas Müller-Gronbach

PLENARY LECTURES

Differential Complexes and Stability of Finite Element Methods

Douglas N. Arnold

IMA, University of Minnesota

In this talk I will describe and illustrate a new approach which has added greatly to our understanding of stability of finite element methods and enabled the development of stable methods for previously intractable problems. This approach is homological, involving differential complexes related to the problem to be solved, discretizations of these complexes obtained by restricting the differential operators to piecewise polynomial subspaces, and commuting projections relating the two. The best known case is the de Rham complex, which underlies both electromagnetic and diffusion problems. In this case there are a large number of possible piecewise polynomial subcomplexes of each order. These can be presented systematically using the Koszul complex. The elasticity equations are related to another differential complex which can be related to the de Rham complex through a subtle homological construction, the discretization of which has led to new stable mixed methods for elasticity.

On Deciding Whether Accurate Linear Algebra Algorithms Exist

Jim Demmel, Ioana Dumitriu, Olga Holtz

Computer Science Division, University of California at Berkeley

Algorithms have recently been derived for highly accurate linear algebra calculations for a variety of structured matrices, i.e. matrices whose entries are defined by families of polynomials. By "highly accurate" we mean a small relative error is guaranteed in the final answer despite any small relative errors made in intermediate computations. Such algorithms have been found for some structured matrices (e.g. Cauchy, polynomial Vandermonde, etc.) but not all (e.g. Toeplitz). We describe progress towards a decision procedure that takes a polynomial (e.g. determinant) and a description of available arithmetic operations performed with 1 rounding error (addition, subtraction, multiplication, and possibly others), and decides whether an accurate algorithm exists.

Counting Points on Curves over Finite Fields

Jan Denef

Department of Mathematics, K.U.Leuven,

Celestijnenlaan 200B, B-3001 Leuven, Belgium

This talk is about fast algorithms to count the number of rational points on a curve C of genus $g \geq 1$ over a finite field $\mathbf{F}_q$ with q elements when $\log q$ is large, and to calculate the zeta function of C over $\mathbf{F}_q$ when $g \log q$ is large. This has important applications to cryptography when g is small, say $g \leq 4$ and $g \log q > 160$. The first significant result is due to Schoof (1985) who obtained an algorithm for elliptic curves with time complexity $O((\log q)^5)$. The last five years much faster algorithms were obtained when the characteristic of $\mathbf{F}_q$ is *small*, culminating for example in $O((\log q)^2)$ time complexity for elliptic curves (Harley) and time $O(g^4(\log q)^3)$ for hyperelliptic curves of arbitrary genus (with q odd) (Kedlaya). Our talk will explain the basic principles (p-adic methods) on which these algorithms are based (Satoh's method and Kedlaya's method) and recent joint work of Castryck, Vercauteren and Denef for curves with given Newton polygon and random coefficients. (In the above complexity estimates factors $\log \log q$ and $\log g$ are neglected for simplicity.)

Sparse Grids for Higher-Dimensional Partial Differential Equations

Michael Griebel

Instuitut für Numerische Simulation, Universität Bonn

The numerical treatment of high(er) dimensional problems suffers in general from the so-called curse of dimensionality. In special cases, i.e. for special function classes, this exponential dependence of $O(n^{-r/d})$ of the achieved accuracy on the invested work n can be substantially reduced. Here, r denotes smoothness and d dimensionality. This is e.g. the case for spaces of functions with bounded mixed derivatives. The associated numerical schemes involve a series expansion in a multiscale basis for the one-dimensional problem. Then, a product construction and a proper truncation of the resulting d-dimensional expansion result in a so-called sparse grid approximation which is closely related to hyperbolic crosses. Here, depending on the respective problem and the 1-dimensional multiscale basis used, a variety of algorithms for higher dimensional problems result which allow to break the curse of dimensionality, at least to some extent, and result in complexities of the order $O(n^{-r}(\log n)^{\alpha(d)})$. In special cases even $\alpha(d) = 0$ can be achieved. This is for example possible if the error is measured in the H^1 seminorm or if the different dimensions as well as their interactions are not equally important and dimension-adaptive strategies are used. The constant in these order estimates, however, is still dependent on d. It also reflects subtle details of the implementation of the respective numerical scheme. In general, the order constant grows exponentially with d. In some cases, however, it can be shown that it decays exponentially with d. This allows to treat quite high dimensional problems in moderate computing time.

We discuss such sparse grid algorithms for the numerical treatment of partial differential equations and related problems in higher dimensions for various applications.

Long-Time Energy Conservation of Numerical Integrators

Ernst Hairer

Section de mathématiques, Université de Genève, Switzerland

The numerical integration of Hamiltonian systems

$$\dot{p} = -\nabla_q H(p,q), \quad \dot{q} = \nabla_p H(p,q)$$

is an important topic in a variety of applications – classical mechanics, astronomy, and molecular dynamics. Especially for integrations over long time intervals it is of utmost importance to preserve various geometric properties of the exact flow, such as symplecticity or the conservation of first integrals and, in particular, of the Hamiltonian (total energy).

This talk is concerned with standard numerical integrators like (partitioned) Runge–Kutta methods or linear multistep methods. It focuses on the question which numerical integrators conserve sufficiently well the total energy of the system without any drift, and it discusses the statement

Hamiltonian system + *symplectic integrator* $\Rightarrow$ *energy conservation.*

In a first part of the talk a few numerical experiments are presented that show the necessity of additional assumptions for the correctness of this statement. It is well-known that the use of classical step size strategies lead to an energy drift, but also the widely used Störmer–Verlet scheme can fail to conserve energy in the presence of high oscillations.

The main part of the talk surveys some recent results, where the symplecticity assumption can be relaxed without destroying the good energy conservation.

- For integrable reversible Hamiltonian systems symmetric methods nearly conserve all action variables and in particular also the total energy.
- For the same class of problems, there exists a reversible step size strategy (integrating controller) which does not destroy the good long-time behaviour of constant step size implementations.
- Numerical integrators that are conjugate to a symplectic method will have the same long-time behaviour as symplectic methods. Certain linear multistep methods are shown to have this property.

References

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P. Chartier, E. Faou & A. Murua, *An algebraic approach to invariant preserving integrators: the case of quadratic and Hamiltonian invariants.* Preprint, February 2005.

E. Hairer & C. Lubich, *Symmetric multistep methods over long times.* Numer. Math. 97 (2004) 699–723.

E. Hairer & G. Söderlind, *Explicit, time reversible, adaptive step size control.* To appear in SIAM J. Sci. Comput. (2006).

Nonsmooth Optimization and Eigenvalues

Adrian Lewis

Cornell University

Variational analysis concerns the geometry and calculus of nonsmooth sets and functions, often viewed from an optimization perspective. Over several decades, variational analysis has matured into a powerful and elegant theory. One rich source of concrete examples involves eigenvalues, either of symmetric matrices, or, in dynamical systems and robust control applications, of nonsymmetric matrices. I will illustrate some central ideas of variational analysis, including convexity and duality, generalized gradients and sensitivity, Lipschitz behavior, and Clarke regularity, using examples from eigenvalue optimization: semidefinite programming, static output feedback, pseudospectra and distances to instability and uncontrollability.

Geometric Representations of Signals and Images

Stéphane Mallat

CMAP, Ecole Polytechnique

Geometric perception of images is a result of complex grouping processes studied in the 1920's by the Gestalt school, on which electro-physiological experiments begin to bring information. The geometry of textures and edges provide essential information for pattern recognition and yet are poorly understood. In video sequences, geometry is at the heart of movement perception. An underlined geometry is also present in the perception of sounds and the "movements" of harmonics and tones. Behind these different forms of "geometry" is there a common mathematical and algorithmic approach that would provide an effective representation of geometric information?

For signals and images, appropriate geometric representations are also required to improve the state of the art of compression, inverse problems and pattern recognition applications. This implies modifying harmonic analysis tools such as Fourier and wavelet transforms. Beginning from this point of view, I will review few approaches that illustrate the difficulties to represent and process the geometry of images, in relation with physiology. The lecture will then focus on hierarchical and multiscale grouping processes, which decompose images on orthogonal bases of "bandelets" having an adaptive geometry. Asymptotic approximation theorems as well as image and sound processing applications will be given.

Discrete Variational Methods

Elizabeth Mansfield

Institute of Mathematics, Statistics and Actuarial Science
University of Kent, Canterbury, CT2 7NF, United Kingdom
E.L.Mansfield@kent.ac.uk

Exact and approximate variational methods are important in the analysis of systems and their numerical models. For smooth systems there is a full *variational complex* (see for example [1]) which allows one to answer questions such as:

- is a given expression a conservation law, that is, is it a divergence and if so, of what is it the divergence?
- what are the specific formulae for Noether's Theorem, giving the conservation law for a given symmetry of the Lagrangian?

These questions are equally important for discrete systems, inherently discrete physical phenomena as well as discretisations. If you want a numerical approximation of a symmetric Lagrangian system to inherit particular conservation laws of the original, as an automatic consequence of how you set up your numerical model, then one way forward is to have a discrete Noether's theorem at your disposal.

Since many of the physically important symmetries involve smooth group actions on the base space, which gets discretized, the first problem to solve is how to obtain an appropriate group action that gives a matching conservation law. The second problem is what mathematical structures are needed to prove a discrete Noether's theorem.

In this talk I shall describe how the algebraic constructions which underpin the smooth variational complex can be matched for difference systems, and describe progress to date of a similar theory for finite element approximations. For difference systems, there is a complete theory which matches amazingly closely that of the smooth [2], despite the fact that there can be no "top down" analogue as there are no de Rham forms, nor contact forms, on an integer lattice. For approximations based on moments with respect to a triangulation of the base space, a variety of subtleties arise [3]. In particular, the set of moments used needs to fit into an "exact" scheme as described by D. Arnold [4].

Last but not least, once you know how the group actions are induced onto the approximation data, you need to know the invariants from which an invariant Lagrangian can be constructed. This can be achieved in a constructive, largely algorithmic fashion, using the theory of moving frames [5].

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[2] P.E. Hydon and E.L. Mansfield, A variational complex for difference equations, *Foundations of Computational Mathematics*, **4**, (2004) 87–217.

[3] E.L. Mansfield and G.R.W. Quispel, Towards a variational complex for the Finite Element Method, to appear in Proceedings, Workshop in Group Theory and Numerical Analysis, Centre de Reserches Mathématiques, Montréal, 2005.

[4] D. Arnold, Differential complexes and stability, Proceedings of the International Congress of Mathematicians, Beijing 2002, Volume I: Plenary Lectures
http://ima.umn.edu/~arnold/papers/icm2002.pdf

[5] M. Fels and P.J. Olver, Moving Coframes I, II *Acta Appl. Math.*, **51** (1998) 161-213; **55** (1999) 127–208.

Hyperbolic 3-Manifolds and Their Computational Aspects

G. Robert Meyerhoff

Math Dept, Boston College, Chestnut Hill, MA, 02467, USA

The volume function is the most natural invariant for hyperbolic 3-manifolds. By the work of Thurston, building on work of Jorgensen and Gromov, the set of volumes of hyperbolic 3-manifolds is well-ordered and of order type ω^ω. In particular, there is a least volume v_1 for hyperbolic 3-manifolds. The leading candidate for a hyperbolic 3-manifold realizing this least volume is the "Weeks Manifold", which is obtained by performing $(5,1),(5,2)$ Dehn surgery on the two components of the Whitehead Link, and has volume 0.942... . As of now, the best lower bound for v_1 is 0.3315, and this result is due to Przeworski, using work of Agol, Gabai-Meyerhoff-NThurston, and Cao-Meyerhoff.

It seems as if the question of finding the low-volume hyperbolic 3-manifold(s) is ripe. It is likely that rigorous computer analysis will play a crucial role in any such proof. In this talk we discuss two ways in which the computer is being used in studying volumes of hyperbolic 3-manifolds. First, combinatorial methods are important in the study of low-volume hyperbolic 3-manifolds and we focus on one such combinatorial argument that leads to a computer analysis. Second, parameter space arguments often arise in these studies and we present one as well.

The work described is joint with D. Gabai, and P. Milley.

Use of Computational Homology in Nonlinear Dynamics

Konstantin Mischaikow

School of Mathematics, Georgia Institute of Technology, Atlanta GA 30030

Much of the fascination and challenge of studying nonlinear dynamical systems arises from the complicated temporal and/or spatial behavior they exhibit. On the level of mathematics this complicated behavior can occur at all scales both in phase space and in parameter space. Somewhat paradoxically, from a scientific perspective, this points to the need for a coherent set of mathematical techniques that is capable of extracting coarse but robust information about the structure of these systems. Furthermore, most of our understanding of specific systems comes from experimental observation or numerical simulations, and thus it is important that these techniques can be applied in a computationally efficient manner. In this talk it will be argued that topological techniques, in particular computational homology, has an important role to play in this endeavor.

How Upper and Lower Complexity Bounds meet in Elimination Theory (Ten Years Later)

Luis M. Pardo

Depto. de Matemáticas, Estadística y Computación
Facultad de Ciencias-Universidad de Cantabria
Avda. Los Castros s/n. E-39071 SANTANDER, Spain.
luis.pardo@unican.es

The talk is viewed as a continuation ("ten years later") of a plenary talk given by the author in AAECC–11 (Paris, 1995). The talk will overview some central statements concerning the complexity of solving systems of multivariate polynomial equations. From the symbolic/universal approach to the numerical/non–universal approach. Some new results concerning the average behavior of the linear and non–linear condition number of singular data will be exhibited (cf.[Beltrán, Pardo, 2005a]). Recent results (that exhibit constructible (and finite) classes of "good initial systems" solving efficiently almost all polynomial equations by linear homotopic methods) will also be introduced (cf. [Beltrán, Pardo, 2005b]).

Numerical Integration in High Dimensions — the Lattice Side of the Story

Ian H. Sloan

University of New South Wales
Sydney, Australia

Numerical integration in high dimensions confronts us with the curse of dimensionality — the number of function values needed to obtain an acceptable approximation can grow exponentially in the number of dimensions d. The exponential increase is clearly inevitable with any form of product integration rule, and for many theoretical settings is now known to be unavoidable no matter how the integration rule is chosen.

It has been known since 1998 that the curse of dimensionality can in principle be overcome within the "weighted Sobolev space" setting introduced by Sloan and Wózniakowki, if the "weights" that describe the behaviour with respect to different variables satisfy a certain (necessary and sufficient) condition.

In that work it was shown that, under the appropriate condition on the weights, there exist integration rules for which the "worst-case error" is bounded independently of d. That 1998 result was non-constructive, giving no clue as to how we might construct "good" integration rules. More recently it has been shown that "good" rules can be found within the much smaller class of (shifted) lattice rules, and even more recently that good rules can be constructed one component at a time.

This talk will review these developments, from early existence proofs and non-constructive methods to recent super-fast constructions of good integration rules in hundreds of dimensions, that may use tens of thousands of sample points.

Algebraic Statistics for Computational Biology

Bernd Sturmfels

UC Berkeley

We discuss recent interactions between algebra and statistics and their emerging applications to computational biology. Statistical models of independence and alignments for DNA sequences will be illustrated by means of a fictional character, DiaNA, who rolls tetrahedral dice with face labels A, C, G, T. For a picture of DiaNA and an on-line version of our book on this subject see

`http://math.berkeley.edu/~lpachter/ascb_book/`

Computational Multiscale Modelling: Fokker–Planck Equations and Their Numerical Analysis

Endre Süli

University of Oxford

The talks surveys recent developments concerning the mathematical analysis and numerical approximation of second-order degenerate and nonselfadjoint partial differential equations. Problems of this kind occur in a number of physical applications where a macroscopic (continuum) model is coupled to a microscopic model comprising a system of stochastic ordinary differential equations.

I shall present new results concerning the existence of global weak solutions to a class of nonlinear partial differential equations that arise in macroscopic-microscopic models of diluted polymers. The model consists of the unsteady incompressible Navier–Stokes equations for the macroscopic variables coupled to a, possibly degenerate, Fokker–Planck equation for the evolution of the probability density function of the stochastic process that captures random variations in the geometry of polymer chains. Similar Fokker–Planck equations also occur as mesoscopic models in molecular biology, on truncating the Taylor expansion of the so-called *master equation* after second-order terms. I shall discuss the construction and the numerical analysis of finite element approximations to second-order degenerate nonselfadjoint partial differential equations, and will also address the computational challenges associated with the solution of high-dimensional Fokker–Planck equations.

Network Games and the Price of Anarchy or Stability

Éva Tardos

Computer Science Department
Cornell University
Upson Hall
Ithaca, New York 14853

In this talk we will consider settings where multiple agents each pursue their own selfish interests, each represented by his own objective function. Traditional algorithms design assumes that the problem is described by a single objective function, and the algorithm designer has the information and power to decide on the outcome. Our goal is to quantify the degradation of quality of solution caused by the selfish behavior of users, compared to a centrally designed optimum.

We will consider simple network games modeling routing, pricing, or network design. Networks that operate and evolve through interactions of large numbers of participants play a fundamental role in many domains, ranging from communication networks to social networks. They can also naturally model the behavior of many physical systems, such as electricity in electric networks, and the distribution of forces in mechanical structures.

Greedy Approximations

V.N. Temlyakov

University of South Carolina, Columbia, SC 29208, USA

The idea of replacing a complex object (target function) by a simpler one (approximant) is widely spread and successfully used in many areas of science including computational mathematics. A specific feature of contemporary theoretical and practical problems is huge and unstructured data sets, which cannot be handled without replacing them by simpler objects. One more new feature, which is important for our motivation, is that now we cannot limit ourselves to the use of well organized and well structured approximation tools. For example, in signal processing the most popular means of approximation are wavelets and the system of Gabor functions. The Gabor system gives more flexibility in constructing an approximant but it is not an orthogonal system. Moreover, it is a redundant (not minimal) system. Thus, in order to address the contemporary needs of computational mathematics we should consider a very general model. As such a model we choose a Banach space X with elements as target functions and an arbitrary system $\mathcal{D}$ of elements of this space as an approximating system. Clearly, in such generality this setting may cover more or less everything. However, in order to obtain meaningful results on approximation we need to impose some restrictions on the Banach space and on the approximating system. The next question is how should an approximant look? The present dominating approach is to form an approximant as a linear combination of m terms from a given system $\mathcal{D}$. Such an approximant is called an m-term approximant with regard (with respect) to $\mathcal{D}$. Introducing the concept of best m-term approximation

$$\sigma_m(f,\mathcal{D}) := \inf_{g_j\in\mathcal{D},c_j;j=1,\dots,m} \|f - \sum_{j=1}^{m} c_j g_j\|, \tag{1}$$

we obtain the lower bound for accuracy of any method providing m-term approximation. The fundamental problem is how to construct a good m-term approximant. It is known that a problem of simultaneous optimization over many parameters (like in (1)) is a very difficult problem. Let us also note that even if we managed to solve (1) this solution has the following disadvantage. The optimal elements, say, $g_1^m(f),\dots,g_m^m(f)$ may depend on m and, therefore, when we go from m to $m+1$ we need to recalculate all $m+1$ elements.

We would like to have a method (an algorithm) of constructing m-term approximants that adds at each step only one new element from $\mathcal{D}$ and keeps elements of $\mathcal{D}$ obtained at the previous steps. Clearly, we are looking for good methods which at a minimum converge for each target function. It turned out that there is one fundamental principal that allows us to build good methods both for arbitrary redundant systems and for very simple well structured bases like the Haar basis. This principal is the use of a greedy step in searching for a new element to be added

to a given m-term approximant. The common feature of all methods of m-term approximation discussed in this talk is the presence of a greedy step. By a greedy step in choosing an mth element $g_m(f) \in \mathcal{D}$ to be used in an m-term approximant, we mean one which maximizes a certain functional determined by information from the previous steps of the algorithm. We obtain different types of greedy algorithms by varying the above mentioned functional and also by using different ways of constructing (choosing coefficients of the linear combination) the m-term approximant from already found m elements of the dictionary.

Smoothed Analysis of Algorithms and Heuristics

Shang-Hua Teng

Boston University/Akamai Technologies Inc.

The theorists have long been challenged by the existence of remarkable Algorithms and heuristics that are known by scientists and engineers to work well in practice, but whose theoretical analyses have been are negative or unconvincing. The root of the problem is that algorithms are usually analyzed in one of two ways: by worst-case or average-case analysis. The former can improperly suggest that an algorithm will perform poorly, while the latter can be unconvincing because the random inputs it considers may fail to resemble those encountered in practice.

We introduce smoothed analysis to help explain the success of some of these algorithms and heuristics. Smoothed analysis is a hybrid of worst-case and average-case analyses that inherits advantages of both. The smoothed complexity of an algorithm is the maximum over its inputs of the expected running time of the algorithm under slight random perturbations of that input, measured as a function of both the input length and the magnitude of the perturbations. If an algorithm has low smoothed complexity, then it should perform well on most inputs in every neighborhood of inputs.

In this talk, we will explain how smoothed analysis can help explain the excellent observed behavior of the simplex method, Gaussian elimination, interior point methods, and some other heuristics.

This is joint work with Daniel Spielman, Arvind Sankar, and John Dunagan.

Propagation, Dispersion, Control and Numerical Approximation of Waves

Enrique Zuazua

Departamento de Matemáticas, Universidad Autónoma, 28049 Madrid. Spain

In this lecture we shall discuss several topics related with numerical approximation of waves.

Control Theory is by now and old subject, ubiquitous in many areas of Science and Technology. There is a quite well-established finite-dimensional theory and many progresses have been done also in the context of PDE (Partial Differential Equations). But gluing these two pieces together is often a hard task from a mathematical point of view.

This is not a merely mathematical problem since it affects modelling and computational issues. In particular, the following two questions arise: Are finite-dimensional and infinite-dimensional models equally efficient from a control theoretical point of view? Are controls built for finite-dimensional numerical schemes efficient at the continuous level?

In this talk we shall briefly analyze these issues for the wave equation as a model example of propagation without damping. We shall show that high frequency spurious oscillations may produce the divergence of the most natural numerical schemes. This confirms the fact that finite and infinite-dimensional modelling may give completely different results from the point of view of control. We shall then discuss some remedies like filtering of high frequencies, multi-grid techniques and numerical viscosity.

Similar questions arise when building numerical approximation schemes for nonlinear Schrödinger equations.

We first consider finite-difference space semi-discretizations and show that the standard conservative scheme does not reproduce at the discrete level the properties of the continuous Schrödinger equation. This is due to high frequency numerical spurious solutions. In order to damp out these high-frequencies and to reflect the properties of the continuous problem we add a suitable extra numerical viscosity term at a convenient scale. We prove that the dispersive properties of this viscous scheme are uniform when the mesh-size tends to zero. Finally we prove the convergence of this viscous numerical scheme for a class of nonlinear Schrödinger equations with nonlinearities that may not be handled by standard energy methods and that require the so-called Strichartz inequalities. Finally, we show that similar convergence results may be obtained by a two-grid algorithm based on the idea of resolving on a fine grid slow oscillations of the initial data and nonlinearity. This is a joint work with Liviu Ignat.

WORKSHOP 1

Foundations of numerical PDEs

ORGANISERS:

Christoph Schwab & Eitan Tadmor

MHD Equations, the div(B) Constraint, and Central Schemes

Jorge Balbas

Department of Mathematics at University of Michigan

Simulations of Magnetohydrodynamical (MHD) flows pose computational challenges that extend beyond the development of discontinuous solutions typical of non-linear hyperbolic systems. In particular, an additional constraint on the magnetic field, $\mathrm{div}(B) = 0$, needs to be satisfied to guarantee the stability of the numerical schemes. We will discuss some of these challenges, the techniques commonly employed to overcome them, and the capability of central schemes to handle them in a rather simple manner. To this end, we present computational results that indicate a remarkable ability of certain "black box" type central schemes to address these difficulties.

Mimetic Finite Differences Methods for Diffusion Problems

Franco Brezzi

Dipartimento di Matematica at Università di Pavia

The talk will overview the basic features of Mimetic Finite Difference Methods, taking as a model problem Darcy's law for filtration in porous media. After a rather general presentation, we will concentrate on the basic ideas that allowed the extension of the method to a very wide class of geometrical situations, including polyhedra with curved faces. This last part will be based on several papers written in collaboration with K. Lipnikov and M. Shashkov from Los Alamos National Laboratory and V. Simoncini from the University of Bologna.

On Constraint Preservation in Finite Element Discretizations of Yang-Mills Equations

Snorre Christiansen

Centre of Mathematics for Applications at University of Oslo

Yang-Mills equations, in their hyperbolic form, are nonlinear wave equations generalizing those of Maxwell. They preserve a nonlinear differential constraint on the initial data, similar to electric charge. Difficulties in preserving such constraints has been perceived to be at the center of numerical instabilities observed in discretizing other evolution equations, such as Einstein's equations of general relativity. We discuss this problem, for finite element discretizations of the Yang-Mills equations, comparing with recent results obtained for the linear (Maxwell) case.

Two-sided Generalized Riemann Solvers: Advantages and Shortcomings

Rosa Donat

Department of Applied Mathematics at Universitat de València

Many modern shock capturing schemes can be classified as characteristic based schemes. These rely on a local diagonalization of the hyperbolic system that involves the Jacobian matrix of the flux function at each cell interface. It is now relatively well known that the interface state used for the computation of the Jacobian can have a significant effect on the fine features of the numerical solution [1]. A new numerical flux function is developed in [2] that takes into account the two physically relevant states at the interface, without ever constructing an artificially mixed interface state. This new flux function reduces to the well known Shu-Osher scheme [3] for scalar conservation laws, but becomes fundamentally different in the case of systems. The resulting numerical scheme can be combined with the ENO-reconstruction technology to produce state of the art HRSC schemes that have been applied in various scenarios. This talk will review recent results that derive from the use of these new schemes, with special emphasis in the advantages and shortcomings found in the simulations.

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Central-Upwind Schemes for Balance Laws. Applications to Multifluid and Multiphase Computations

Alexander Kurganov

Mathematics Department at Tulane University

First I will briefly describe Godunov-type central-upwind schemes for hyperbolic systems of conservation and balance laws. These schemes are simple, universal and, at the same time, high-resolution methods that can be applied as a 'black-box' solver to many different problems. However, applications to balance laws typically require a special source term discretization, which should guarantee (a perfect) balance between the flux and the source terms. This will be illustrated on the example of the Saint Venant system of shallow water equations with nonflat bottom topography. Then I will focus on two more complicated applications: to compressible two-phase flows and to multi-layer shallow water equations, for which a treatment of nonconservative products seems to be the most challenging part in designing a reliable numerical method.

Some Questions in Computational Chemistry

Claude Le Bris

CERMICS at École Nationale des Ponts et Chaussées

We will make an overview of various techniques used in computational chemistry for the calculations of electronic structure. Such calculations require the solution of some nonlinear eigenvalue problems. In particular, advanced techniques for large systems will be addressed.

High-resolution Finite Volume Methods and Approximate Riemann Solvers for Hyperbolic Systems

Randall LeVeque

Department of Applied Mathematics at University of Washington

High-resolution finite volume methods for hyperbolic systems of PDEs are often based on Godunov's method, with the addition of second-order correction terms that are limited in some manner to avoid nonphysical oscillations and sharpen the resolution of steep gradients or discontinuities such as shock waves. This is often coupled with upwinding based on a characteristic decomposition or the solution of the Riemann problem between the neighboring states. A variety of slope limiting or flux limiting methods have been proposed and successfully used. The wave-propagation method is a general formulation of such methods that limits the waves resulting from the Riemann solution and uses the resulting waves to define the high-resolution correction terms. This approach has allowed the development of general-purpose software that applies to a wide range of hyperbolic problems, including problems that are not in conservation form and to quasi-steady balance laws where the flux gradient is nearly balanced by a source term. In practice approximate Riemann solvers are often used, ranging from very simple approximations such as the HLL solver to more sophisticated and expensive solvers that better mimic important aspects of the exact solution. Several of these can be unified by interpreting them in the framework of "relaxation Riemann solvers" that are related to relaxation schemes for hyperbolic systems.

Mesh Adaptivity and Artificial Diffusion in Hyperbolic Problems

Charalambos Makridakis

Department of Applied Mathematics at University of Crete

We consider finite element, finite difference schemes and adaptive strategies for the approximation of nonlinear hyperbolic systems of conservation laws. It is known that finite elements are not a very popular choice for computing singular solutions of hyperbolic problems. When applied directly to the system they will result computational solutions with oscillatory character close to shocks and/or not convergent approximations. This well known phenomenon is of course related to the fact that direct finite element discretizations behave like dispersion approximations. Similar behavior is observed in the study of related dispersive difference schemes approximating conservation laws.

To overcome this difficulty in using standard schemes, several modifications have been proposed in the literature by adding artificial viscosity and / or extra stabilization terms in the schemes. The higher order versions of these methods are complicated and with poor theoretical backup. Recently many of these schemes have been tested with various mesh adaptation methods. Our motivation was to consider schemes designed to be used in conjunction with appropriate mesh refinement. We will show that mesh refinement strategies can change our view on the application of many of the known schemes. This is because the mesh distribution influences not only the accuracy of the scheme but also its stability behavior.

In this talk we (1) consider new finite element schemes for HCL designed to be used with mesh adaptivity. (2) discuss adaptive strategies for shock computations based on estimator functionals or a posteriori error control. (3) conclude to new and rather unexpected observations for the behavior of Entropy Conservative difference schemes. Classical dispersive-type finite element and finite difference schemes are also considered. We then try to explain why many of these schemes when used in conjunction with appropriate adaptive strategies yield computational solutions with surprisingly stable behavior.

We conclude that the qualitative effect of mesh adaptivity is the responsible mechanism and not the 'resolution' of boundary layers/shocks. These conclusions are also applicable to convection - diffusion problems where similar numerical problems occur.

Boundary Concentrated FEM

Jens Melenk

Mathematics at University of Reading

It is well-known for elliptic problems with smooth coefficients that the solution is smooth in the interior of the domain; low regularity is only possible near the boundary. The hp-version of the finite element method (hp-FEM) with variable order polynomial degree distribution allows us to exploit this observation to get optimal (in the sense of n-widths) approximation methods by using meshes where the element size grows proportionally to the element's distance to the boundary and the approximation order is suitably linked to the element size. In this way most degrees of freedom are concentrated near the boundary, and whence comes the name of this variant of the hp-FEM.

A focus of this talk, will be (near) optimal solution techniques for the arising linear system. The first approach we discuss is based on multilevel techniques for this variant of the hp-FEM. The second approach considered is based on the concept of H-matrices, recently introduced by W. Hackbusch.

The boundary concentrated FEM is variant of the hp-FEM; many of the implementation issues discussed for it apply in fact to the hp-FEM in general.

Finite Element Methods for Moving Surfaces and Applications to Stressed Epitaxial Films, Shape Optimization, and Image Processing

Ricardo Nochetto

Mathematics and IPST at University of Maryland

Using shape differential calculus, which expresses variations of bulk and surface energy with respect to domain changes, and Euler implicit time discretization, we formulate gradient flows for these energies which yield geometric laws for the motion of domain boundaries (curves or surfaces). We next present a semi-implicit variational formulation which requires no explicit parametrization of the surface, and is sufficiently flexible to accommodate several scalar products for the computation of normal velocity, depending on the application. This leads to linear systems of lower order elliptic PDE to solve at each time step, in both the surface and bulk. We develop adaptive finite element methods (AFEM), and propose a Schur complement approach to solve the resulting linear SPD systems.

We first apply this idea to surface diffusion, namely to the geometric motion of a surface with normal velocity proportional to the surface Laplacian of mean curvature, and couple it with elasticity in the bulk; this is a simple model for stressed epitaxial films. We present several numerical experiments for surface diffusion including pinch-off in finite time and topological changes. We also present preliminary computations for the coupled system exhibiting formation of dislocations. We next discuss applications to shape optimization and image processing, that is to the minimization of functionals subject to differential constraints, and present preliminary simulations. We discuss time and space adaptivity to handle the multiscale nature of these problems, as well as mesh generation, mesh distortion and mesh smoothing.

This work is joint with E. Baensch, G. Dogan, P. Morin, and M. Verani.

Bregman Iteration and the Dual of BV in Inverse Problems in Imaging and elsewhere

Stanley Osher

Department of Mathematics at University of California, Los Angeles

We shall review some new results obtained with: M. Burger, S. Kindermann, D. Goldfarb, O. Scherzer, W. Yin and J.J. Xu in image processing and general inverse problems. State-of-the-art image restoration methods result from easy to explain nonlinear optimization and functional analysis considerations.

Recent Advances in Computational Ferromagnetism

Andreas Prohl

Seminar für Angewandte Mathematik at ETH Zürich

Micromagnetics is a continuum variational theory describing magnetization pattern in ferromagnetic media. Its multiscale nature due to different inherent spatio-temporal physical and geometric scales, together with nonlocal phenomena and a nonconvex side-constraint leads to a rich behavior and pattern formation. This variety of effects is also the reason for severe problems in constructing reliable numerical methods, which are reviewed in this talk.

High Order Finite Volume Schemes for Balance Laws with Stiff Source

Giovanni Russo

Dipartimento di Matematica ed Informatica at Università di Catania

Finite difference methods are usually preferred over their finite volume counterparts for the development of high order schemes for the numerical solution of hyperbolic systems of balance laws with a stiff source. The unstaggered version of such schemes is usually developed by a method-of-line approach: the derivative of the flux is discretized in a conservative way, using some high order nonlinear reconstruction on the flux, to prevent spurious oscillations, and the system of PDE's is reduced to a large system of ODE's, which contains a non stiff part (the flux term) and a stiff part (the relaxation). At this point, the system of ODE's can be solved by some Implicit-Explicit (IMEX) scheme, in which the source term is treated implicitly, while the flux term is treated explicitly. The advantage of finite difference is that the source term does not couple the cells, and therefore the implicit step can be efficiently solved, treating each cell separately. The same procedure cannot be applied with a finite volume scheme, since the cell average of the source is not equal to the source evaluated at the cell average, and the source term couples the cells. In our finite volume methods, a different approach to time discretization, called Central Runge Kutta (CRK), treats the numerical solution (cell average) with a conservative scheme for the flux, while the stage values at the edge of each cell (pointwise) are treated by a non conservative scheme, in which the source term is decoupled from the other cells, and therefore implicit schemes can be effectively used for the source. During the talk the development of finite difference on staggered grids will be also discussed, and high order finite volume methods for stiff balance laws will be illustrated both in the staggered and unstaggered version.

Moving Mesh Methods for Computational Fluid Dynamics

Tao Tang

Department of Mathematics at Hong Kong Baptist University

This talk will describe some recent developments on moving mesh methods. In particular, we review their applications to computational fluid dynamics. The following website contains some materials related to this talk:
`http://www.math.hkbu.edu.hk/~ttang/MMmovie`

Locking-free Finite Element Methods Based on the Hu-Washizu Formulation for Linear and Geometrically Nonlinear Elasticity

Barbara Wohlmuth

Institut für Angewandte Analysis und Numerische Simulation at Universität Stuttgart

It is well-known that the low-order elements based on four-noded quadrilaterals or eight-noded hexahedra have two drawbacks in finite element computation. The first one is the locking effect in nearly incompressible case; in other words, they do not converge uniformly with respect to the Lamé parameter λ. The second one is that these standard elements lead to poor accuracy in bending-dominated problems when coarse meshes are used. In the first part of the talk, we examine the classical Hu-Washizu mixed formulation for plane and three dimensional problems in linear elasticity with the emphasis on behavior in the incompressible limit. The classical continuous problem is embedded in a family of Hu-Washizu problems parameterized by a scalar α for which $\alpha = \lambda/\mu$ corresponds to the classical formulation, with λ and μ being the Lamé parameters. We discuss the uniform well-posedness in the incompressible limit of the continuous problem for $\alpha = -1$. Finite element approximations are based on the choice of piecewise bilinear or trilinear approximations for the displacements on quadrilateral or hexahedral meshes. Conditions for uniform convergence are made explicit. These conditions are shown to be met by particular choices of bases for stresses and strains that include well-known bases as well as newly constructed ones. Though a discrete version of the spherical part of the stress exhibits checkerboard modes, we establish a λ-independent optimal a priori error estimates for the displacement and for the postprocessed stress. Furthermore, starting from a suitable three-field formulation we introduce a two-field mixed formulation for geometrically nonlinear elasticity with Saint-Venant Kirchhoff law. In the second part, we demonstrate the performance of our approach through different numerical examples.

WORKSHOP 2

Geometric integration and computational mechanics

ORGANISERS:

Begoña Cano, Debra Lewis & Brynjulf Owren

Some questions about symplectic and multi-symplectic discretizations

Thomas J. Bridges

Department of Mathematics and Statistics, University of Surrey, Guildford, Surrey GU2 7XH UK

The talk will discuss two questions in geometric integration. The first question is: what is the appropriate geometric integrator for simulating overturning and breaking water waves? The widely used governing equations for modelling water waves are Hamiltonian, and therefore one would expect that symplectic integrators would be appropriate for time integration. When the surface is a graph, the symplectic structure is canonical. But for general surfaces, for example breaking waves, one needs a coordinate-free Hamiltonian formulation, and this was first proposed by Benjamin & Olver (1982). This Hamiltonian structure is no longer canonical, and new ideas from geometric integration are needed. Mathematically, the problem reduces to tracking a two-dimensional surface in 3−space, and has similarities with curvature-driven surface flow.

The second question is: how does one combine multi-symplectic structures on a manifold with "difference forms" or "discrete differential forms"? It has recently been shown that on any smooth Riemannian manifold, there is a "canonical multi-symplectic structure" on the total exterior algebra bundle. This structure gives a coordinate-free formulation of a large class of nonlinear elliptic PDEs (or hyperbolic PDEs if the manifold is pseudo-Riemannian). The canonical multi-symplectic structure is characterized in terms of differential forms on the *base manifold*, rather than the fibre. Therefore a natural way to discretize this structure is to use the theory of discrete differential forms or difference forms. The talk will discuss progress with developing this idea.

Conserved quantities of some Hamiltonian wave equations after full discretization

B. Cano

Departamento de Matemática Aplicada, Universidad de Valladolid, C/Doctor Mergelina, s/n, 47011, Valladolid, Spain

Hamiltonian PDEs have some invariant quantities, which would be good to conserve with the numerical integration. In this talk, we concentrate on the nonlinear wave and Schröedinger equations. Under hypotheses of regularity and periodicity, we study how a symmetric space discretization makes that the space discretized system also has some invariants or 'nearly' invariants which well approximate the continuous ones. The good approximation of them after time integration is studied under some assumptions when using symplectic Runge-Kutta methods or symmetric linear multistep methods for second-order systems.

Explicit Magnus expansions for nonlinear equations

Fernando Casas

Departament de Matemàtiques, Universitat Jaume I, 12071 Castellón, Spain

Nowadays the Magnus expansion constitutes a widespread tool to construct approximate solutions of time dependent systems of linear differential equations. From a numerical point of view, the resulting algorithms have proved to be competitive with other, more conventional numerical schemes with respect to accuracy and computational effort, whilst preserving the geometric structure of the original system.
In this talk we develop and analyse new explicit Magnus expansions for the nonlinear equation

$$Y' = A(t, Y)Y, \qquad Y(0) = Y_0 \in \mathcal{G},$$

where $\mathcal{G}$ is a matrix Lie group. In particular, integration methods up to order four are presented in terms of integrals which can be either exactly evaluated or replaced by conveniently adapted quadrature rules. The structure of the algorithm allows to use to change the step size and even the order along the integration process, thus improving its efficiency. Several examples are considered, including isospectral flows and highly-oscillatory nonlinear differential equations.
This is a joint work with Arieh Iserles.

Eulerian and Semi-Lagrangian exponential integrators for convection dominated problems

E. Celledoni

Department of Mathematical Sciences, NTNU, N-7491 Trondheim, Norway

Exponential integrators have been proposed and studied in the literature especially when applied to parabolic PDEs which in their semidiscretized form present a dominating linear stiff part and a nonlinear non-stiff part. Typically in the exponential integrator the exact integration of the linear stiff part is performed.

We propose here a new class of integration methods particularly suited for convection diffusion problems with dominating convection. These methods are exponential integrators and their peculiarity is that they allow for the computation of exponentials of the linearized convection term.

The main reason for developing this type of methods is that as it turns out they can be applied to the numerical integration of the considered PDEs in a semi-Lagrangian fashion. The main challenge in the numerical approximation of convection dominated phenomena is to avoid the occurrence of spurious oscillations in the numerical solution, (numerical dispersion), without adding diffusion. This task is achieved nicely by semi-Lagrangian methods. In these methods linear convective terms are integrated *exactly* by computing first the characteristics corresponding to the gridpoints of the adopted discretization, and then producing the numerical approximation via a simple although expensive interpolation procedure.

Some results on numerical propagation when integrating Hamiltonian relative periodic orbits

Angel Durán

Departamento de Matemática Aplicada, Facultad de Ciencias, Universidad de Valladolid, P/ Prado de la Magdalena s/n, 47005 Valladolid, Spain

Many Hamiltonian systems that appear in physical applications (such as rigid bodies, N-body problems or molecular problems) possess a supplementary structure provided by a symmetry groups.

One of the points in which the Hamiltonian system is affected by the symmetry group concerns to the analysis of relative equilibrium solutions and relative periodic solutions. They respectively project to equilibria and periodic solutions of the Hamiltonian reduced by the symmetry group. Some theoretical works have analyzed existence and stability of this kind of solutions [4,5,6].

In a numerical sense, it is interesting to study the influence of the symmetries when approximating to these solutions or to small perturbations. In [2,1] some results about error growth in the case of relative equilibrium integration have been obtained, within the context of geometric integration [3]. In this talk, we make a first approach to the numerical analysis of relative periodic orbits. We try to study the structure of the error in order to obtain some conclusions about the behaviour of the integrators.

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Dispersion properties of conservative discretizations for wave equations

Jason Frank

CWI, P.O. Box 94079, 1090 GB Amsterdam, Netherlands

The dispersion properties of multisymplectic box schemes are contrasted with those of more traditional symmetric finite difference methods. We show that MS box schemes (and more generally symmetric Runge-Kutta box schemes) preserve the sign of group velocity, implying that energy always flows in the right direction. This has important consequences for adaptivity, since grid nonuniformities inevitably lead to an exchange of energy between characteristic modes in finite difference methods, and this is responsible for the generation of numerical reflections. We show that all symmetric RK box schemes are reflection-free, whereas only some cell-centered FD schemes are, and all such methods are necessarily implicit.

This is joint work with Brian Moore and Sebastian Reich.

On an isospectral Lie–Poisson flow and numerical computation of faithful Lie-algebra representations

A. Iserles

Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences, University of Cambridge, Wilberforce Rd, Cambridge CB3 0WA,UK.

Our point of departure is the matrix flow $X' = [N, X^2]$, $X(0) \in \mathrm{Sym}(n)$, $N \in \mathfrak{so}(n)$, which is subject to two distinct group actions, by similarity and by congruence. We demonstrate that it is isomorphic to an integrable Lie–Poisson flow and explore its invariants and Casimirs.

As is well known in geometric integration, there are important advantages in solving Lie–Poisson systems in the dual to underlying Lie algebra. However, in our case the Lie algebra is *a priori* unknown: we are given just its structure constants. Hence the task of determining a faithful representation: essentially, rendering the Ado theorem constructively in our special case. Although there exists a general symbolic algorithm for the computation of faithful representations, due to Willem de Graaf, it leads in our case to excessively large matrices which render computation in the dual very expensive indeed. Instead, we treat the task in hand as a problem in numerical linear algebra and present a very effective algorithm based on complex-valued Cholesky factorization.

Using a different approach, we demonstrate that the underlying Lie algebra is isomorphic to the symplectic algebra.

This work has been done in collaboration with Tony Bloch, Jerry Marsden and Tudor Ratiu.

Long time averaging for molecular dynamics simulations

Frédéric Legoll

IMA, University of Minnesota, Minneapolis and CERMICS-ENPC, Paris

Many properties of chemical systems (such as the pressure inside a liquid, or radial distributions) are defined as phase space averages of functions depending on the state of the system. A common way to compute these averages is to use Molecular Dynamics and to compute time averages on long trajectories.

When considering systems at constant temperature, the problem amounts to finding a dynamics which is ergodic for the canonical (or Gibbs) measure. Many different methods have been proposed in this vein, some of them based on deterministic dynamics (Hamiltonian or not), some of them based on stochastic differential equations (such as the Langevin equation). We will review theoretical properties of some of these methods and compare their numerical efficiency on some simple examples.

New Time Reversible and Volume Preserving Multiple Scale Integrators

B. Leimkuhler

Department of Mathematics, University of Leicester, University Road, Leicester, LE1 7RH, UK.

For Hamiltonian systems, it is well established that symplectic methods have desirable long term stability properties, and methods within this class are now in widespread use for applications in molecular dynamics and astronomical simulation. Symplecticness can be a restrictive requirement in designing integrators for systems with components that evolve on a wide range of spatial and time scales. While less well established from a theoretical perspective, time-reversible/volume preserving methods appear to offer a practical, more flexible alternative to symplectic methods in some cases. In this talk I will describe a new approach to designing time-reversible volume preserving multiscale integrators.

This talk describes joint work with Zhidong Jia.

Lie Group and Elementary Differential Variational Integrators with Applications to Full Body Problems

Melvin Leok

Department of Mathematics, University of Michigan
2074 East Hall, 530 Church Street
Ann Arbor, MI 48109-1043, USA.
mleok@umich.edu http://www.math.lsa.umich.edu/~mleok/

The study of attitude dynamics and control has been motivated by applications in robotics, space flight systems, ground-based pointing and tracking systems. In spite of the extensive literature on these problems, there has been little attention in the engineering literature given to efficient and accurate computational approaches that respect the geometry underlying these problems.

We introduce a variational integrator that performs computations at the level of the Lie algebra, and updates the solution using the matrix exponential. Consequently, the attitude automatically evolves on the rotation group embedded in the space of matrices, without the need for reprojection onto the manifold, or the use of constraints.

We will also discuss the construction of elementary differential variational integrators, which are obtained by considering discrete Lagrangians arising from quadrature rules that incorporate derivative information. These schemes can be interpreted as generalizations of symplectic Runge–Kutta methods, which utilize derivatives of the Hamiltonian vector field in propagating the numerical solution. This provides a general framework for constructing variational integrators that overcome the barrier order associated with traditional Runge–Kutta schemes, and provides a systematic method of constructing symplectic methods with s stages, but with order of accuracy greater than $2s$. This work is in collaboration with Taeyoung Lee and N. Harris McClamroch.

Partial connections and geometric integration

Debra Lewis[a], Nilima Nigam[b], and Peter Olver[c]

[a]Institute for Mathematics and Its Applications, University of Minnesota, MN 55455, USA, and Mathematics Department, University of California, Santa Cruz, Santa Cruz, CA 95064, USA

[b]Department of Mathematics and Statistics, McGill University, Montreal, H3A 2K6, Canada

[c]Department of Mathematics, University of Minnesota, MN 55455, USA

A partial connection associated to a group action on a manifold is an assignment of a complement to the tangent space to the group orbit at each point that is 'as smooth and as equivariant as possible'. Partial connections possess many of the essential features of connections on principal bundles, but can be defined not only on manifolds with nonfree actions, but on manifolds with multiple orbit types. Partial connection forms can be combined with Lie group numerical methods, yielding schemes for the solution of IVPs on manifolds with general group actions. In the presence of sufficient isotropy, partial connections determining conservative schemes can be constructed.

Fourier analysis on groups applied to spectral element discretizations of PDEs

H. Munthe-Kaas

Department of Informatics, University of Bergen, N-5020 Bergen, Norway

In this talk we will discuss some applications of Fourier analysis on groups in the solution of PDEs. Equivariant discretizations yields structured matrices that can be block diagonalized using the Generalized Fourier Transform (GFT). An example being equivariant discretisations for PDEs on spheres, where the use of GFT over the icosahedral symmetry group yields fast algorithms for computing matrix exponentials, which is a basic computation for Lie group integrators.

This leads to the important question of how to construct spectral element discretizations based on triangular subdivisions. By taking the quotient of a compact abelian group with a finite group of automorphisms, we obtain families of multivariate generalizations of Chebyshev polynomials on certain non-separable domains. This yields an approximation theory for triangular domains suitable for the construction of spectral element bases. We will show that these bases share most of the beautiful properties of classical Chebyshev expansions, such as near-optimal Lebesgue constants for the interpolation error and the existence of fast transforms for expansions and spectral differentiation.

An algebraic approach to conservation of first integrals in numerical integration

Ander Murua

Informatika Fakultatea, KZAA saila, Donostia-San Sebastian, Spain

We present an algebraic approach to the exact preservation of first integrals in terms of the series expansions of the numerical integrators. We also explore the existence of modified first integrals, which explains the near preservation of first integrals observed in some cases. We first consider results obtained for B-series and P-series methods (a joint work with Philippe Chartier and Erwan Faou), and then show how to extended our approach to very general classes of numerical integrators.

Riemannian Geometry of Neural Networks for Unsupervised Learning

Yasunori Nishimori

Neuroscience Research Institute
National Institute of Advanced Industrial Science and Technology (AIST),
AIST Tsukuba Central 2, 1-1-1, Umezono, Tsukuba, Ibaraki 305-8568, Japan
e-mail: y.nishimori@aist.go.jp

Most learning machines assume a fixed structure plus modifiable learning parameters. Information geometry deals with a geometric object arising from a set of learning machines with a fixed structure. For instance, a set of neural networks for unsupervised learning often forms a manifold such as the Stiefel, the Grassmann, or more general flag manifold. Thus learning with those neural networks is regarded as an optimization problem on such manifolds. To solve this, we present Riemannian optimization methods on $O(n)$-homogeneous spaces utilizing quasi-geodesics. Applications include minor component analysis, independent component analysis, and independent subspace analysis. This is joint work with Shotaro Akaho.

On Geometric Integrators for the Nonlinear Schrödinger equation

B. Owren

Department of Mathematical Sciences, NTNU, N-7491 Trondheim, Norway

The nonlinear Schrödinger equation

$$iu_t + u_{xx} = \lambda |u|^2 u$$

with periodic boundary conditions is considered. A number of geometric integrators have been developed for this system, many of them are summarized in [Islas et al., Geometric Integrators for the nonlinear Schrödinger equation, J. Comp. Phys. 173 (2001)]. One may use a multisymplectic formulation and use multisymplectic discretisations as e.g. the box schemes. Or one may use various time discretisations obtained from the Ablowitz-Ladik system which is a completely integrable ODE system. These integrators seem to work well for a moderate number of spatial degrees of freedom, but in the limit, several difficulties occur. There are nonlinear systems to be solved in each time step, and as their size grows, the convergence deteriorates substantially. Unless very strong convergence criteria are imposed, one typically observes numerical resonance for moderate values of the time step. More recently, a conservative scheme has been developed in [Besse, A relaxation scheme for the nonlinear Schrödinger equation. SIAM J. Numer. Anal. 42 (2004). This scheme is formulated and analysed for the abstract Cauchy problem, where no spatial discretisation is introduced. The author shows that the numerical scheme is well-posed in H^s for sufficiently large s. But also this method seems to experience difficulties when the scheme is implemented with a a large number of spatial points.

The Discrete Moser-Veselov Algorithm for the Free Rigid Body, Revisited

A. Zanna

Department of Informatics, University of Bergen, N-5020 Bergen, Norway

We revisit the Moser-Veselov description of the free Rigid Body in body coordinates, which, in the $3x3$ case, can be implemented as an explicit, second order, integrable approximation of the continuous solution. By backward error analysis, we study the modified vector field which is integrated exactly by the discrete algorithm. We deduce that the discrete Moser-Veselov (DMV) is well approximated to higher order by time-reparametrizations of the continuous equations (modified vector field). We use the modified vector field to scale the initial data of the DMV to improve the order of the approximation and show the equivalence of the DMV and the RATTLE algorithm. Numerical integration with these preprocessed initial data is several orders of magnitude more accurate of the original DMV and RATTLE approach.

This talk is based on a joint work with Robert MacLachlan

WORKSHOP 3

Information-based complexity

ORGANISERS:

Leszek Plaskota & Ian Sloan

A Lower Bound for the Sturm-Liouville Eigenvalue Problem on a Quantum Computer

Arvid J. Bessen

bessen@cs.columbia.edu
Department of Computer Science
Columbia University

We study the complexity of approximating the lowest eigenvalue of a Sturm-Liouville differential equation on a quantum computer. This general problem includes the special case of computing the ground state energy of a quantum system for a given potential.

Recently Papageorgiou and Woźniakowski proved that quantum computers could achieve exponential speedups compared to classical computers for certain potentials. Let $\mathbb{L}_q$ be the differential operator for the Sturm-Liouville problem. Papageorgiou's and Woźniakowski's method uses the (discretized) unitary propagator $\exp(i\mathbb{L}_q)$ as a query. They showed that if the operator $\exp(ip\mathbb{L}_q)$ (a "power query") is computable in cost comparable to $\exp(i\mathbb{L}_q)$ for any integer p, one can solve the Sturm-Liouville problem with $\mathcal{O}(\log \epsilon^{-1})$ power queries.

In this paper we will prove a matching lower bound of $\Omega(\log \epsilon^{-1})$ power queries, therefore showing that $\Theta(\log \epsilon^{-1})$ power queries are sufficient and necessary. Our proof is based on a frequency analysis technique, which examines the probability distribution of the final state of a quantum algorithm and the dependence of its Fourier transform on the input.

An illustrated story of fast construction of embedded lattice sequences for weighted spaces

Ronald Cools[a], Frances Y. Kuo[b], Dirk Nuyens[c], and Ian H. Sloan[d]

[ac]Department of Computer Science, K.U.Leuven, Belgium

[bd]School of Mathematics, University of New South Wales, Sydney, Australia

This talk delves deeper into the practical aspects of the construction of embedded lattice sequences as introduced in the talk by Frances Kuo. We optimize for the "best" component-by-component lattice rules with embedded point sets with a number of points equal to 2^m, for a given $m_{\min}$ and $m_{\max}$. Due to the nature of the component-by-component algorithm this lattice rule is also extensible in the dimension, and thus can be used from $d = 1$ up to $d = d_{\max}$ (which can be chosen arbitrary).

The matrix-vector formulation of the component-by-component construction algorithm enabled us to construct lattice rules in time $\mathcal{O}(d_{\max} n \log(n))$, for n points in d dimensions, $1 \le d \le d_{\max}$. This fast construction was first established for n prime and later extended for general n. For the embedded lattice sequences we exploit the particularly useful embedded structure present in the matrix. This enables us to also construct embedded lattice sequences in a fast way with virtually no additional cost, i.e. in time $\mathcal{O}(d_{\max} 2^{m_{\max}} \log(2^{m_{\max}}))$ for a lattice rule up to dimension $d_{\max}$ with a number of points from $2^{m_{\min}}$ up to $2^{m_{\max}}$. This result holds for function spaces with product weights as well as for order dependent weights of finite order.

Optimal Approximation of Elliptic Problem by Linear and Nonlinear Mappings II: Wavelet Methods and the Poisson Equation

Stephan Dahlke[a], Erich Novak[b], Winfried Sickel[c]

[a]Philipps-Universität Marburg, FB12 Mathematik und Informatik, Hans-Meerwein Straße, Lahnberge, 35032 Marburg, Germany, e-mail: dahlke@mathematik.uni-marburg.de

[b]Friedrich-Schiller-Universität Jena, Mathematisches Institut, Ernst-Abbe-Platz 2, 07743 Jena, Germany, e-mail: novak@math.uni-jena.de

[c]Friedrich-Schiller-Universität Jena, Mathematisches Institut, Ernst-Abbe-Platz 2, 07743 Jena, Germany, e-mail: sickel@math.uni-jena.de

This talk is concerned with optimal approximations of the solutions of elliptic boundary value problems. After briefly recalling the fundamental concepts of optimality, we shall especially discuss best n-term approximation schemes based on wavelets. We shall mainly be concerned with the Poisson equation in Lipschitz domains. It turns out that wavelet schemes are suboptimal in general, but nevertheless they are superior to the usual uniform approximation methods. Moreover, for specific domains, i.e., for polygonal domains, wavelet methods are in fact optimal. These results are based on regularity estimates of the exact solution in a specific scale of Besov spaces.

Obtaining $O(N^{-2+\varepsilon})$ Convergence for Quadrature Rules Based on Digital Nets

L.L. Cristea[a], J. Dick[b], G. Leobacher[a], and F. Pillichshammer[a]

[a] Institut für Finanzmathematik, Universität Linz, Altenbergstraße 69, A-4040 Linz, Austria

[b] School of Mathematics, University of New South Wales, Sydney 2052, Australia

In this talk we investigate multivariate integration in reproducing kernel Sobolev spaces for which the second partial derivatives are square integrable. As quadrature points for our quasi-Monte Carlo algorithm we use digital (t, m, s)-nets over $\mathbb{Z}_2$ which are randomly digitally shifted and then folded using the baker's transformation. For this QMC algorithm we show that the root mean square worst-case error converges with order $2^{m(-2+\varepsilon)}$ for any $\varepsilon > 0$, where 2^m is the number of points. A similar result holds for lattice rules as shown by Hickernell.

On the Randomized and Quantum Complexity of Elliptic PDE

Stefan Heinrich

Department of Computer Science
University of Kaiserslautern
D-67653 Kaiserslautern, Germany

We discuss recent results on the complexity of solving elliptic partial differential equations in the randomized and quantum setting. Matching, up to logarithmic factors, upper and lower bounds are obtained. A key ingredient is a new Monte Carlo algorithm for approximating weakly singular integral operators.

On the Solvability of Ill-Posed Problems in two Models of Computation

Peter Hertling

Fakultät für Informatik, Universität der Bundeswehr München, 85577 Neubiberg, Germany

Can a computational problem given by an unbounded linear transformation be solvable? Pour-El and Richards showed that the answer is no in the framework of computability theory, and Werschulz showed that the answer is no in the worst case setting in Information-Based Complexity. In fact, Pour-El and Richards showed that there exist partial differential equations with computable initial conditions that have non-computable solutions. We explain this phenomenon and related results by Brattka and by Weihrauch and Zhong. On the other hand, due to work by Kon, Ritter, and Werschulz, as well as Vakhania, it is known that every measurable unbounded linear operator is bounded on the average, for all Gaussian measures. Hence, due to a result by Werschulz, every linear ill-posed problem is solvable in the average case setting in Information-Based Complexity, for any Gaussian measure. We discuss these results from the viewpoint of computability theory. We also discuss the solvability of other ill-posed problems.

Global optimization without Lipschitz constant

Matthias Horn

Mathematisches Institut, Universität Jena, Ernst-Abbe-Platz 2, D-07743 Jena

We consider the global optimization problem for a family of function classes which allow several minima and which demand only Lipschitz continuity for smoothness. We give upper and lover bounds for the error numbers of these classes. We present an algorithm using only function values as information about the objective. This algorithm has no information about the Lipschitz constant and still has the optimal convergence rate.

Solving Ordinary Differential Equations – From Worst Case to Quantum Setting

Bolesław Kacewicz

Faculty of Applied Mathematics, University of Science and Technology, Cracow, Poland

The solution of ordinary differential equations is an important topic in numerical analysis. It has been widely studied within the information-based complexity framework. In this talk, we shall discuss complexity results for initial- and boundary-value problems. We start by showing matching upper and lower complexity bounds in the worst-case setting. We show that the worst-case results can be used to analyze the problems in the asymptotic setting. We further discuss initial-value problems in the randomized and quantum settings. It turns out that the problem complexity is reduced by switching to these less conservative settings. We explain some aspects of the complexity reduction, and present problems that still remain open.

A Continuous Complexity Analysis of Support Vector Machines

Mark A. Kin
Department of Mathematics and Statistics, Boston University, Boston MA 02215, mkon@bu.edu

In this talk we will ablaze support vector machine (SVM) learning algorithms from the standpoint of continuous complexity theory. Such a continuous algorithmic analysis is appropriate to study the complexity of SVM's, which are formulated in the language of continuous operations (function evaluation and the basic arithmetic operations). We will study information and algorithmic complexity separately, analyzing optimality properties. We will also analyze a scaling between the two. Such a scaled relationship is motivated by some currently studied paradigms in which increased information (e.g., examples) calls for increased algorithmic complexity (e.g., polynomial order of the approximating function). This scaling analysis will yield error bounds on our algorithms. We will apply the algorithms studied to a machine learning analysis bioinformatic data.

New Upper Bounds on the Star Discrepancy of (t, m, s)-Nets and (t, s)-Sequences

Peter Kritzer[1]

Department of Mathematics
University of Salzburg, Austria

A popular way of measuring the quality of distribution of a point set of N points in the s-dimensional unit cube is to consider its star discrepancy D_N^*. The star discrepancy of (t, m, s)-nets and (t, s)-sequences, which are among the most widely known classes of s-dimensional quasi-Monte Carlo point sets, was studied extensively by Niederreiter. In the case of (t, m, s)-nets, for example, he showed that for the star discrepancy of any (t, m, s)-net in base b with $m > 0$ we have

$$ND_N^* \leq B(s, b)b^t(\log N)^{s-1} + O\left(b^t(\log N)^{s-2}\right),$$

where $N = b^m$, the constant in the O-notation is independent of N, and where $B(s, b)$ is a term depending only on s and b. A similar result exists for the star discrepancy of (t, s)-sequences. Whereas it is believed that the order of magnitude in N in Niederreiter's bounds cannot be improved, it has been shown by several authors that the constants in the leading term (in the case of (t, m, s)-nets, $B(s, b)$) can be replaced by smaller constants for special choices of t, s, and b.

Here, we present improved upper bounds on the star discrepancy of (t, m, s)-nets and (t, s)-sequences that are not limited to special choices of t, m, s, and b but hold for arbitrary values of these parameters. In our investigations, we are mainly interested in the constants occurring in the leading terms. The new upper bounds on the star discrepancy of nets and sequences are of a similar form as those mentioned above, which makes it easy to compare them to earlier results. We also study the asymptotic behavior of the constants in the leading terms for increasing dimension s and present some numerical results.

Constructing good embedded lattice sequences with millions of points

Ronald Cools[a], Frances Y. Kuo[b], Dirk Nuyens[c], and Ian H. Sloan[d]

[ac]Department of Computer Science, K. U. Leuven, Belgium

[bd]School of Mathematics, University of New South Wales, Sydney, Australia

Lattice rules are a family of equal-weight cubature formulas for approximating high-dimensional integrals. By now it is well established that good generating vectors for lattice rules having n points can be constructed component-by-component for integrands belonging to certain function spaces. However, though the lattice rules constructed this way are extensible in dimension, they are not extensible in n, in the sense that changing n means the generating vector would need to be constructed anew. In this talk I will start with a review of recent advances in this research area and then introduce a new algorithm for constructing good generating vectors for embedded lattice sequences. Our generating vectors can be used with n ranging from roughly a thousand to a million!

[1]Research supported by the Austrian Science Foundation (FWF), Projects P17022-N12 and S8311-MAT

Summation of noisy orthogonal series

Peter Mathe

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany

We shall discuss the issue of numerical reconstruction of some function, when noisy Fourier coefficients are given and the level of noise is known. For bounded deterministic noise this is a classical problem, while for statistical noise this is an instance of regression. The quality of any method of reconstruction depends on the underlying smoothness. We discuss the situation, when smoothness is measured in some weighted Hilbert space with respect to the Fourier system. This allows to treat problems varying in the range of very mildly ill-posed to severely ill-posed ones. Some geometric features are highlighted.

Weighted quadrature formulas and approximation by zonal function networks on the sphere

H. N. Mhaskar

[a]Department of Mathematics, California State University, Los Angeles, California, 90032, U.S.A.

Let $q \geq 1$ be an integer, $\mathbb{S}^q$ be the unit sphere embedded in the Euclidean space $\mathbb{R}^{q+1}$. A Zonal Function (ZF) network with an activation function $\phi : [-1, 1] \to \mathbb{R}$ and n neurons is a function on $\mathbb{S}^q$ of the form $\mathbf{x} \mapsto \sum_{k=1}^n a_k \phi(\mathbf{x} \cdot \xi_k)$, where a_k's are real numbers, ξ_k's are points on $\mathbb{S}^q$. We consider the activation functions ϕ for which the coefficients $\{\hat{\phi}(\ell)\}$ in the appropriate ultraspherical polynomial expansion decay as a power of $(\ell + 1)^{-1}$. We construct ZF networks to approximate functions in the Sobolev classes on the unit sphere embedded in a Euclidean space, yielding an optimal degree of approximation in terms of n, compared with the nonlinear n-widths of these classes. Our networks are explicitly defined as linear operators, and do not require training in the traditional sense. In the case of uniform approximation, our construction utilizes values of the target function at scattered sites. The approximation bounds are used to obtain error bounds on a very general class of quadrature formulas that are exact for the integration of high degree polynomials with respect to a weighted integral. The bounds are better than those expected from a straightforward application of the Sobolev embeddings.

Error bounds for weak approximation of diffusion processes

Thomas Müller-Gronbach[a], Steffen Dereich[b], and Klaus Ritter[c]

[a]Department of Mathematics, Otto-von-Guericke-University Magdeburg, 39106 Magdeburg, Germany

[b]Institute of Mathematics. Berlin University of Technology, 10623 Berlin, Germany

[c]Department of Mathematics, Darmstadt University of Technology, 64289 Darmstadt, Germany

We consider the problem of approximating the expectation $E(f(X))$, where X is a diffusion process with values in $C([0,1])$ and $f : C([0,1]) \to \mathbb{R}$ is Lipschitz continuous. We study deterministic as well as randomized algorithms based on a fixed number of functional evaluations. Exploiting recent results on quantization of diffusion processes we establish lower and upper bounds on the corresponding minimal worst case errors.

Optimal Approximation of Elliptic Problems by Linear and Nonlinear Mappings I: General Concepts

Stephan Dahlke[a], Erich Novak[b], Winfried Sickel[c]

[a]Philipps-Universität Marburg, FB12 Mathematik und Informatik, Hans-Meerwein Straße, Lahnberge, 35032 Marburg, Germany, e-mail: dahlke@mathematik.uni-marburg.de

[b]Friedrich-Schiller-Universität Jena, Mathematisches Institut, Ernst-Abbe-Platz 2, 07743 Jena, Germany, e-mail: novak@math.uni-jena.de

[c]Friedrich-Schiller-Universität Jena, Mathematisches Institut, Ernst-Abbe-Platz 2, 07743 Jena, Germany, e-mail: sickel@math.uni-jena.de

We study the optimal approximation of the solution of an operator equation $\mathcal{A}(u) = f$ by four types of mappings: a) linear mappings of rank n; b) n-term approximation with respect to a Riesz basis; c) approximation based on linear information about the right hand side f; d) continuous mappings. We consider worst case errors, where f is an element of the unit ball of a Sobolev or Besov space $B^r_q(L_p(\Omega))$ and $\Omega \subset \mathbb{R}^d$ is a bounded Lipschitz domain; the error is always measured in the H^s-norm. The respective widths are the linear widths (or approximation numbers), the nonlinear widths, the Gelfand widths, and the manifold widths. As a technical tool we also study the Bernstein numbers. Our main results are the following. If $p \geq 2$ then the order of convergence is the same for all four classes of approximations. In particular, the best linear approximations are of the same order as the best nonlinear ones. The best linear approximation can be quite difficult to realize as a numerical algorithm since the optimal Galerkin space usually depends on the operator and of the shape of the domain Ω. For $p < 2$ there is an essential difference, nonlinear approximations are better than linear ones. Also in this case it turns out, however, that linear information about the right hand side f is optimal. As a main theoretical tool we study best n-term approximation with respect to an optimal Riesz basis and related nonlinear widths. The main results are about approximation, not about computation. However, we also discuss consequences of the results for the numerical complexity of operator equations.

Mean Square L_2-Discrepancy of Randomized Digital Nets in Prime Base

Ligia L. Cristea[a], Josef Dick[b], and Friedrich Pillichshammer[c]

[a] Institut für Finanzmathematik, Universität Linz, Altenbergerstraße 69, A-4040 Linz, Austria.

[b] School of Mathematics, University of New South Wales, Sydney 2052, Australia.

[c] Institut für Finanzmathematik, Universität Linz, Altenbergerstraße 69, A-4040 Linz, Austria.

We study the mean square weighted L_2-discrepancy of randomized digital (t, m, s)-nets in prime base p. The randomization method considered here is a digital shift of depth m, i.e., for each coordinate the first m digits of each point are shifted by the same shift whereas the remaining digits in each coordinate are shifted independently for each point. We give a formula for the mean square weighted L_2-discrepancy using the generating matrices of the digital net and we prove an upper bound on this discrepancy. Using this bound we find that there exist digital nets in prime base whose mean square weighted L_2-discrepancy is best possible in the order of magnitude in N, the number of points, (in the sense of Roth's lower bound). Further we investigate how the constant of the leading term depends on the choice of the base p. We show that the best result can be obtained for $p = 2$.

Adaption makes it easy to integrate functions with unknown singularities

Leszek Plaskota[a], and Grzegorz W. Wasilkowski[b]

[a]Faculty of Mathematics, Informatics, and Mechanics, Warsaw University, Poland

[b]Department of Computer Science, University of Kentucky

We study numerical integration $I(f) = \int_0^T f(x)\,dx$ for functions f with *singularities*. Nonadaptive methods are inefficient in this case, and we show that the problem can be efficiently solved by *adaptive* quadratures at cost similar to that for functions with no singularities.

Consider first a class $\mathcal{F}_r$ of functions whose derivatives of order up to r are continuous and uniformly bounded for any but one singular point. We propose adaptive quadratures Q_n^*, each using at most n function values, whose *worst case* errors $\sup_{f\in\mathcal{F}_r} |I(f) - Q_n^*(f)| = \Theta(n^{-r})$. On the other hand, the worst case error of nonadaptive methods is $\Omega(n^{-1})$.

These worst case results do not extend to the case of functions with two or more singularities; however, adaption shows its power even for such functions in the *asymptotic setting*. That is, let F_r^∞ be the class of r-smooth functions with arbitrary (but finite) number of singularities. Then a generalization of Q_n^* yields adaptive quadratures Q_n^{**} such that $|I(f) - Q_n^{**}(f)| = O(n^{-r})$ for any $f \in F_r^\infty$. In addition, we show that for any sequence of nonadaptive methods there are 'many' functions in F_r^∞ for which the errors converge no faster than n^{-1}.

Results of numerical experiments are also presented.

An implicit Euler scheme with non-uniform time discretization for stochastic heat equations

Thomas Müller-Gronbach[a] and Klaus Ritter[b]

[a]Universität Madgeburg, Germany

[b]Technische Universität Darmstadt, Germany

We present a new algorithm for solving stochastic heat equations on the spatial domain $[0,1]^d$. We derive an error bound, which depends on d, on the number of evaluations of one-dimensional components of the driving Brownian motion W, and on the decay of eigenvalues of the covariance of W. This bound matches known lower bounds that are valid for every algorithm. Hence the new algorithm is asymptotically optimal.

Functions with the Maximum Effective Dimension

Shu Tezuka

Faculty of Mathematics, Kyushu University

We introduce a broad class of functions on the d-dimensional unit hypercube which have the maximum effective dimension d, then prove that there exists a quadrature rule with N points whose convergence rate is $O(N^{-1})$ for the integration of any function in the class.

Randomly-shifted lattice rules for unbounded integrands

Frances Y. Kuo[a], Ian H. Sloan[a], Greg W. Wasilkowski[b], and Ben J. Waterhouse[a]

[a] School of Mathematics, University of New South Wales, Sydney NSW 2052, Australia

[b] Computer Science Department, University of Kentucky, Lexington, KY, USA

We study the problem of multivariate integration over $\mathbb{R}^d$ with integrands of the form $f(x)\rho(x)$ where ρ is a probability density function. Our study is motivated by problems in mathematical finance, where unbounded integrands over $[0,1]^d$ can arise as a result of using transformations to map the integral to the unit cube. We assume that the functions f belong to some weighted Hilbert space. We carry out a worst-case analysis in this space and show that good randomly-shifted lattice rules can be constructed component-by-component to achieve a worst-case error of order $O(n^{-1/2})$.

Optimal Monte Carlo and Quantum Algorithms for Parametric Integration

Carsten Wiegand

Fachbereich Informatik
Technische Universität Kaiserslautern
D-67653 Kaiserslautern, Germany
wiegand@informatik.uni-kl.de

In our talk we summarize the development of the last three years in the topic of parametric integration. We present matching upper and lower bounds for different function classes and different settings. For Sobolev spaces we complete earlier results of Heinrich in the Monte Carlo setting. In the quantum setting we show matching upper and lower bounds for sufficiently smooth functions. Finally we explain the main ideas to achieve the results and discuss some open problems for the remaining cases in the quantum setting.

On the optimal convergence rate of universal and non-universal algorithms for multivariate integration and approximation

Michael Griebel[a] and Henryk Woźniakowski[b]

[a]Institut für Numerische Simulation, Universität Bonn, Wegelerstrasse 6, D-53113 Bonn, Germany, griebel@ins.uni-bonn.de

[b] Department of Computer Science, Columbia University, New York, NY 10027, USA, and Institute of Applied Mathematics, University of Warsaw, ul. Banacha 2, 02-097 Warszawa, Poland,
henryk@cs.columbia.edu

We discuss the optimal rate of convergence of algorithms for multivariate integration and approximation of d-variate functions. We consider functions which belong to reproducing kernel Hilbert spaces $H(K_d)$. Here K_d is an arbitrary kernel whose all partial derivatives up to order r satisfy a Hölder-type condition with exponent 2β. We study algorithms which use n function values and analyze their rate of convergence as n tends to infinity. Without loss of generality it is enough to consider linear algorithms. We focus on *universal* algorithms, whose weights and sample points are only dependent on d, r and β but not on the specific kernel K_d, and *non-universal* algorithms whose weights and sample points may depend additionally on the specific kernel K_d.

We prove that the optimal (largest possible) rate of convergence of universal algorithms is $(r+\beta)/d$. This holds for both, multivariate integration and approximation. Furthermore, we prove that the optimal rate of convergence of non-universal algorithms is $1/2+(r+\beta)/d$ for multivariate integration and $a+(r+\beta)/d$ with $a \in [1/(4+4(r+\beta)/d), 1/2]$ for multivariate approximation. Hence, the optimal rate

of convergence of universal algorithms suffers from the curse of dimensionality, i.e., it goes linearly with d^{-1} to zero and can be arbitrarily small if d is large relative to $r+\beta$, whereas the optimal rate of convergence of non-universal algorithms does *not* suffer from the curse of dimensionality since it is always at least $1/2$ for multivariate integration, and $1/4$ for multivariate approximation. This is the price we have to pay for using universal algorithms. On the other hand, if $r+\beta$ is large relative to d then the optimal rates of convergence for universal and non-universal algorithms are approximately the same.

We also consider the case when we have the additional knowledge that the kernel K_d has product structure, $K_{d,r,\beta} = \otimes_{j=1}^{d} K_{r_j,\beta_j}$. Here K_{r_j,β_j} are some univariate kernels which are r_j times continuously differentiable and whose derivatives up to order r_j satisfy a Hölder-type condition with exponent $2\beta_j$. In this case, the optimal order of convergence of universal algorithms is $q := \min_{j=1,2,\ldots,d}(r_j+\beta_j)$ for both, multivariate integration and approximation. For non-universal algorithms the optimal rate of convergence is $1/2+q$ for multivariate integration and $a+q$ with $a \in [1/(4+4q), 1/2]$ for multivariate approximation. The optimal rate of convergence of universal algorithms for product kernels now depends on d only through the minimum of local regularities $r_j+\beta_j$. If we assume, for example, that $r_j \geq 1$ or $\beta_j \geq \beta > 0$ for all j, then the optimal rate is at least $\min(1,\beta)$, and the curse of dimensionality is *not* present for product kernels. This shows that the product form of reproducing kernels is essential and breaks the curse of dimensionality even by universal algorithms.

WORKSHOP 4

Learning theory

ORGANISERS:

Steve Smale, David McAllester,

Tomaso Poggio & Gábor Lugosi

Kernels as Features: On Kernels, Margins, and Low-dimensional Mappings

Avrim Blum

Carnegie Mellon University, Pittsburgh PA 15213-3891, USA.

Kernel functions are typically viewed as providing an implicit mapping of points into a high-dimensional space, with the ability to gain much of the power of that space without incurring a high cost if data is separable there by a large margin. However, the Johnson-Lindenstrauss lemma suggests that in the presence of a large margin, a kernel function can also be viewed as a mapping to a *low*-dimensional space if we can generate random vectors in the implicit space to perform the random projection. In this talk, I will discuss how we can efficiently produce such low-dimensional mappings, given only black-box access to the kernel function. That is, given just a program that computes $K(x, y)$ on inputs x,y of our choosing, can we construct an explicit (small) set of features that effectively capture the power of the implicit high-dimensional space? Our method requires black-box access to the underlying data distribution (i.e., unlabelled examples) and can be viewed as a way of converting any given kernel into a distribution-dependent feature set; we also show that without access to the data distribution, then this is not possible in general for an arbitrary black-box kernel.

This is joint work with Nina Balcan and Santosh Vempala.

Model selection and fast rates for regularized least-squares

Andrea Caponnetto

CBCL, Massachusetts Institute of Technology, 45 Carleton St., Cambridge, MA
and DISI, University of Genoa, Via Dodecaneso 35, Genova, Italy

We present some new results concerning the problem of model selection for regularized least-squares on a reproducing kernel Hilbert space.

We first describe the relevant minimax problems associated to a suitable class of priors. The prior condition is twofold, the conditional probability distribution on the input space is assumed to be known and moreover the regression function must belong to some subspaces defined in terms of the kernel and the marginal distribution.

Then both lower and upper bounds are derived, showing that optimal rates for the minimax problem are determined by the spectrum of an integral operator. The corresponding choice for the regularization parameter is expressed by the effective dimension of the problem, we show that this quantity can be effectively estimated by unlabelled data.

Finally particular attention is reserved to the setting of semi-supervised learning and generalization to multi-task learning.

Regularized Least Squares for Classification

Nicolò Cesa-Bianchi

DSI, Università di Milano, Italy

We present a number of recent results concerning the theoretical and empirical performance of algorithms for learning regularized least-squares classifiers. The behavior of these algorithms is analyzed under various assumptions on the generation of the data (individual sequences, i.i.d., probabilistic linear model) and relatively to various learning tasks (binary classification, hierarchical classification, label efficient learning).

This research has been carried out jointly with C. Gentile, A. Conconi, and L. Zaniboni.

Adaptivity in Learning

Ronald A. DeVore

CSCAMM, U. of Maryland

We shall discuss adaptive on line algorithms for the regression problem in machine learning. Our emphasis on algorithms that can be proven to be optimal in the sense of either probability or expectation. This is joint work with Peter Binev, Albert Cohen, Wolfgang Dahmen, and Vladimir Temlyakov.

Algorithmic Trading in Modern Markets

Michael Kearns

Computer and Information Science University of Pennsylvania

In the last decade, the ubiquity of the Internet and related technologies has radically altered fundamental aspects of our traditional financial markets. Especially influential has been the real-time revelation of important low-level transaction data (often referred to as market microstructure). For the past several years, we have been engaged in a research program examining the algorithmic challenges and opportunities presented by these changes.

In this talk, I will describe theoretical research on various aspects of modern markets. These include on-line algorithms with competitive ratio guarantees for trading based on limit order data, and a "universally profitable" algorithm for trading in a rich class of Markovian price evolution models. These models are designed to capture common financial parlance such as uptrends and downtrends, and support and resistance levels.

Generalization Bounds for Structured Classification

David McAllester

Toyota Technological Institute at Chicago
1427 East 60th Street
Chicago Il. 60637

In structured classification one is to assign a structured label to an input. Examples include labelling a word string with part of speech tags, labelling a word string with a parse tree, labelling the pixels of an image with depth or labelling each pixel with a class such as cloud or ground. In the last few years large margin classifiers have been constructed for structured classification. This talk will discuss large margin generalization bounds for structured classification where the dependence on the number of class labels C is $O(\sqrt{\log \log C})$.

A Geometric Perspective on Learning Theory and Algorithms

Partha Niyogi

The University of Chicago, Hyde Park, Chicago, IL 60637.

Increasingly, we face learning problems in very high dimensional spaces. We proceed with the intuition that although natural data lives in very high dimensions, they have relatively few degrees of freedom. One way to formalize this intuition is to model the data as lying on or near a low-dimensional manifold embedded in the high dimensional space. This point of view leads to a new class of learning algorithms that are "manifold motivated" and a new set of theoretical questions that surround their analysis. A central construction in these algorithms is a graph or simplicial complex that is data-derived and we will relate the geometry of these to the geometry of the underlying manifold. Applications to embedding, clustering, classification, and semi-supervised learning will be considered.

Learning: new theoretical results and old applications

Tomaso Poggio

M.I.T. Center for Biological and Computational Learning, Computer Science and Artificial Intelligence Laboratory and McGovern Institute for Brain Research

The problem of learning is one of the main gateways to making intelligent machines and to understanding how the brain works. In this talk I will give a brief overview of recent work on learning theory, including new results on predictivity and stability of the solution of the learning problem. I will then describe recent efforts in developing machines that learn for applications such as visual recognition, computer graphics and bioinformatics.

Relevant papers can be downloaded from
`http://www.ai.mit.edu/projects/cbcl/publications/all-year.html`

AdaBoost: The Expected but Untrue, and the Unexpected but True

Cynthia Rudin

New York University, 4 Washington Place, Room 809, New York, NY 10003

When the AdaBoost algorithm was introduced by Freund and Schapire in 1997, it became immediately popular for its empirical success in solving classification problems. Some fundamental convergence properties of AdaBoost remained unsolved until very recently, so I would like to describe the bizarre twists in the recent history of AdaBoost that have led to our current understanding. I will focus on two topics, a theory which was widely believed yet false, and an amazing property which no one could have guessed.

The Expected but Untrue: Since the development of AdaBoost eight years ago, there have been at least 3 large-scale empirical studies and several theoretical results to indicate that AdaBoost maximizes the "margin". This fundamental question regarding AdaBoost's convergence was finally solved last year, via the analysis of AdaBoost as a dynamical system. Not only does the nonlinear iterated map defined by AdaBoost yield rich dynamics such as stable cycles and chaos, it also provides the key to understanding the convergence of AdaBoost's margins. The result is the opposite of what was thought to be true; AdaBoost does not necessarily maximize the margin.

The Unexpected but True: AdaBoost was designed for solving specifically the classification problem. It is quite surprising that AdaBoost is equally superior for a completely different application: the bipartite ranking problem. In fact, a very recent proof shows that AdaBoost asymptotically achieves the same AUC as RankBoost in the case where the constant hypothesis is included in the set of AdaBoost's weak classifiers.

Portions of this talk are joint work with Corinna Cortes, Ingrid Daubechies, Mehryar Mohri, and Robert E. Schapire.

Learning theory algorithms and estimates without covering numbers

Steve Smale

Toyota Technological Institute at Chicago 1427 East 60th St. Chicago, IL 60637 also University of Chicago, Mathematics and Computer Science Departments, Univ of California, Berkeley, Mathematics and Economics Departments

Functional Methods for Learning Theory

Alessandro Verri

DISI - Università di Genova, Genova (I)

This talk presents some results of learning theory obtained through functional methods. Starting from the close relation between learning theory and the theory of regularization of ill-posed problems we discuss several mathematical properties of regularization networks, an important class of learning algorithms. The optimal choice of the regularization parameter is also discussed.

Stable message-passing and convex surrogates for learning in Markov random fields

Martin Wainwright

[a]Department of EECS, and Department of Statistics, UC Berkeley, Berkeley, CA, USA

Key statistical problems that arise in applications of Markov random fields (MRFs) are computing means and marginal distributions (e.g., for performing prediction) and estimating model parameters from data. Although easily solved for trees, these problems are intractable for general MRFs, which motivates the use of approximate methods. For instance, the sum-product or belief propagation algorithm is widely used in many fields (e.g., statistics, communication theory, computer vision) to compute approximate means/marginals in MRFs with cycles.

We consider the problem of learning MRF parameters from data, with the ultimate goal of using the estimated MRF to predict or smooth a new set of noisy observations. We propose a computationally efficient approach, based on constructing convex surrogates to the likelihood, such that the initial parameter estimation and subsequent prediction are coupled. As a particular example, we construct a surrogate likelihood based on a convexified Bethe approximation. Interestingly, even though this surrogate is implicitly defined, the function value and its derivatives can be computed efficiently by a tree-reweighted version of the belief propagation algorithm. We then prove that the parameter estimates obtained by maximizing the surrogate likelihood are asymptotically Gaussian but inconsistent. Nonetheless, this inconsistency turns out to beneficial in the prediction setting. In particular, we provide theoretical bounds on the performance loss of our computationally tractable method relative to the unattainable Bayes optimum, which depend on the graph structure and potential strength. These bounds hinge on the Lipschitz stability of the tree-reweighted message-passing updates.

Learning Variable Covariances via Gradients

Sayan Mukherjee[a] and Ding-Xuan Zhou[b]

[a]Institute of Genome Sciences and Policy, Duke University, sayan@stat.duke.edu

[b]Department of Mathematics, City University of Hong Kong, mazhou@cityu.edu.hk

Learning theory studies learning function relations from samples. In this talk we are interested in the learning of function gradients from their sample values. A least-square type learning algorithm based on the Tikhonov regularization in reproducing kernel Hilbert spaces is proposed. We show with error bounds that the output function converges to the gradient of the regression function as the sample size becomes large. Hence variable selection and estimation of covariation can be expected. An efficient method is provided to reduce the size of the linear system when the number of variables is much larger than the sample size. Some applications of our algorithm to gene expression analysis will be mentioned.

A theoretical comparison of regularized classifiers

Sara van de Geer

Mathematical Institute, University of Leiden

In binary classification, the problem is to predict a label $Y \in \{\pm 1\}$ given a feature $X \in \mathcal{X}$. A classifier is of the form $\text{sign}(f(X))$, where $f : \mathcal{X} \to \mathbf{R}$ is some function of X. In fact, we will also call f itself a classifier. Using a training set $\{(X_i, Y_i)\}_{i=1}^n$ of i.i.d. copies of (X, Y), the aim is to construct a classifier $\hat{f}_n$ with small prediction error. To this end one may consider various model classes $\mathcal{F}$ as candidate classifiers. various loss functions and also various complexity penalties.

We will study the case where $\mathcal{F}$ is a subset of a linear space, say

$$\mathcal{F} \subset \{f = \sum_{j=1}^m \alpha_j \psi_j : \alpha \in \mathbf{R}^m\},$$

where $\psi_j : \mathcal{X} \to \mathbf{R}$ $(j = 1, \ldots, m)$ are given base functions. Examples of base functions are those corresponding to a kernel representation, the base functions may be $\{\pm 1\}$-valued base classifiers (in the case of averaging classifiers), or they may form an orthogonal system. Examples of loss functions are: exponential, logit, or hinge loss (support vector machines). Examples of penalties are L_2 norms (e.g. induced by a kernel), ℓ_1 norms on the coefficients, penalties based on the dimensionality or other measures of complexity.

We will put these various choices in a single framework, and derive inequalities for the excess risk of the classifier $\hat{f}_n$. Our results depend on the margin behavior, in particular on the margin parameter κ as introduced by Tsybakov (Ann. Statist. 2004). We illustrate that a comparison of exponential loss or logit loss with hinge loss, depends on the (unknown) smoothness of the regression $\eta(X) = P(Y = 1|X)$. For kernel vector machines, an optimal tuning of the smoothing parameters may require knowing the margin parameter κ. We also give an example where hinge loss yields a classifier that is both adaptive to κ as well as to the "smoothness" of the boundary of Bayes classifier.

Defensive forecasting for decision making

Vladimir Vovk

Computer Learning Research Centre
Department of Computer Science
Royal Holloway, University of London
Egham, Surrey TW20 0EX
United Kingdom

We consider how to make probability forecasts of binary labels corresponding to vector objects. The basic observation is that for any continuous gambling strategy used for detecting disagreement between the forecasts and the actual labels, there exists a forecasting strategy whose forecasts are ideal as far as this gambling strategy is concerned. A forecasting strategy obtained in this way from a gambling strategy demonstrating a weak law of large numbers, which we call the K29 algorithm, is analyzed without making any assumptions about the way the data is generated. The algorithm is shown to be well calibrated and to have high resolution for big enough data sets and for a suitable choice of its parameter, a kernel on the Cartesian product of the forecast space $[0, 1]$ and the object space. With a given utility function, good probability forecasts make it possible to make good decisions by maximizing expected utility. Applying this expected utility maximization principle to the forecasts produced by the K29 algorithm leads to surprising new results in prediction with expert advice, with the pool of experts comprising a functional space.

WORKSHOP 5

Optimization

ORGANISERS:

Raphael Hauser, James Renegar & Philippe Toint

Smoothing techniques on Jordan algebras

Michel Baes

CORE and INMA, Université catholique de Louvain, Belgium. E-mail: baes@core.ucl.ac.be.
A research fellowship from the Belgian National Fund for Scientific Research is gratefully acknowledged by the author.

A spectral function on a formally real Jordan algebra is a real-valued function that depends only on the eigenvalues of its argument. One convenient way to create such a function is to start from a function $f : \mathbb{R}^r \mapsto \mathbb{R}$ that is symmetric in the components of its argument, and to define the function $F(u) := f(\lambda(u))$ where $\lambda(u)$ is the vector of eigenvalues of u. A particular example of this construction is given by functions of symmetric matrices that only depend on the eigenvalues of its argument. In this talk, we positively answer an open question proposed in the PhD thesis of H. Sendov : "is it possible to compute the hessian of spectral functions on Jordan algebras ?" We then exploit this result to investigate how the powerful smoothing techniques of Yu. Nesterov can in some instances be extended to Jordan algebras. In particular, we propose a new algorithm to minimize the sum of Euclidean norms and we perform its complexity analysis.

Optimizing the Roots of Polynomials

James V. Burke[a], Adrian S. Lewis[b], and Michael L. Overton[c]

[a]Department of Mathematics, University of Washington, Seattle, WA 98195, USA.

[b]School of Operations Research and Industrial Engineering, Cornell University, Ithaca, NY 14853, USA.

[c]Courant Institute of Mathematical Sciences, New York University, New York, NY 10012, USA.

A number of problems from the stability theory for continuous and discrete dynamical systems can be cast as a problem requiring the optimization of some function of the roots of polynomials. Although these functions are often non-Lipschitzian and extended real-valued, they possess other properties that aid in their analysis. In this talk, we present results from variational analysis and numerical optimization that are useful in the analysis such problems. The ideas are presented in the context of a particular problem known as the Blondel Belgian Chocolate Problem for Polynomial Stabilization.

Overcoming Some Disadvantages of a Mehrotra-Type Primal-Dual Corrector Interior Point Algorithm for Linear Programming

Coralia Cartis

Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford OX1 3QD, UK
ccartis@comlab.ox.ac.uk

Linear Programming (LP) is the most widely used mathematical model for real world applications that involve optimization. In the past fifteen years, Interior Point Methods (IPMs) have become highly successful in solving LP problems, especially large-scale ones, while enjoying good theoretical convergence and complexity properties. Nevertheless, for the IPM that is implemented in most codes, the Mehrotra Predictor-Corrector (MPC) algorithm, no global convergence or complexity theory is available. We construct a similar algorithm to the MPC algorithm, the Primal-Dual Corrector (PDC), and show that the PDC may fail to converge to a solution of the LP problem in both exact and finite arithmetic. Moreover, we present two ways of modifying the PDC algorithm that lead to theoretically reliable and practical methods.

Analyticity of Central Path for Homogeneous Cone Programming

Chek Beng Chua

Department of Combinatorics and Optimization
University of Waterloo, Canada

The central paths for optimization problems over homogeneous cones can be viewed as weighted central paths for the appropriate representing semidefinite programming problems. We show that the weighted central paths for semidefinite programming problems are analytic at their limit points, and hence conclude the analyticity of the central paths for homogeneous cone programming problems at their limit points.

On level-2 condition numbers

Felipe Cucker and Dennis Cheung

Dept. of Mathematics, City University of Hong Kong

Condition numbers measure the effect perturbations on the input data have on the computed result. Given a certain input data, one might thus be interested on computing its condition number. This is a new computational problem (induced by the original one) and has itself a condition number. Demmel called it a level-2 condition number. We prove that for a large class of problems (including, e.g., all decision problems), the level-2 condition number coincides (essentially) with the original one.

Reduction of symmetric semidefinite programs using the regular *-representation

Etienne de Klerk[a], Dima V. Pasechnik[b], and Alexander Schrijver[c]

[a]Tilburg University, The Netherlands / University of Waterloo, Canada

[b]Tilburg University, The Netherlands

[c]CWI, Amsterdam, The Netherlands

We consider semidefinite programming problems on which a permutation group is acting. We describe a general technique to reduce the size of such problems, exploiting the symmetry. The technique is based on a low-order matrix *-representation of the commutant (centralizer ring) of the matrix algebra generated by the permutation matrices. We apply it to extending a method of de Klerk et al. that gives a semidefinite programming lower bound to the crossing number of complete bipartite graphs.

Implementation of Infinite-dimensional Interior-point method for solving multi-criteria Linear-Quadratic control problem

L. Faybusovich[a], T. Mouktonglang[b], and T. Tsuchiya[c]

[a] Department of Mathematics, University of Notre Dame, Notre Dame, IN, 46556, USA

[b]Department of Mathematics, University of Notre Dame, Notre Dame, IN, 46556, USA

[c] The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-Ku, Tokyo,106-8569, Japan

We describe an implementation of an infinite-dimensional primal-dual algorithm based on the Nesterov–Todd direction. Several applications to both continuous and discrete-time multi-criteria linear-quadratic control problem and linear-quadratic control problem with quadratic constraints are described. Numerical results show a very fast convergence (typically, within 3-4 iterations) to optimal solutions.

Randomized Methods for (Continuous) Deterministic Optimization and Associated Complexity Analysis

Robert M. Freund

MIT Sloan School of Management, 50 Memorial Drive, Cambridge, MA 02139, USA,
rfreund@mit.edu

In contrast to conventional continuous optimization algorithms whose iterates are computed and analyzed deterministically, randomized methods rely on stochastic processes and random number/vector generation as part of the algorithm and/or its analysis. Whereas randomization in algorithms has been a part of research in discrete optimization for the last 20 years, randomization has played at most a minor role in the analysis of algorithms in continuous convex optimization, at least until recently. In this talk we will discuss three recent randomization-based algorithms for convex optimization with very good complexity bounds: a method by Bertsimas and Vempala based on cuts at the center of mass, a method similar to the classic perceptron algorithm due to Dunagan and Vempala, and a method for projectively transforming a poorly behaved problem to one with good complexity bounds. It is most interesting that randomization in a variant of the perceptron algorithm converts a very inefficient algorithm to one that is very efficient with high probability.

Interior-Point ℓ_2-penalty Methods for Nonlinear Programming: Global and Local Convergence

Donald Goldfarb[a] and Lifeng Chen[b]

[a]Columbia University

[b]Columbia University

We propose two line search primal-dual interior-point methods that approximately solve a sequence of equality constrained barrier subproblems. To attain feasibility they force an ℓ_2 exact penalty to zero whenever the steps generated tend to zero. Under standard assumptions, our methods have strong global and local convergence properties. For example, any limit point of the iteration sequence is either a KKT point of the problem (as long as MFCQ holds), or a Fritz-John point of the feasibility problem.

Homogeneous cone programming

Osman Güler

Homogeneous cones include include symmetric cone programming (such as the semidefinite programming and second order cone programming) but go considerably beyond it in several respects. In this talk, we continue our ongoing investigation into these cones on two fronts: the structure theory of the homogeneous cones, and then the development of efficient interior-point methods on them. A proper understanding of the structure theory, from several different but related viewpoints (left-symmetric algebras, Jordan algebras, Siegel cone construction, etc.) is an essential step for the development of efficient interior-point methods.

Hyperbolic van der Waerden conjecture, its proof and algorithmic applications

Leonid Gurvits

Los Alamos National Laboratory, USA, New Mexico.

The van der Waerden conjecture states that the permanent of $n \times n$ doubly stochastic matrix A satisfies the inequality $Per(A) \geq \frac{n!}{n^n}$ (VDW bound) and was finally proven (independently) by D.I. Falikman and G.P. Egorychev in 1981. They both shared Delbert Ray Fulkerson prize in 1982 .

It was for more than XX years the most important conjecture about permanents. The VDW bound is the simplest and most powerful bound on permanents and therefore among the simplest and most powerful general purpose bounds in combinatorics. We introduce and prove a vast generalization of the VDW conjecture:
Consider a homogeneous polynomial $p(z_1, ..., z_n)$ of degree n in n complex variables. Assume that this polynomial satisfies the property:

$|p(z_1, ..., z_n)| \geq \prod_{1 \leq i \leq n} Re(z_i)$ *on the domain* $\{(z_1, ..., z_n) : Re(z_i) \geq 0, 1 \leq i \leq n\}$.

We prove that $|\frac{\partial^n}{\partial z_1 ... \partial z_n} p| \geq \frac{n!}{n^n}$.

Our generalization not only affects the world of permanents, but also has important implications concerning PDEs, stability and control theory, complexity theory. Besides, our proof is much shorter and conceptually simpler than original proofs as of the van der Waerden conjecture for permanents as well of the Bapat's conjecture on mixed discriminants, proved by the author. The paper with the proof is available at `http://lanl.arxiv.org/abs/math.CO/0504397`. See also the paper at `http://xxx.lanl.gov/abs/math.CO/0404474` .

Treating equality constraints in PENNON

Michal Kocvara[a] and Michael Stingl[b]

[a]UTIA Prague
[b]University of Erlangen

We will present a new approach to the treatment of inequality constraints in the code PENNON. The code, designed to solve nonlinear optimization problems as well as semidefinite programs, is based on the generalized augmented Lagrangian algorithm. The original algorithm, due to R. Polyak and Ben-Tal/Zibulevsky, was designed for problems with inequality constraints. The equality constraints were either treated by classic augmented Lagrangian approach or by rewriting an equality as two inequalities. Neither of these approaches was truly satisfying and the presence of equality constraints sometimes disqualified the code, otherwise very efficient for inequality constraints. We propose a new (within our framework) approach to equality constraints. Numerical tests will demonstrate its superiority the the currenly used ones.

Exploiting Sparsity in Sums of Squares Relaxations of Polynomial Optimization Problems

M. Kojima[a], S. Kim[b], M. Muramatsu[c] and H. Waki[d]

[a]Tokyo Institute of Technology, Meguo-ku, Tokyo 152-8552 Japan

[b]Ewha Women's University, Seoul 120-750 Korea

[c]The University of Electro-Communications, Chofu-Shi, Tokyo 182-8585 Japan

[d] Tokyo Institute of Technology, Meguo-ku, Tokyo 152-8552 Japan

Sums of squares (SOS) relaxations are very powerful numerical methods for computing global optimal solutions of polynomial optimization problems (POPs). SOS relaxations are converted into semidefinite programs (SDPs) in numerical approaches. It is known that the size of a POP and the required accuracy for an optimal solution of the POP affect the size of the resulting SDP very much. Very large-scale SDPs are obtained as the size of the original POP and the accuracy increase. The sparsity of the problem should be exploited to apply SOS relaxations to larger-scale POPs. We introduce a correlative sparsity representing a certain sparse structure of a POP, which is used to derive the sparse sets of the supports of SOS polynomials for efficient SOS relaxations. Some numerical results on sparse POPs are also reported to show the improved performance of the proposed sparse SOS relaxation.

SOS approximations of nonnegative polynomials

Jean B. Lasserre

LAAS-CNRS, Toulouse, France

We prove that every nonnegative polynomial f is almost a sum of squares (s.o.s.), that is, one exhibits a sequence of polynomials $\{f_j\}$, where each f_j is obtained from f by adding marginal (monomial) squares with small coefficients. The resulting sequence of approximations $\{f_j\}$ converges to f in the ℓ_1 norm of coefficients. We also show that a polynomial f, nonnegative on an arbitrary real variety V, can also be approximated in the ℓ_1 norm, using the same type of perturbation, and each approximation f_j has a simple certificate of positivity on V. Importantly, no compactness assumption on V is required. As a consequence for optimization, one thus obtains simplified converging SDP-relaxations for minimizing f on V.

Strengthened semidefinite bounds for codes

Monique Laurent
CWI, Amsterdam, The Netherlands

We consider the problem of computing the maximum size $A(n,d)$ of a binary code with word length n and minimum distance d. That is, $A(n,d)$ is equal to the maximum cardinality of a stable set in the graph $G(n,d)$ with node set $\mathcal{P}$, the collection of all subsets of $\{1,\ldots,n\}$, and with edges the pairs (I,J) for which $|I\Delta J| = 1,\ldots,d-1$, for $I,J \in \mathcal{P}$.

Exploiting symmetries of the graph $G(n,d)$, Delsarte [1973] formulated a bound which can be computed via a linear program of size n, based on diagonalizing the (commutative) Bose-Mesner algebra of the Hamming graph. Schrijver [2004] proposed a stronger bound which can be computed via a semidefinite program of size $O(n^3)$, based on block-diagonalizing the (noncommutative) Terwiliger algebra.

We consider a hierarchy of semidefinite bounds obtained by applying a construction of Lasserre (in terms of moment matrices and the dual theory of sums of squares of polynomials). Using a result of de Klerk, Pasechnik, Schrijver [2005] about the regular $*$-representation of matrix $*$-algebras, we show that at any *fixed* stage in the hierarchy, the bound can be computed (to any precision) in time polynomial in n. The first bound is Lovász' theta number which coincides with Delsarte's bound, while the second bound in the hierarchy refines Schrijver's bound. We propose some refinements of Schrijver's bound whose computation also involves semidefinite programs of size $O(n^3)$.

Strategies for updating the barrier parameter in interior-point methods for nonlinear programming

Paul ARMAND[a], Joël BENOIST[b], and Dominique ORBAN[c]

[a]Université de Limoges, France

[b]Université de Limoges, France

[c]École Polytechnique de Montréal, Québec, Canada

We propose a unified framework for the update of the barrier parameter in interior-point methods for nonlinear programming. In this framework, the original primal-dual system is augmented to incorporate explicitly an updating function. We analyze local convergence properties and recover known updating strategies as special cases. We report preliminary numerical experiments on linear and nonlinear problems.

Computing the stability number of a graph via linear and semidefinite programming

Javier Peña[a], Juan Vera[b], and Luis Zuluaga[c]

[a]Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, jfp@andrew.cmu.edu

[b]Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, jfp@andrew.cmu.edu

[c]Faculty of Administration, University of New Brunswick, PO Box 4400, Fredericton, NB E38 5X9, CANADA, lzuluaga@unb.ca

We discuss properties of certain novel linear and semidefinite programming lifting approximation schemes for computing the stability number of a graph proposed by De Klerk and Pasechnik. These lifting approximations are based on a formulation of the stability number via copositive programming and a hierarchy of approximations to the copositive cone inspired by Polya's Theorem for polynomials that are non-negative over the non-negative orthant. Our results highlight several interesting connections between the linear and semidefinite programming liftings, the stability number of a graph, and properties of non-negative polynomials.

Geometry of Sample Sets in Derivative Free Optimization

Katya Scheinberg

IBM Watson

Many methods for derivative free optimization rely on sample models of the objective function (and constraints), instead of Taylor models. The quality of a sample model depends on the quality of the sample set. In the case of polynomial interpolation there are Taylor-like bounds on the err or between the interpolating polynomial and the true function. These bounds involve a constant which is connected to the basis of Lagrange polynomials for the given set of interpolation points. These polynomials play an important role in identifying a quality of an interpolation set. We give a geometric (simple) interpretation of these polynomials and connect their properties with the condition number of a certain Vandermonde matrix. We then show how we can algorithmically control this condition number, keeping it uniformly bounded in a derivative free optimization framework, hence keeping the quality of the interpolation bound from deteriorating. We then show how the notion of Lagrange polynomials and their properties can be, simply but elegantly, extended to the case of the least square regression. We do the same for the underdetermined interpolation, but with some caution.

The Clarke Generalized Jacobian of the Projection onto the Cone of Positive Semi-definite Matrices

Jérôme Malick[a] and Hristo Sendov[b]

[a]INRIA, Montbonnot, F-38334 Saint Ismier, France. Email: jerome.malick@inria.fr

[b]Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario, Canada N1G 2W1. Email: hssendov@uoguelph.ca

In this talk we are interested in the differentiability and non-smooth properties of the projection map onto the cone of positive semi-definite matrices. We present a concise formula for the Clarke generalized Jacobian of that map at an arbitrary symmetric matrix X. In order to do that we consider the projection as the gradient of a *spectral function*. In general, a spectral function F is a real valued function on a symmetric matrix argument with the property $F(U^T XU) = F(X)$ for every X in its domain and every orthogonal U.

Projectively related optimization problems and interior-point methods

Michael J. Todd

School of Operations Research, Rhodes Hall, Cornell University, NY 14853, USA

We discuss a pair of projectively related optimization problems, with one representing the search for either an optimal solution or a certificate of unboundedness for the other. We then present correspondences between their feasible solutions, optimal solutions, central paths, and the Newton steps to follow these paths. In some cases the relationship between these Newton steps has a sign switch, so that a Newton step for one corresponds to an "anti-Newton" step for the other.

The Theory and the Reach of Interior-Point Methods Continue to Expand!

Levent Tunçel

University of Waterloo

The modern theory of interior-point methods started with Karmarkar's seminal work in 1984. Since then a very comprehensive theory and a very impressive range of applications followed; moreover, many deep connections between the theory of interior-point methods and other branches of research in mathematics have been established.

In this talk, after introducing some of this general, modern theory of interior-point methods, I will turn to some very recent joint work with A. Nemirovski and discuss how to approximate the feasible region of the structured convex optimization problem at hand by a family of convex sets for which we have efficient (if the accuracy of the approximation is not too high) self-concordant barriers. Access to such barrier functions allows the development of efficient interior-point algorithms (and their computational complexity analysis) for the underlying convex optimization problems.

On the Rapid Convergence of Simulated Annealing

Adam Kalai[a] and Santosh Vempala[b]

[a]Toyota Technological Institute, Chicago IL, USA. Email: kalai@cs.uchicago.edu

[b]MIT, Cambridge MA, USA. Email: vempala@math.mit.edu

Simulated annealing is a general-purpose optimization technique that has met success in practice. The standard justification given for its success is that it avoids "getting stuck in local minima."

We prove that simulated annealing has a second advantage. Surprisingly, for at least some problems, simulated annealing also *speeds up* the descent and convergence to a minimum (local or global). In particular, we analyze annealing for convex optimization, where every local minimum is a global minimum. Annealing, using distributions from the Boltzmann-Gibbs family, leads to a fast convergence to the minimum. In fact, it improves the best-known bounds on the query complexity of the general problem of minimizing a linear function over a convex set, where the set is specified by a membership oracle. We also show that under certain general conditions, the Boltzmann-Gibbs distributions are optimal for annealing.

Adaptive Barrier Parameter Strategies For Nonlinear Programming

Andreas Wächter[a], Jorge Nocedal[b], and Richard Waltz[c]

[a]IBM T.J. Watson Research Center, Yorktown Heights, NY, USA

[b]Northwestern University, Evanston, IL, USA

[c]Northwestern University, Evanston, IL, USA

The effectiveness of interior methods is largely dependent on the strategy for updating the barrier parameter. In the past, efficient updating procedures have been developed and analyzed for interior methods within the context of linear and convex programming. In this presentation we examine various adaptive choices of the barrier parameter within interior algorithms for general nonlinear nonconvex optimization. A framework for enforcing global convergence is presented. The practical performance of the approaches is demonstrated on numerical results using the IPOPT and KNITRO software packages.

Robust algorithms for large sparse linear and semidefinite programming

Hua Wei and Henry Wolkowicz

Department of Combinatorics and Optimization, University of Waterloo, Canada

Current paradigms for search directions for primal-dual interior-point methods for linear programming (LP) and semidefinite programming (SDP) use: (i) form the linearization of the optimality conditions at the current estimate; (ii) form and solve the Schur complement/normal equation for the dual variable dy; (iii) back solve to complete the search direction. These steps result in loss of sparsity and ill-conditioning/instability, in particular when one takes long steps and gets close to the boundary. In addition, for SDP, one usually needs to symmetrize the linearization of the optimality conditions at the current estimate, which introduces further instability. This has resulted in the almost exclusive use of direct, rather than iterative methods, for the linear system.

We look at alternative paradigms based on least squares, an inexact Gauss-Newton approach, and a matrix-free preconditioned conjugate gradient method. This avoids the ill-conditioning in the nondegenerate case. We emphasize exploiting structure in large sparse problems. In particular, we look at large LPs and at SDP relaxations of the: Max-Cut; Quadratic Assignment; Theta function; Nearest Correlation Matrix; and Nearest Euclidean Distance Matrix problems.

Convex Quadratic Programming with Parameters

Stephen Wright

Department of Computer Sciences, University of Wisconsin, 1210 W. Dayton Street, Madison, WI 53706.

Linear and convex quadratic programs that involve parameters (either a scalar or a small-dimensional vector) arise in many applications, including control, statistics, and machine learning. Frequently, users wish to solve these problems for many values of these parameters. We discuss several applications, discuss the nature of the solution sets (especially their dependence on the parameters), and present several algorithmic approaches.

WORKSHOP 6

Special Functions

ORGANISERS:

Mourad Ismail, Guillermo Lopez & Ed Saff

A positivity result for Bessel polynomials

Christian Bergh[a] and Christophe Vignat[b]

[a]Dept. Mathematics, Universitetsparken 5, Copenhagen Ø, Denmark

[b]Laboratoire des Images et Signaux, Grenoble, France

Up to scaling the Student t-distribution with parameter $\nu > 0$ has the density

$$f_\nu(x) = \frac{A_\nu}{(1+x^2)^{\nu+\frac{1}{2}}}, \qquad A_\nu = \frac{\Gamma(\nu+\frac{1}{2})}{\sqrt{\pi}\Gamma(\nu)},$$

and for $\nu = n + \frac{1}{2}$, $n = 0, 1, \ldots$, we say that the distribution has $2n+1$ degrees of freedom. The Fourier transform of the density f_ν is an even function, which for $\xi \geq 0$ can be expressed as

$$k_\nu(\xi) := \int_{-\infty}^{\infty} f_\nu(x) \mathrm{e}^{-ix\xi} \mathrm{d}x = \frac{2^{1-\nu}}{\Gamma(\nu)} \xi^\nu K_\nu(\xi),$$

where K_ν is the modified Bessel function of the third kind, also called the Macdonald function.

For $\nu = n + \frac{1}{2}$ it is well-known that we have

$$k_{n+\frac{1}{2}}(u) = \exp(-u) q_n(u), \qquad u \geq 0, \tag{2}$$

where q_n is a polynomial of degree n, proportional to the Bessel polynomial θ_n of [1].

If X and Y are independent random variable with Student t-distributions of $2n+1$ and $2m+1$ degrees of freedom, we prove that for $a \in [0,1]$ the random variable $aX + (1-a)Y$ has the distribution with density

$$\sum_{k=\min(n,m)}^{n+m} \beta_k^{(n,m)}(a) f_{k+\frac{1}{2}}(x),$$

where the coefficients $\beta_k^{(n,m)}(a)$ are non-negative with sum 1. The main point is the non-negativity of the coefficients, which was conjectured in [2]. From an analytical point of view 2 is equivalent to the formula

$$q_n(au) q_m((1-a)u) = \sum_{k=\min(n,m)}^{n+m} \beta_k^{(n,m)}(a) q_k(u).$$

References

[1] E. Grosswald, *Bessel Polynomials*, Lecture Notes in Mathematics **698**, Springer, 1978.

[2] Glenn A. Walker, John G. Saw, *The distribution of linear combinations of t-variables*, J. Amer. Stat. Association. **73**, 876-878 (1978).

Connection between Sobolev orthogonal polynomials on the bounded interval and the circle. Asymptotics on the real line

E. Berriochoa[a], A. Cachafeiro[b], and J. García[c]

[a]Dpto. Matemática Aplicada I, Facultad de Ciencias, Universidad de Vigo, Ourense, Spain,

[b]Dpto. Matemática Aplicada I, E.T.S.I. Industriales, Universidad de Vigo, 36310 Vigo, Spain,

[c]Dpto. Matemáticas, I.E.S. Torrente Ballester, Pontevedra, Spain

In this talk we show the connection between Sobolev orthogonal Laurent polynomials on the unit circle and Sobolev orthogonal polynomials on a bounded interval of the real line. As consequence we deduce the strong outer asymptotics for Sobolev orthogonal polynomials with respect to an inner product of the following type

$$\langle f(x), g(x)\rangle_{s_\mu} = \int_{-1}^{1} f(x)g(x)\mathrm{d}\mu_0(x) + \int_{-1}^{1} f'(x)g'(x)\mathrm{d}\mu_1(x),$$

under suitable conditions for the second measure.

Moreover, we give sufficient conditions on the measures of orthogonality for obtaining the asymptotic behavior of the support of the measures.

Matrix interpretation of multiple orthogonal polynomials

Amilcar Branquinho

Departamento de Matemática
Faculdade de Ciências e Tecnologia
Universidade de Coimbra
3001-454 Coimbra
Portugal

In this talk we present the general theory of multiple orthogonal polynomials. Our departure point is the three-term recurrence relation, with matrix coefficients, satisfied by a sequence of vector polynomials. Connection with operator theory and constructive theory of approximation are presented.

From the Schur parameters to the support of the orthogonality measure

María José Cantero, Leandro Moral, and Luis Velázquez

Departamento de Matemática Aplicada, Universidad de Zaragoza, Spain

In this talk we expose some new results about the orthogonality measure of orthogonal polynomials on the unit circle.

Given a sequence of Schur parameters, we develop a method to approach the support of the orthogonality measure. The method is based on the relation between this support and the eigenvalues of finite unitary truncations of the related five0diagonal representation.

We analyze the convergence of these eigenvalues to the support of the measure, which can be seen as the study of the limit points of zeros of para-orthogonal polynomials.

In particular, we answer and improve a conjecture of L. Golinskii concerning the relation between the strong limit points of such zeros and the support of the measure. Other interesting properties concerning interlacing of zeros are also presented.

Scalar polynomials associated with rational and algebraic solutions of the Painlevé equations

Peter A Clarkson

Institute of Mathematics, Statistics & Actuarial Science
University of Kent
Canterbury, CT2 7NF, UK
P.A. Clarkson@kent.ac.uk

In this talk I shall discuss special polynomials associated with rational and algebraic solutions for the third and fifth Painlevé equations. The six Painlevé equations (P_I–P_{VI}) are six nonlinear ordinary differential equations that have been the subject of much interest in the past thirty years, which have arisen in a variety of applications such as random matrices. Further they may be thought of as nonlinear special functions. Rational solutions of the Painlevé equations are expressible in terms of the logarithmic derivative of certain special polynomials. For P_{II} these polynomials are known as the *Yablonskii-Vorob'ev polynomials*, first derived in the 1960's by Yablonskii and Vorob'ev. The locations of the roots of these polynomials have highly regular triangular structure in the complex plane. In this talk I shall describe the analogous special polynomials associated with rational and algebraic solutions of P_{III} and P_V. It is shown that their roots also have highly regular structure and other properties of these polynomials will be discussed.

The zeros of linear combinations of orthogonal polynomials

Kathy Driver

University of Witwatersrand
Johannesburg, South Africa

Let $\{p_n\}$ be a sequence of monic polynomials with p_n of degree n, that are orthogonal with respect to a suitable Borel measure on the real line. Stieltjes showed that if $m < n$ then there are $m+1$ zeros $x_1, \ldots, x_{m+1}$ of p_n with $x_1 < \cdots < x_{m+1}$ such that each interval (x_j, x_{j+1}) contains a zero of p_m. Our main theorem proves a similar result with p_m replaced by a linear combination of $p_1, \ldots, p_m$. The interlacing of the zeros of linear combinations of two and three adjacent orthogonal polynomials is also discussed.

Orthogonal matrix polynomials satisfying second-order differential equations

Antonio J. Durán

Departamento de Análisis Matemático, Universidad de Sevilla
Apdo (P.O. BOX) 1160, 41080 Sevilla, Spain

In the last three years a large class of families of orthogonal matrix polynomials $(P_n)_n$, satisfying second-order differential equations of the form

$$P_n''(t)A_2(t) + P_n'(t)A_1(t) + P_n(t)A_0 = \Gamma_n P_n(t)$$

has been introduced. Here A_2, A_1 and A_0 are matrix polynomials (which do not depend on n) of degrees less than or equal to 2, 1 and 0, respectively, and Γ_n are Hermitian matrices.

These families of orthogonal polynomials are among those that are likely to play in the case of matrix orthogonality the role of the classical families in the case of scalar orthogonality.

The purpose of this talk is to show an overview of these examples, in particular we will discuss some of the many differences among the matrix and the scalar case, such as the role of the Rodrigues' formula or uniqueness of the differential equation.

Quasi-linear Stokes phenomenon for *tronquée* Painlevé Transcendents. The Riemann-Hilbert approach

Alexander Its

Indiana University, Purdue University, Indianapolis, USA

Using the Isomonodromy-Riemann-Hilbert approach, we study the quasi-linear Stokes phenomenon for the second Painlevé equation $y_{xx} = 2y^3 + xy - \alpha$. We show that the representation of the Painlevé functions in terms of the solution of the associated Riemann-Hilbert-Birkhoff inverse monodromy problem can be directly used to obtain a precise description of the exponentially small jumps in the s-called tronquée solution of the Painlevé equation, i.e. the solutions which approach α/x or $\sqrt{-x/2}$ as $|x| \to \infty$ in the appropriate sectors. We will also evaluate the asymptotics of the coefficients of the formal power series, which asymptotically represent the solution. In addition, we will describe the asymptotic behavior of the tronquée solutions in the full neighborhood of infinity A brief history and the results of the global asymptotic analysis of the Painlevé equations via the Riemann-Hilbert method will be presented as well. The talk is based on my joint work with A. Kapaev and (in the part concerning the increasing tronquée solution) on a recent work of A. Kapaev.

The Askey scheme as a four-manifold with boundary

Tom H. Koornwinder

University of Amsterdam, Korteweg-de Vries Institute

In an earlier note [1] I discussed certain multi-parameter limits between some families of hypergeometric orthogonal polynomials. This dealt with the limit passage from Racah to Hahn and all families in the Askey scheme below Hahn. In the lecture I will extend these results to the full Askey scheme. In my point of view, part of the asymptotics associated with the Askey scheme can be phrased in terms of a four-manifold with boundary.

[1] T. H. Koornwinder, *Uniform multi-parameter limit transitions in the Askey tableau*, arXiv:math.CA/9309213, 1993.

Orthogonal Polynomials, Padé Approximants, and Sets of Minimal Capacity

Herbert Stahl

TFH-Berlin/ FB II,
Luxemburger Str. 10
D-13353 Berlin
e-mail: stahl@tfh-berlin.de

The connection between orthogonal polynomials, Padé approximants, and continued fractions is classical. If the approximation task is concerned with a Markov. a Stieltjes, or a Hamburger fraction, which are the well studied cases of the early days of the theory, then the associated orthogonal polynomials live on the real axis or an subsets of the real axis. If one, however, wants to study Padé approximants for a broader class of functions, then this often leads away from the real axis into the complex plane, and instead of real intervals, one has to deal with the sets of minimal capacity. The complements of these sets are called extremal domains, and they typically are domains in which diagonal Padé approximants converge. In this respect, extremal domains are for Padé approximants what disks are for power series. The associated orthogonal polynomials live on the complementary sets, which are the sets of minimal capacity. They are of special structure and have very particular properties.

In this talk we address results about these sets of minimal capacity and their associated extremal domains. Topics of discussion will be their unique existence under rather weak assumptions, special properties, and the relationship with similar concepts in geometric function theory. If time allows, we will also turn some attention to the problem of determining specific parameters of the quadratic differentials that arise in the description of such sets of minimal capacity.

The ABC of Hyper Recursions II

Nico M. Temme[a], Amparo Gil[b], and Javier Segura[b]

[a]CWI, Amsterdam

[b]University of Cantabria, Santander

The Gauss hypergeometric functions

$$f_n = {}_2F_1(a+\varepsilon_1 n, b+\varepsilon_2 n; c+\varepsilon_3 n; z),$$

with fixed $\varepsilon_j = 0, \pm 1$ (not all ε_j equal to zero) satisfy a second-order linear difference equation with respect to n of the form

$$A_n f_{n-1} + B_n f_n + C_n f_{n+1} = 0. \qquad (*)$$

Because of symmetry relations and functional relations for the Gauss functions, many of the 26 cases can be transformed into each other.

In a previous paper we have given a set of five basic difference equations of the form $(*)$ from which all other cases can be obtained, and we have described the domains in the complex z-plane where minimal and dominant solutions of the difference equations have to be determined.

In this talk we consider for each of the five basic equations $(*)$ (and for each domain of these equations) a set of six Gauss functions (as used in the theory of hypergeometric differential equations). We have found that of these six functions always one unique minimal solution (up to a multiplicative constant) exists and the five dominant solution. Asymptotic and numerical methods are used to verify the nature of these solutions.

Asymptotics of Orthogonal Polynomials — The Riemann–Hilbert Approach

R. Wong

Department of Mathematics
City University of Hong Kong
83 Tat Chee Avenue
Kowloon, HONG KONG

There are various ways to find asymptotic behaviour of orthogonal polynomials. For instance, one may use the Liouville–Green (or WKB) approximation in differential equation theory, the steepest decent method for contour integrals, or three-term recurrence relations. In this lecture, I will mention a new approach based on two-dimensional Riemann–Hilbert problems.

WORKSHOP 7

Approximation theory

ORGANISERS:

Martin Buhmann & Juan Manuel Pena

Compactly supported radial basis functions and spherical averaging

B. J. C. Baxter

School of Economics, Mathematics and Statistics
Birkbeck College, University of London
England

A radial basis function approximation has the form

$$s(x) = \sum_{k=1}^{n} a_k \, \phi(x - b_k), \qquad x \in \mathcal{R}^d, \tag{1}$$

where $\phi\colon \mathcal{R}^d \to \mathcal{R}$ is some given (usually radially symmetric) function, $(a_k)_1^n$ are real coefficients, and the centres $(b_k)_1^n$ are points in $\mathcal{R}^d$. We investigate some properties of ϕ when it has the distributional Fourier transform $\widehat{\phi}(\xi) = c\|\xi\|^{-2m}$, where c is a real number and m is a positive integer. The class of such functions includes the Euclidean norm $\phi(x) = \|x\|$ when the dimension d is odd and the thin plate spline $\phi(x) = \|x\|^2 \log \|x\|$ when $d = 2$. Our key observation is that it is easy to construct a signed measure μ on $\mathcal{R}^d$ for which the function $\hat{\mu}(\xi)\widehat{\phi}(\xi)$ is an entire function of exponential type. Hence the Paley-Wiener theorem allows us to conclude that the convolution $\mu * \phi$ is a compactly supported function. Further, we find that it is possible to choose the coefficients $(a_k)_{k=1}^n$ and the centres $(b_k)_{k=1}^n$ such that the *spherical average* As of s is compactly supported, where the spherical average of a continuous function $f : \mathcal{R}^d \to \mathcal{R}$ is defined by the equation

$$Af(x) = \int_{O_d} f(Ux) \, d\sigma(U), \qquad x \in \mathcal{R}^d. \tag{2}$$

Here O_d denotes the group of real $d \times d$ orthogonal matrices and σ_d denotes the group invariant probability measure on O_d. We also apply this insight to construct a new class of compactly supported radial basis functions, as well as suggesting a new technique for the evaluation of European options.

Classification of planar nodal sets satisfying a geometric characterization

J. M. Carnicer and C. Godés

Departamento de Matemática Aplicada. Universidad de Zaragoza.

The geometric characterization (GC) identifies sets of nodes such that the Lagrange polynomials can be factorized as products of polynomials of first degree giving rise to simple interpolation formulae. The number of lines containing more points than the degree can be used to classify such configurations of nodes. A particular instance of GC sets of nodes are generalized principal lattices, which are sets of points obtained as intersection of three families of lines. We explore the role of generalized principal lattices in the classification and of GC sets and its relation with algebraic cubic pencils of lines.

On the Error of Local RBF approximation of Scattered Data

Oleg Davydov

Department of Mathematics, University of Strathclyde, 26 Richmond St, Glasgow G1 1XH, Scotland, UK

We discuss the estimates for the error of approximation of scattered data using radial basis functions locally, in particular in the first stage of a two-stage method.

[1] O. Davydov, A. Sestini and R. Morandi, Local RBF approximation for scattered data fitting with bivarite splines, *in* "Trends and Applications in Constructive Approximation," (D. H. Mache, J. Szabados, and M. G. de Bruin, Eds.), ISNM Vol. 151, Birkhäuser, 2005.

Interpolation points and interpolating formulas on the square

Stefano de Marchi

Department of Computer Science, University of Verona (Italy)

In this talk we present new interpolation points on the square. For these points we also discuss the growth of the corresponding Lebesgue constant and we shall be also interested in the construction of the corresponding interpolating formulas.

Extremal trigonometric and univalent polynomials

Dimitar K. Dimitrov

DCCE, IBILCE, UNESP, São José do Rio Preto, SP, Brazil

We discuss some relations between positive summability kernel that are fundamental in Approximation Theory and algebraic polynomials which play an important role in Geometric Function Theory. The starting point of this piece of research is a surprising identity which reveals a tight connection between positive trigonometric polynomials discovered by Fejér in 1915 and univalent algebraic polynomials constructed by Suffridge in 1969. Another relation, involving kernels arising from a problem for approximation of smooth functions and and univalent polynomials which obey the so-called asymptotic Koebe quarter theorem, is also considered.

General Uncertainty Principles and Signal Recovery

Say Song Goh[a], and Tim N. T. Goodman[b]

[a]Department of Mathematics, National University of Singapore, 10, Kent Ridge Crescent, Singapore 119260

[b]Department of Mathematics, University of Dundee, Dundee DD1 4HN, Scotland, UK

We give a general uncertainty principle for operators on Banach spaces. This has many special cases, including uncertainty principles for the measures of sets on which a (periodic) function has approximate support in time and frequency. These results have application to stable recovery of noisy signals which are missing information on a given set.

Constrained spline approximation: back to basics?

Kirill Kopotun

Department of Mathematics, University of Manitoba, Winnipeg, Manitoba, R3T 2N2, Canada

Several results on equivalence of moduli of smoothness of univariate splines will be discussed. For example, for any $1 \le k \le r+1$, $0 \le m \le r-1$, and $0 < p \le \infty$, we have

$$n^{-\nu}\omega_{k-\nu}(s^{(\nu)}, n^{-1}, [-1,1])_p \sim \omega_k(s, n^{-1}, [-1,1])_p\,, \quad 1 \le \nu \le \min\{k, m+1\}\,,$$

where $s \in C^m[-1,1]$ is a piecewise polynomial of degree $\le r$ on a quasi-uniform (*i.e.*, the ratio of lengths of the largest and the smallest intervals is bounded by a constant) partition. Similar results for Chebyshev partitions and weighted Ditzian-Totik moduli of smoothness will also be presented. These results allow considerable simplification of various proofs in the area of Constrained Approximation by polynomials and splines, and help to determine directions for future research.

Fourier-Padé Approximation for Angelesco systems

M. Bello Hernández[a], G. López Lagomasino[b], and J. Mínguez[a]

[a]Univ. La Rioja, c/ Luis de Ulloa s/n - 26004, Logroño, Spain

[b]Univ. Carlos III de Madrid, Avda. Universidad, 30 - 28911, Leganés, Spain

In this talk we consider linear Fourier-Padé approximation for Angelesco systems of functions. This construction is similar to that of Hermite-Padé approximation. Instead of considering power series expansions of the functions in the system, we take their expansion in a series of orthogonal polynomials.
Diagonal sequences of Fourier-Padé approximation were studied by A. A. Gonchar, E. A. Rakhmanov, and S. P. Suetin in [1] when the function to be approximated is of Markov type; that is, the Cauchy transform of a measure supported on the real line. There, the authors obtain the exact rate of convergence of diagonal sequences of linear Fourier-Padé approximation in terms of the equilibrium measure of a certain potential theoretic problem. We generalize the results of that paper by considering instead of one Markov function a system of such functions given by measures whose supports do not intersect and study their simultaneous approximation. These systems are called of Angelesco type. In this situation, vector potential theory is used.

References

[1] A. A. Gonchar, E. A. Rakhamanov, S. P. Suetin, On the rate of convergence of Padé approximants of orthogonal expansions. *Progress in approximation theory* (Tampa, FL, 1990), 169-190, *Springer Ser. Comput. Math.*, **19**, Springer, New York, 1992.

How big can a smooth function get if it is zero on an h-spaced set

Robert Brownlee[a], Jeremy Levesley[b], and Xingping Sun[c]

[a]University of Leicester, Leicester, UK.

[b]University of Leicester, Leicester, UK.

[c]University of Southwest Missouri, Missouri, US.

Error estimates for interpolation using radial basis functions require a technical argument. We are interested in proving error estimates by embedding the problem in an appropriate space of continuous functions, and reading off the error directly from the continuity index. In order to make progress with this line of reasoning we first consider the natural problem of a function which is zero on a finite h-spaced set, with bounded kth order derivatives, and ask the question of how big such a function can get in a neighbourhood of the points. We can show that the function can get no bigger than Ch^k for some constant C which depends on the original function. We also extend this result for functions in a Zygmund space (a scale of spaces with a continuous index).

Widths and shape preserving widths of s-monotone functions

D. Leviatan

School of mathematics, Tel Aviv University, Tel Aviv, Israel

Let I be a finite interval, $s \geq 0$, and $r \geq 1$. Denote by $\Delta_+^s L_q$ the subset of all functions $y \in L_q$ such that the s-difference $\Delta_\tau^s y(\cdot)$ is nonnegative on I, $\forall \tau > 0$. Further, denote by $\Delta_+^s W_p^r$, the class of functions x on I with the seminorm $\|x^{(r)}\|_{L_p} \leq 1$, such that $\Delta_\tau^s x \geq 0$, $\tau > 0$. We obtain two-sided estimates of the shape preserving widths

$$d_n\left(\Delta_+^s W_p^r, \Delta_+^s L_q\right)_{L_q} := \inf_{M^n \in \mathcal{M}^\setminus} \sup_{x \in \Delta_+^s W_p^r} \inf_{y \in M^n \cap \Delta_+^s L_q} \|x - y\|_{L_q},$$

where $\mathcal{M}^\setminus$ is the set of all linear manifolds M^n in L_q, $\dim M^n \leq n$, such that $M^n \cap \Delta_+^s L_q \neq \emptyset$.

We will compare the above widths with the Kolmogorov widths

$$d_n\left(\Delta_+^s W_p^r\right)_{L_q} := \inf_{M^n \in \mathcal{M}^\setminus} \sup_{x \in \Delta_+^s W_p^r} \inf_{y \in M^n} \|x - y\|_{L_q},$$

and with the widths $d_n(W_p^r)_{L_q}$.

Among the new results we present are

Theorem. Let r and s be integers such that $s > r + 1 \geq 2$, and let $1 \leq p, q \leq \infty$ be so that $(r, p, q) \neq (1, 1, \infty)$. Then

$$d_n(\Delta_+^s W_p^r)_{L_q}^{kol} \asymp n^{-s+\min\{1/q', 1/2\}}, \quad n \geq s.$$

However,

$$d_n\big(\Delta_+^s W_p^r, \Delta_+^s L_q\big)_{L_q}^{kol} \asymp n^{-2}, \quad n \geq s.$$

Multivariate Bernstein bases and evaluation algorithms in barycentric coordinates

Esmeralda Mainar[a] and Juan Manuel Peña[b]

[a]Departamento de Matemáticas, Estadística y Computación. University of Cantabria, Spain

[b] Departamento de Matemática Aplicada. University of Zaragoza, Spain

We perform backward and forward error analysis of the multivariate de Casteljau algorithm for the evaluation of polynomials in barycentric coordinates. We present some numerical experiments in order to show stability properties of this algorithm.

Logarithmic Asymptotic of Contracted Sobolev orthogonal polynomials on the real line

G. López Lagomasino[a], Francisco Marcellán[b], and H.Pijeira[c]

[a]Departamento de Matemáticas, Universidad Carlos III de Madrid (Spain)

[b]Departamento de Matemáticas, Universidad Carlos III de Madrid (Spain)

[c]Universidad de Matanzas, Cuba

For a wide class of Sobolev type norms with respect to measures with unbounded support on the real line, the contracted zero distribution and the logarithmic asymptotic of the corresponding rescaled Sobolev orthogonal polynomials is given.As an illustrative example we will show some asymptotic results for the case of Sobolev orthogonal polynomials associated with Freud weights.

On Chebyshevian spline subdivision schemes

Marie-Laurence Mazure
LMC-IMAG, Université Joseph Fourier, BP53
38041 Grenoble Cedex 9, France

A classical formula gives the blossom of the derivative of a polynomial function in terms of its own blossom. We first extend this formula to the Chebyshevian framework. We then show how to exploit it to analyse the convergence of Chebyshevian spline subdivision schemes.

Nonlinear Diffusion and Wavelet Shrinkage in Signal Processing

Gerlind Plonka[a] and Gabriele Steidl[b]

[a]Department of Mathematics, University of Duisburg-Essen, Campus Duisburg, 47048 Duisburg, Germany

[b]Faculty of Mathematics and Computer Science, D7, 27, University of Mannheim, 68131 Mannheim, Germany

Nonlinear diffusion filtering is a powerful tool for denoising of signals and images. The choice of nonlinear diffusion filters leads to impressive results by removing insignificant, small-scale variations while preserving important features such as discontinuities. But most approaches for numerical evaluation are based on the simplest finite difference discretization, the so-called Euler-forward scheme. This scheme requires very small time steps in order to be stable making the procedure very time-consuming. One way to address the problem is to take a semi-implicit discretization which does not suffer from any time step restriction. The disadvantage of the implicit approach is that each iteration step requires to solve a tridiagonal system of equations. Further, in numerical experiments it is observed that the implicit technique tends to smooth the signal much stronger than the explicit one and the deviation from the true solution becomes weaker with increasing step size. Recently, the close connection between numerical schemes of soft wavelet shrinkage and nonlinear diffusion has been established. In particular, it has been shown that the Euler-forward explicit discretization of the diffusion equation with total variation (TV) diffusivity and a single Haar wavelet shrinkage step coincide. Based on this idea one can derive now wavelet shrinkage functions from existing diffusivity functions by expressing the explicit discretization of nonlinear diffusion in terms of Haar wavelet shrinkage on a single spatial level. However, wavelet filter banks are usually applied with multiple scales. The multiscale shrinkage with Haar wavelets can be interpreted as application of the nonlinear explicit diffusion scheme to hierarchical signals. In this talk we present a new explicit discretization of the diffusion equation which is inspired by (and can be understood as) application of a translation–invariant multiscale wavelet filter bank with shrinkage. The new discretization admits greater time steps than the original Euler-forward scheme and is much more efficient. We shall prove that the new scheme satisfies a lot of discrete scale-space criteria as the average level invariance, a certain extremum principle and convergence to spatial average.

Stability of B-splines on bounded domains

Ulrich Reif

Fachbereich Mathematik, Technische Hochschule Darmstadt (Germany)

Tensor product B-splines loose their stability properties when restricted to smoothly bounded domains. In this talk, we discuss two different possibilities to stabilize the basis and demonstrate their properties at hand of several examples.

The first approach, called extension, is based on a specific linking of unstable outer B-splines to stable inner B-splines. The resulting basis has a reduced number of degrees of freedom, but retains full approximation order. It is particularly well suited for scattered data approximation on bounded domains.

The second approach, called normalization, employs a scaling procedure to achieve stability. The construction is even simpler as in the first case and, in addition, yields nestedness of the resulting spline spaces under knot refinement. Applications considered here include FE approximations of the Helmholtz equation and the Stokes equation.

Spline quasi-Interpolating projectors on a three direction mesh of the plane

T. Lyche[a], C. Manni[b], and P. Sablonnière[c]

[a]University of Oslo

[b]University of Roma Tor Vergata

[c]INSA Rennes

We study two classes of spline quasi-interpolants (abbr. QI) which are also projectors on spaces of polynomial splines defined on a three direction mesh of the plane. These operators have the general form

$$Qf = \sum_{\alpha \in \mathbb{Z}^2} \mu_\alpha(f)\, B_\alpha$$

where B_α are box-splines and $\mu_\alpha(f)$ are linear functionals. QI of the first class are called discrete quasi-interpolating projectors (abbr. dQIP) because the $\mu_\alpha(f)$ are linear combinations of values $f(\beta)$ at integer points β in some neighbourhood of $\alpha \in \mathbb{Z}^2$. QI of the second class are called integral quasi-interpolating projectors (abbr. iQIP) because the $\mu_\alpha(f)$ are linear combinations of weighted mean values $\int f B_\gamma$ where γ lies in some neighbourhood of $\alpha \in \mathbb{Z}^2$. For splines of total degrees not greater than 10, we give the expressions of QIP of both classes and their infinity norms. Moreover, we prove the existence and unicity of iQIP for splines of any regularity. As far as possible, we give constructive proofs and algorithms which allow the computation of all types of operators in both classes.

Efficient reduction

Juan Manuel Peña[a] and Tomas Sauer[b]

[a]Depto. Matemática Aplicada, Fac. Ciencias, Universidad de Zaragoza, E–50009 Zaragoza, Spain

[b]Lehrstuhl für Numerische Mathematik, Justus–Liebig–Universität Gießen, Heinrich–Buff–Ring 44, D–35392 Gießen, Germany

Reduction is the multivariate counterpart of division with remainder, decomposing a polynomial into a part belonging to certain ideal and a part belonging to a normal form space with respect to this ideal. This operation is crucial for the computation of Gröbner- and H–bases.

In order to obtain term order free H–bases which are capable of presenting symmetries of the original problem, it is useful to use a reduction process based on orthogonal projections to homogeneous spaces generated by the leading forms of a finite set of polynomials. This seemingly innocent operation turns out to be of very high complexity when the number of variables increases. In this talk, we present an algorithm which performs the projections in a way that is not only more efficient but also numerically stable and thus allows for algebraic operations with coefficients given in floating point form.

Triangulations of spherical polygons and applications

N. Boal[a], V. Domínguez[b], and F.–J. Sayas[a]

[a]Dep. Matemática Aplicada, C.P.S., Universidad de Zaragoza, Zaragoza (Spain)

[b]Dep. Matemática e Informática, Universidad Pública de Navarra, Pamplona (Spain)

In 1985 an icosahedral triangulation of the sphere was introduced by Baumgardner and Frederickson. This kind of triangulation has been extensively employed in the oceanographical community, since it gives very uniform–looking partitions of the sphere. In the original paper, the method consisted of defining curved barycentric coordinates for arbitrary spherical triangles by doing a sequence of dyadic partition of the triangles and assuming that the process led to a limit by a continuity argument.

We prove here several results related to this triangulation, that confirm in theory what can was numerically checked originally. First of all, we prove that the sequence of transformations leads to a continuous Lipschitz but non–smooth parameterization of any spherical triangle. We also prove that in the sequence of inscribed polyhedra resulting from the successive application of the partition procedure all the triangles have roughly the same area, that is, the minimum area bounds the maximum area from above (with an adequate constant independent of the level of the partition) and that this quasi–uniformity applies also to the lengths of the sides.

Finally, we show how to take advantage of this method for automatic numerical quadrature on spherical polygons based on extrapolation.

Resolution of Gibbs' phenomenon, from global to semi-global

Jared W. Tanner[a], Eitan Tadmor and Anne Gelb

[a]Department of Statistics, Stanford University

Spectral projections enjoy high order convergence for globally smooth functions. However, a single discontinuity introduces $O(1)$ spurious oscillations near the discontinuity and reduces the high order convergence rate to first order, Gibbs' Phenomena. Although a direct expansion of the function in terms of its global moments yields this low order approximation, high resolution information is retained in the global moments. Two techniques for the resolution of the Gibbs' phenomenon are discussed, filtering and reprojection methods. An adaptive filter with optimal joint time-frequency localization is presented, which recovers a function from its N term Fourier projection within the error bound $\exp(-Nd(x))$, where $d(x)$ is the distance from the point being recovered to the nearest discontinuity. Symmetric filtering, however, must sacrifice accuracy when approaching a discontinuity. To overcome this limitation, Gegenbauer postprocessing was introduced by Gottlieb, Shu, et al., which recovers a function from its N term Fourier projection within the error bound $\exp(-N)$. An extension of Gegenbauer postprocessing with improved convergence and robustness properties is presented, the robust Gibbs complements. Filtering and reprojection methods will be put in a unifying framework, and their properties such as robustness and computational cost contrasted.

Best Approximation of Random Data

A. Torokhti[a], P. Howlett[a], and C. Pearce[b]

[a]School of Mathematics and Statistics, University of South Australia, SA 5095, Australia

[b]Department of Applied Mathematics, University of Adelaide, SA 5005, Australia

We propose and justify a new approach to constructing optimal *nonlinear* transforms of random vectors.

The *linear* reduced-rank transforms have successfully been applied to the solution of problems related to data compression, filtering, clustering, feature selection, forecasting, etc. Such a diversity of applications has stimulated a considerable increase in the study of reduced-rank transforms in recent decades. In this paper, a further advance in the development of reduced-rank transforms is presented.

A motivation for the proposed approach arises from the following observation. In general, the reduced-rank transform consists of the three companion operations which are filtering, compression and reconstruction. Filtering and compression are performed simultaneously to estimate a reference signal $\mathbf{x}$ with m components from noisy observable data $\mathbf{y}$ and to reduce that to a shorter vector $\hat{\mathbf{x}}$ with η components, $\eta < m$. Components of $\hat{\mathbf{x}}$ are often called principal. The quotient η/m is called the compression ratio. Reconstruction returns a vector $\tilde{\mathbf{x}}$ with m components so that $\tilde{\mathbf{x}}$ should be close to the original $\mathbf{x}$. It is natural to perform these three operations so that the reconstruction error and the related computational burden are minimal.

As a result, the performance of the reduced-rank transform is characterised by three issues which are (i) associated accuracy, (ii) compression ratio, and (iii) computational work.

For a given compression ratio, the Karhunen-Loève transform (KLT) minimizes the reconstruction error over the class of all *linear* reduced-rank transforms. Nevertheless, it may happen that the accuracy and compression ratio associated with the KLT are still not satisfactory. In such a case, an improvement in the accuracy and compression ratio can be achieved by a transform with a more general structure than that of the KLT. In particular, such *non-linear* transforms have been studied in [1, 2], where the transform structures have been developed from the generalised operator Volterra polynomials. We note that the transforms [1, 2] imply a substantial computational burden associated with the large number of terms caused by the underlying Volterra polynomial structure.

Our objective is to justify a new transform that may have both accuracy and compression ratio better than those of the known transforms. An associated objective is to find a way to reduce the computational work compared with that implied by the transforms studied in [1, 2].

References

[1] P. G. Howlett, A. P. Torokhti, C. E. M. Pearce, A Philosophy for the Modelling of Realistic Non-linear Systems, *Proc. of Amer. Math. Soc.*, vol. 132, pp. 353-363, 2003.

[2] A. Torokhti and P. Howlett, Constructing Fixed Rank Optimal Estimators with Method of Recurrent Best Approximations, *J. Multivariate Analysis,* vol. 86, pp. 293-309, 2003.

Sobolev Error Estimates for Scattered Data Interpolation via Radial Basis Functions

Holger Wendland

Institut für Numerische und Angewandte Mathematik, Lotzestr. 16-18, Universität Göttingen, Germany

Error estimates for scattered-data interpolation via radial basis functions for target functions in the associated reproducing kernel Hilbert space have been known for a long time. Recently, these estimates have been extended to apply to certain classes of target functions generating the data which are outside of the associated RKHS. However, these classes of functions still were not "large" enough to be applicable to a number of practical situations. In this talk I will address Sobolev-type error estimates on compact regions of $\mathbb{R}^n$ when the RBFs have Fourier transforms that decay algebraically. These error estimates finalize the theory for such basis functions.

If time permits I will present numerical tests to corroborate the theoretical results.

This talk is based on joint work with Francis J. Narcowich and Joseph D. Ward from Texas A&M University.

WORKSHOP 8

Computational algebraic geometry

ORGANISERS:

Teresa Krick & Andrei Gabrielov

On Arithmetic Aspects of Sparse Resultants

Carlos D'Andrea

Department of Mathematics & The Miller Institute for Basic Research in Science.
University of California, Berkeley 94720 USA
cdandreamath.berkeley.edu

Let $\mathcal{A}_0, \ldots, \mathcal{A}_n \subset \mathbb{Z}^n$ be finite sets of integer vectors, and $\mathrm{Res}_{\mathcal{A}_0,\ldots,\mathcal{A}_n} \in \mathbb{Z}[U_0, \ldots, U_n]$ the associated mixed sparse resultant, which is a polynomial in $n+1$ groups $U_i := \{U_{ia}; a \in \mathcal{A}_i\}$ of $m_i := \#\mathcal{A}_i$ variables each.

Resultants are of fundamental importance for solving systems of polynomial equations and therefore have been extensively studied. Recent research has focused on arithmetic aspects of this polynomial such as its *height* and its *Mahler measure*

The absolute height of a polynomial $g = \sum_\alpha c_\alpha U^\alpha \in \mathbb{C}[U]$ is defined as $H(g) := \max\{|c_\alpha|,\, \alpha \in \mathbb{N}^m\}$, where $U = \{U_{ia}, i = 0, \ldots, n,\, a \in \mathcal{A}_i\}$ and $m := m_0 + \ldots + m_n$. Its (logarithmic) height is

$$h(g) := \log H(g) = \log \max\{|c_\alpha|,\, \alpha \in \mathbb{N}^m\}.$$

The Mahler measure of g is defined as

$$m(g) := \frac{1}{(2\pi\mathbb{I})^{n+1}} \int_{\mathbb{T}^{n+1}} \log|g(U_0, \ldots, U_n)| \frac{\mathrm{d}U_0}{U_0} \ldots \frac{\mathrm{d}U_n}{U_n},$$

where for $j = 0, \ldots, n$, $\frac{\mathrm{d}U_j}{U_j}$ is short for $\prod_{a\in\mathcal{A}_j} \frac{\mathrm{d}U_{ja}}{U_{ja}}$, and

$$\mathbb{T}^{n+1} = \{(z_0, \ldots, z_n) \in \mathbb{C}^{n+1} \,|\, |z_0| = \cdots = |z_n| = 1\}$$

is the $n+1$-torus.

Even though there are explicit upper bounds for both the height and the Mahler measure of resultants, very little seems to be known about the problem of computing explicitly both the height and the Mahler measure of the resultant.

In this talk, we will show how these two objects are related, and also some non trivial heights (joint work with Kevin Hare) and Mahler measures (joint work with Matilde Lalin) of resultants.

Some Examples in Computational Algebraic Geometry

Wolfram Decker
Universität des Saarlandes
Fachrichtung Mathematik
Postfach 151 150
D-66041 Saarbrücken

Systems of polynomial equations arise throughout mathematics, science, and engineering. Their solution sets are geometric objects which may be studied using algebraic means. Problems originating from this study are handled in algebraic geometry, a mathematical discipline of its own.

As a result of the historical development of algebraic geometry, there is nowadays a multitude of theoretical and highly abstract techniques for the qualitative and quantitative study of solution sets, without actually studying the equations at the first place.

The development of powerful computers and effective computer algebra algorithms at the end of the twentieth century, however, brings the study of concrete examples via their equations back to the center of interest. The novel computational methods make algebraic geometry accessible to experiments. The experimental method, which has proven highly successful in number theory, now also adds to the toolbox of the algebraic geometer.

In my talk, I discuss several examples of how explicit computations may help to do research in algebraic geometry. On my way, I will present code in two computer algebra systems, `Singular` and `Macaulay2`, discussing also some new features of these systems.

Binomial Complete Intersections

Eduardo Cattani[a] and Alicia Dickenstein[b]

[a]Department of Mathematics and Statistics. University of Massachusetts. Amherst, MA 01003, USA

[b]Departamento de Matemática, FCEyN. Universidad de Buenos Aires. (1428) Buenos Aires, Argentina

A binomial ideal in a polynomial ring is an ideal generated by binomials. Binomial ideals are quite ubiquitous in very different contexts particularly those involving toric geometry and its applications, and in the study of semigroup algebras. While binomial ideals are quite amenable to Gröbner and standard bases techniques, they also provide some of the worst-case examples in computational algebra, such as the Mayr-Meyer ideals. Thus, we are interested in algorithms that allow us to obtain information about binomial ideals purely in terms of the data defining them.

In this talk we will discuss how to determine when a binomial ideal is a zero-dimensional complete intersection and, if so, how we may compute the total number of solutions and the total multiplicity of solutions in the coordinate subspaces. These problems arise naturally in the computation of sparse discriminants and in the study of hypergeometric systems of differential equations, which we will briefly discuss.

Lower bounds in some problems of real Schubert calculus

Alexandre Eremenko, Andrei Gabrielov

Department of Mathematics, Purdue University
150 N. University St., West Lafayette, IN 47907-2067, USA

We single out some problems of Schubert calculus of subspaces of codimension 2 that have the property that all their solutions are real whenever the data are real. For other similar problems we prove non-trivial lower bounds for the number of real solutions. Our arguments explore the connection between subspaces of codimension 2 and rational functions of one variable.

Singularities May Reduce Complexities ... Unfortunately, not Always

Marc Giusti
Marc Giusti, LIX, École polytechnique, 91128 Palaiseau, France

In the first part of the talk, we will illustrate this paradigm in case of the following problem: find for each connected component of a real algebraic variety a sample point.

In the second part, we shall show how hidden degenerations of mathematical objects like varieties or polynomials generate obstructions for the efficiency of elimination algorithms.

Joint works with Bernd Bank, Joos Heintz, Guy Mbakop and Luis Miguel Pardo.

Tropical Geometry and Asymptotic Enumeration of Real Rational Curves

Ilia Itenberg

IRMA
Université Louis Pasteur
7, rue René Descartes
67084 Strasbourg Cedex
France

The goal of the talk is to give a brief introduction to tropical geometry and to present its applications in real enumerative geometry. In tropical geometry, complicated algebro-geometric objects are replaced by rather simple piecewise-linear ones. An important link between the complex algebraic world and the tropical one is given by Mikhalkin's correspondence theorem. This theorem together with the discovery of the Welschinger invariants (these invariants can be seen as a real analog of the Gromov-Witten invariants) give rise to several results concerning the enumeration of real rational curves on algebraic surfaces.

On the Computation of Resolvent Representations

Lisi D'Alfonso, Gabriela Jeronimo, and Pablo Solernó

Departamento de Matemática, Facultad de Ciencias Exactas y Naturales,
Universidad de Buenos Aires

The notion of a resolvent representation of a prime differential ideal in a ring of differential polynomials was introduced by Ritt as a tool toward an algebraic elimination theory in the realm of differential equations. Roughly speaking, a resolvent representation of a prime differential ideal provides a parametrization of the generic zeros of the ideal by the zeros of a single irreducible differential polynomial.

The talk will deal with the computation of resolvent representations of prime differential ideals associated with a class of ordinary first order multivariate polynomial differential systems. We will present upper bounds on the order and degree of the polynomials involved in a resolvent representation and we will exhibit a probabilistic algorithm which obtains this resolvent representation within time polynomial in the natural syntactic parameters and the degree of a certain algebraic variety related to the input system.

Unlike the previous methods, our algorithm does not require the computation of Gröbner bases or characteristic sets. Based on the computation of algebraic eliminating polynomials, our approach enables us to obtain complexity estimates in terms of a geometric invariant, which are more precise than those depending only on syntactic parameters.

Insolvability of Equations in Finite Terms

Askold Khovanskii
University of Toronto, Department of Mathematics, Canada

Abel, Galois, Liouville, Picard, Vessiot, Kolchin and others found a lot of results about solvability and insolvability of equations in finite terms. According to them, algebraic equations are usually not solvable by means of radicals. Ordinary linear differential equations and holonomic systems of linear differential equations in partial derivatives are not usually solvable by quadratures. Galois theory belongs to algebra. In fact results about insolvability of differential equations belong to differential algebra (and are also purely algebraic). About 30 years ago I constructed a topological version of Galois theory for functions in one complex variable. According to it, there are topological restrictions on the way the Riemann surface of a function representable by quadratures covers the complex plane. If the function does not satisfy these restrictions, then it is not representable by quadratures. Beside its geometric clarity the topological results on nonrepresentability of functions by quadratures are stronger than the algebraic results. Recently I have constructed a multi-dimensional topological version of Galois theory.

Quadratic Newton Iteration for Systems with Multiple Zeroes and Clusters of Zeroes

Grégoire Lecerf

CNRS UMR 8100, laboratoire de Mathématiques, Université de Versailles
Saint-Quentin-en-Yvelines, 45 avenue des États-Unis, 78035 Versailles, France

Newton's iterator is one of the most popular components of polynomial equation system solvers, either from the numeric or symbolic point of view. This iterator usually handles smooth situations only (when the Jacobian matrix associated to the system is invertible). This is often a restrictive factor. Generalizing Newton's iterator is still a challenging problem: how to design an efficient iterator with a quadratic convergence even in degenerate cases?

In this talk, we will present a symbolic algorithm for a I-adic topology when the ideal I can be chosen generic enough: compared to a smooth case we prove quadratic convergence with a small overhead that grows with the square of the multiplicity of the root.

Then we will present a numeric generalization of this algorithm to analytic maps. We will restrict to situations where the analytic map has corank one at the multiple isolated zero, which has embedding dimension one in the frame of deformation theory. These situations are the least degenerate ones and therefore most likely to be of practical significance. More generally, we define clusters of embedding dimension one. We provide a criterion for locating such clusters of zeroes and a fast algorithm for approximating them, with quadratic convergence. In case of a cluster with positive diameter our algorithm stops at a distance of the cluster which is about its diameter. These results are in the vein of the α-theory for simple zeroes, that was initiated by M. Shub and S. Smale in the beginning of the eighties.

The numeric generalization is joint work with Marc Giusti, Bruno Salvy and Jean-Claude Yakoubsohn.

On Computational Complexity of the Hilbert Polynomial

Peter Bürgisser and **Martin Lotz**

Department of Mathematics, University of Paderborn, D-33095 Paderborn, Germany
email: {pbuerg,lotzm}@upb.de

We study the computational complexity of the problem of computing the Hilbert polynomial of a complex projective variety V, given by a list of homogeneous input polynomials. In case V is smooth and equidimensional, we show that the problem can be reduced in deterministic polynomial time (in the sparse size of the input polynomials) to the problem of counting the number of complex common zeros of a finite set of multivariate polynomials. The reduction algorithm is based on ideas from complexity theory (elimination of generic quantifiers) and on a formula expressing the coefficients of the Hilbert polynomial in terms of the degrees of certain degeneracy loci. This formula is derived from well-known results in Schubert calculus and the Hirzebruch-Riemann-Roch theorem. The problem of counting the number of solutions of a system of polynomial equations can be solved in polynomial space. The general problem of computing the Hilbert polynomial of a homogeneous ideal can be shown to be at least as hard as the homogeneous ideal membership problem, a problem known to be polynomial-space hard.

Reference:
P. Bürgisser and M. Lotz. *The complexity of computing the Hilbert polynomial of smooth equidimensional complex projective varieties.*
www.arxiv.org/abs/cs/cs.CC/0502044.

Subdivision Methods for Solving Polynomial Equations

B. Mourrain & J.P. Pavone
GALAAD, INRIA, BP 93, 06902 Sophia Antipolis, France

We present a new algorithm for solving a system of polynomials, in a domain of $\mathbb{R}^n$. It uses a powerful reduction strategy based on univariate root finder using Bernstein basis representation and *Descarte's rule*. We analyse the behavior of the method, from a theoretical point of view, shows that for simple roots, it has a local quadratic convergence speed and gives new bounds for the complexity of approximating real roots in a box of $\mathbb{R}^n$. The improvement of our approach, compared with classical subdivision methods, is illustrated on geometric modelling applications such as computing intersection points of implicit curves, self-intersection points of rational curves, and on classical benchmarks.

On the Size of the Solutions of Sparse Polynomial Systems

Martin Sombra

Departament d'Algebra i Geometria, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain

We present sharp estimates for the size of the solution set of a sparse polynomial system, both in the function field and in the arithmetic settings. The obtained estimates make appear a new combinatorial invariant, the *mixed integral* of a family of concave functions. These results can be seen as analogues of the Bernstein-Kushnirenko theorem for rings of dimension 1.

The Nearest Multivariate System with Given Root Structure

Agnes Szanto

North Carolina State University, USA

Let $f_1, \ldots, f_s$ be polynomials in the variables $x_1, \ldots, x_n$ with finitely many common roots. Assume that either

(a) $f_1, \ldots, f_s$ is an over-constrained system (more equation than variables) which has k common roots,

or

(b) $f_1, \ldots, f_s$ has roots with multiplicities, which can be described by the vanishing of certain derivatives of $f_1, \ldots, f_s$ in the roots

However, even small perturbation of the coefficients can destroy completely the above root structures. This is the reason that in numerical computations handling the above systems is a major challenge: convergence to the solution is slow and the output is unreliable, or no output is returned.

In this talk we propose iterative methods, which for a given (perturbed) system $F_1, \ldots, F_s$ and given root structure, computes the nearest system $f_1, \ldots, f_s$ which has roots with the given structure. The method also computes the common roots of $f_1, \ldots, f_s$ simultaneously.

This is a joint work with Scott Pope (NCSU), Olivier Ruatta (Université Limoges) and Mark Sciabica (NCSU).

Flattening of Morphisms and Applications to Singularities over Perfect Fields

Orlando Villamayor

Universidad Autonoma de Madrid

Let A be a noetherian domain and M a finitely generated A-module. A theorem of Lipman provides, in a natural manner, a projective birational morphism $f : X \to Spec(A)$ with the property that the coherent sheaf $f^*(M)$, modulo torsion, is locally free over X. Furthermore, the morphism f is universal with this property.

In the particular case in which M is an ideal in A, then, as expected, the morphism f is simply the blow-up of $Spec(A)$ at M.

In general there is a procedure to compute an ideal I, in terms of a presentation of the A-module M, so that the universal morphism f is the blow-up at I. Despite the fact that the computation of I is difficult from the point of view of its complexity, some examples will be presented.

We will discuss how these morphisms relate with Grothendieck's theorem of flattening of projective morphisms.

Finally we will indicate some suggestive applications of these techniques to the study of singularities over perfect fields.

Topological Complexity of Definable Sets

Nicolai Vorobjov

Department of Computer Science, University of Bath, Bath, BA2 7AY, UK

The talk will explain the recent progress in proving upper bounds on Betti numbers of semialgebraic and sub-Pfaffian sets defined by first-order formulae with and without quantifiers (joint work with Andrei Gabrielov).

WORKSHOP 9

Computational number theory

ORGANISERS:

Alan Lauder & Jonathan Pila

Integer factorization: a progress report

Daniel J. Bernstein

Department of Mathematics, Statistics, and Computer Science,
The University of Illinois at Chicago,
MC 249,
851 S. Morgan Street,
Chicago, IL 60607–7045

There have been several recent improvements to the number-field sieve. I'll explain some of what's going on.

Applications of pairings in cryptography

Steven D. Galbraith
Mathematics Department, Royal Holloway University of London, Egham, Surrey TW20 0EX, UK.

I will survey some applications of pairings on elliptic curves over finite fields in public key cryptography. I will also present some new results on efficient implementation of pairings. Some of the results presented may include joint work with James McKee, Paula Valença, Victor Rotger, Jordi Pujolas, Paulo Barreto, Colm O hEigeartaigh and Mike Scott.

Explicit determination of the dihedral Galois group of irreducible polynomials

M. Ángeles Gómez-Molleda

Dpto. Álgebra, Geometría y Topología
Universidad de Málaga
29071 Málaga, Spain
gomezma@uma.es

Algorithms are known to determine whether the Galois group of an irreducible polynomial $f \in \mathbb{Q}[x]$ of odd degree is dihedral. We present a characterization of dihedral Galois groups that allows to extend the algorithms to even degree polynomials.

Deformation and the cohomology of Monsky and Washnitzer

Hendrik Hubrechts

Celestijnenlaan 200B, 3001 Heverlee (Belgium)

In order to calculate the zeta function of a hyperelliptic curve over finite characteristic, it is possible to combine the deformation theory of Dwork with the MW cohomology. The idea is to use an equation of the form $Y^2 = Q(X, \Gamma)$, where the situation for $\Gamma = 0$ is known, and the goal is to find the matrix of Frobenius for the other members of the family.

Following a suggestion of Lauder, we determine how to construct the necessary convergence criterions to make this theory work. We will briefly mention how to use the theory to calculate the number of points, with an explanation of the main advantage of the method, namely a reduction in the space complexity.

Solvability of a system of polynomial equations over a finite field

Neeraj Kayal

Department of Computer Science, IIT Kanpur, India 208016

We investigate the complexity of the following polynomial solvability problem: given a finite field $\mathbb{F}_q$ and a set of polynomials $f_1, f_2, \cdots, f_m \in \mathbb{F}_q[x_1, x_2, \cdots, x_n]$ determine the $\mathbb{F}_q$-solvability of the system $f_1 = f_2 = \cdots = f_m = 0$. That is, we want to determine whether there exists a point $\mathbf{a} \in \mathbb{F}_q^n$ such that

$$f_1(\mathbf{a}) = f_2(\mathbf{a}) = \cdots = f_m(\mathbf{a}) = 0$$

This problem is easily seen to be **NP**-complete even when the field size q is as small as 2 and the degree d of each polynomial is bounded by 2. We examine the deterministic complexity of this problem when the number of variables, n, is fixed. We show that for a fixed n, there is a *deterministic* algorithm for this problem whose running time is bounded by a polynomial in d, m and $\log q$.

Perfect powers having equal digits but one

Omar Kihel

Brock University, St. Catharines, ON, Canada

The problem of finding all perfect powers that have equal digits was initiated by R. Oblath. The problem is now completely solved. A. Gica and L. Panaitopol determined all squares having all digits equal but one. They asked to solve the problem for higher powers. We will consider the question for cubes in base 10 and discuss the problem for other basis. This is a joint work with F. Luca.

Effective p-adic estimates in rigid cohomology

Alan Lauder

Mathematical Institute, 24-29 St Giles, Oxford OX1 3LB, U.K.

Rigid cohomology is a Weil cohomology theory for varieties over finite fields. It has a very explicit description in terms of differential forms which has made it useful for the (machine) computation of zeta functions of varieties over finite fields, i.e., for calculating the number of solutions to polynomial equations over finite fields. One problem which arises in such computations is to determine the (p-adic) accuracy needed to ensure the final answer is correct. The purpose of this talk is to explain both the problem and also the results which have been obtained on it to date.

Computations with L-functions associated to Hecke eigenforms

Atul Pokharel[a], and Michael Rubinstein[b]

[a]Princeton University, Princeton, NJ, USA

[b]University of Waterloo, Waterloo, Ontario, Canada

I'll discuss some algorithms and experiments related to the computation of L-functions associated to Hecke eigenforms for the full modular group.

The Riemann zeta-function – an atlas of the real world –

Rasa Šleževičienė-Steuding[a] and Jörn Steuding[b]

[a]Rasa Šleževičienė-Steuding
Department of Physics and Mathematics, Šiauliai University
Višinskio str. 19, 77 156 Šiauliai, Lithuania
rasa.slezeviciene@fm.su.lt

[b]Jörn Steuding
Departamento de Matemáticas, Universidad Autónoma de Madrid
C. Universitaria de Cantoblanco, 28 049 Madrid, Spain
jorn.steuding@uam.es

Voronin's universality theorem states, roughly speaking, that any non-vanishing analytic function $g(s)$, defined on some sufficiently small disk, can be uniformly approximated by shifts of the Riemann zeta-function $\zeta(s+i\tau)$ in the critical strip; moreover the set of these approximating shifts τ has positive lower density. This remarkable property has impact on the zero-distribution, e.g., the Riemann hypothesis (i.e., all non-real zeros of $\zeta(s)$ lie on the critical line $\mathrm{Re}\, s = 1/2$) is true if and only if the zeta-function can approximate itself uniformly (in the sense of Voronin's theorem). The known proofs of Voronin's universality theorem are ineffective, giving neither an estimate for the first approximating shift τ nor bounds for the positive lower density, at least not for functions $g(s)$ satisfying the general conditions of Voronin's theorem. Recently, Garunkštis proved an effective version for a 'small' class of functions $g(s)$, and, in particular, an explicit bound for the first approximation τ. Hence, it makes sense to ask for an algorithm which detects this τ. We present such an algorithm which terminates in finite time, at least for the aforementioned class of functions $g(s)$.

Generalizing Kedlaya's order counting based on Miura theory

Joe Suzuki

Department of Mathematics, Graduate School of Science, Osaka University

K. Kedlaya proposed a method to count the number of $\mathbb{F}_q$-rational points on a hyper-elliptic curve, using the Leschetz fixed points formula in Monsky-Washinitzer Cohomology. The method has been extended to super-elliptic curves (Gaudry and Gürel) immediately, to characteristic two hyper-elliptic curves, and to C_{ab} curves (J. Denef, F. Vercauteren). Based on Miura theory, which is associated with how a curve is expressed as an affine variety, this paper applies Kedlaya's method to so-called strongly telescopic curves which are no longer plane curves and contain super-elliptic curves as special cases.

The discrete logarithm problem on algebraic tori

Robert Granger[a] and Frederik Vercauteren[b]

[a]University of Bristol, Department of Computer Science,
Merchant Venturers Building, Woodland Road,
Bristol, BS8 1UB, United Kingdom

[b]Department of Electrical Engineering
University of Leuven
Kasteelpark Arenberg 10, B-3001 Leuven-Heverlee, Belgium

Using a recent idea of Gaudry and exploiting rational representations of algebraic tori, we present an index calculus type algorithm for solving the discrete logarithm problem that works directly in these groups. Using a prototype implementation, we obtain practical upper bounds for the difficulty of solving the DLP in the tori $T_2(\mathbb{F}_{p^m})$ and $T_6(\mathbb{F}_{p^m})$ for various p and m.

WORKSHOP 10

Multiresolution and adaptivity in numerical PDEs

ORGANISERS:

Wolfgang Dahmen, Bob Russell & Endre Süli

Scale-free adaptive methods for partial differential equations

Chris J. Budd

University of Bath, UK

One of the key qualitative features of the solution of a nonlinear partial differential equation is the evolution of behaviour on many different length scales. Examples are the continuum of length scales in turbulence modelling, the formation of shocks in fluid mechanics and the appearance of singularities in nonlinear optics. Traditional numerical methods perform poorly in such situations as the discretisation imposes an artificial length scale on the problem. Ideally, to give the correct qualitative behaviour a numerical method should perform equally well at all length scales. I will show in this talk that by using a mixture of ideas from Lie group theory to meteorology it is possible to construct adaptive numerical methods which can effectively capture many different scales of behaviour. I will illustrate this with examples from optics and combustion.

An adaptive multiscale semi-Lagrangian method for the Vlasov-Poisson equation

Martin Campos[a], Albert Cohen[b], and Michel Mehrenberger[c]

[a]Laboratoire J.L.Lions, Université P. & M. Curie, Paris

[b]Laboratoire J.L.Lions, Université P. & M. Curie, Paris

[c]Département de mathématiques, Université Louis Pasteur, Strasbourg

An adaptive multiscale scheme is proposed for the discretization of the Vlasov-Poisson equation. This scheme is based on a semi-Lagrangian discretization : at each time step, the solution is transported and re-interpolated on a grid which differs from the previous one. The strategy for transporting the grid is designed in such a way that the anticipated interpolation error is controlled by a prescribed tolerance in the uniform norm, which allows us to derive rigorous error estimates. Numerical results illustrate the ability of the scheme to capture the filamental structures developed by the solution with an optimal balance between accuracy and complexity.

Convergence Analysis of Adaptive MFEM and NFEM

Carsten Carstensen[a] and Ronald H.W. Hoppe[bc]

[a]Inst. of Math., Humboldt Univ. of Berlin, D-10099 Berlin, Germany

[b]Dept. of Math., Univ. of Houston, Houston, TX 77204-3008, U.S.A.

[c]Inst. of Math., Univ. of Augsburg, D-86159 Augsburg, Germany

We are concerned with the development, analysis and implementation of adaptive mixed and nonconforming finite element methods (MFEM and NFEM) for second order elliptic PDEs. In case of standard conforming P1 approximations, such methods have been considered previously in [3]. The methods presented in this contribution provide an error reduction and thus guarantee convergence of the adaptive loop which consists of the essential steps 'SOLVE', 'ESTIMATE', 'MARK', and 'REFINE'. Here, 'SOLVE' stands for the efficient solution of the finite element discretized problems. The following step 'ESTIMATE' is devoted to the a posteriori error estimation of the global discretization error. A greedy algorithm is the core of the step 'MARK' to indicate selected elements for refinement, whereas the final step 'REFINE' deals with the technical realization of the refinement process itself. The analysis is carried out for the Poisson equation with homogeneous Dirichlet boundary data as a model problem and discretization by the lowest order Raviart-Thomas and Crouzeix-Raviart finite elements. Important tools in the convergence proof are the reliability of the estimator, a strong discrete local efficiency, and quasi-orthogonality properties. The proof does not require regularity of the solution nor does it make use of duality arguments.

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Towards multilevel meshless methods

Wolfgang Dahmen[a], Shai Dekel[b], and Pencho Petrushev[c]

[a]Institut für Geometrie und Praktische Mathematik, RWTH Aachen, 52056 Aachen, Germany

[b]Department of Mathematics, University of Tel Aviv, Tel Aviv, Israel

[c]Department of Mathematics, University of South Carolina, Columbia, SC 29208, U.S.A.

A scale of nonlinear smoothness spaces is presented that is capable among other things to capture anisotropic features. It can be based on multiscale collection s of atoms that are products of polynomials and smooth compactly supported cutoff functions forming on each scale a partition of unity. It is indicated under which circumstances representations in terms of such atoms characterize the above mentioned smoothness classes and related best N-term approximation. Possible applications concern meshless methods for the numerical treatment of elliptic boundary value problems based on such collections of atoms. Possible ways of deriving preconditioners from such norm equivalences are discussed that work, in particular, for adaptive approximations.

A new class of anisotropic spaces

Ronald A. DeVore

CSCAMM, U. of Maryland

It is the contention of this talk that classical smoothness spaces such as Sobolev and Besov spaces are not sufficient for measuring regularity for some problems in nonlinear PDEs and image processing. We propose to develop new classes of smoothness spaces based on level sets. These spaces should be truly anisotropic and have no coordinate axes bias. A starting definition for these spaces will be given and some of their elementary properties including computability will be discussed.

Fast and Reliable Methods for Determining the Evolution of Uncertain Parameters in Differential Equations

Donald Estep[a] and David Neckels[b]

[a]Department of Mathematics
Colorado State University
Fort Collins, CO 80523

[b]Department of Mathematics
Colorado State University
Fort Collins, CO 80523

A very common problem in science and engineering is the determination of the effects of uncertainty or variation in parameters and data on the output of a deterministic nonlinear operator. The Monte-Carlo Method is a widely used tool for understanding such effects that employs random sampling of the input space in order to produce a pointwise representation of the output. It is a robust and easily implemented tool. Unfortunately, it generally requires sampling the operator very many times at a significant cost. Moreover, it provides no robust measure of the error of information computed from a particular representation. We present an alternative approach for ascertaining the effects of variations and uncertainty in parameters in a differential equation that is based on techniques borrowed from a posteriori error analysis for finite element methods. The information from our approach can be used either to create a higher order method or produce an error estimate for information computed from a given representation. In the latter case, this provides the basis for adaptive sampling. Both the higher order method and the adaptive sampling methods are generally orders of magnitude faster than Monte- Carlo methods in a variety of situations.

Discontinuous Galerkin Finite Element Methods for Second–Order Elliptic PDEs: A Posteriori Error Estimation and Adaptivity

Paul Houston

Department of Mathematics, University of Leicester, Leicester LE1 7RH, UK
Paul.Houston@mcs.le.ac.uk

In recent years there has been considerable interest in the mathematical design and practical application of nonconforming finite element methods that are based on discontinuous piecewise polynomial approximation spaces; such approaches are referred to as discontinuous Galerkin (DG) methods. The main advantages of these methods lie in their conservation properties, their ability to treat a wide range of problems within the same unified framework, and their great flexibility in the mesh-design. Indeed, DG methods can easily handle non-matching grids and non-uniform, even anisotropic, polynomial approximation degrees, which makes them ideally suited for application within adaptive finite element software.

In this talk we present an overview of some recent developments concerning the *a posteriori* error analysis and adaptive mesh design of h– and hp–version DG finite element methods for the numerical approximation of second–order elliptic boundary value problems. In particular, we consider the derivation of computable upper and lower bounds on the error measured in terms of an appropriate (mesh–dependent) energy norm. The proofs of the upper bounds are based on rewriting the method in a non-consistent manner using polynomial lifting operators and employing an appropriate decomposition result for the underlying discontinuous spaces. Applications to the numerical approximation of second–order linear elliptic problems, including Poisson's equation, Stokes equations, nearly–incompressible elasticity, and the time harmonic eddy current problem, as well as second–order quasilinear boundary value problems, which typically arise in the modelling of non-Newtonian flows, will be considered. Numerical experiments confirming the reliability and efficiency of the proposed *a posteriori* error bounds within an automatic mesh refinement algorithm employing both local mesh subdivision and local polynomial enrichment will be presented.

This research has been carried out in collaboration with Dominik Schötzau (University of British Columbia), Thomas Wihler (University of Minnesota), Ilaria Perugia (University of Pavia), and Endre Süli (University of Oxford).

Metric tensors for anisotropic mesh adaptation

Weizhang Huang

Department of Mathematics, University of Kansas

Significant improvements in accuracy and efficiency can be gained if a mesh is properly chosen such that the shape, size, and orientation of its elements adapt to the features of the physical solution. Such an anisotropic mesh is often generated in practice as an isotropic one in the metric determined by a tensor specifying the shape, size, and orientation of elements throughout the physical domain. It is thus crucial to choose an appropriate metric tensor in anisotropic mesh generation and adaptation. In this talk, metric tensors for this purpose are presented based on error estimates for polynomial preserving interpolation on simiplicial elements. Two dimensional numerical results are given to demonstrate the ability of the metric tensor to produce anisotropic meshes with correct mesh concentration and good overall quality.

Adaptive Wavelet Methods for Linear-Quadratic Elliptic Control Problems

Angela Kunoth

Inst. f. Angewandte Mathematik, Wegelerstr. 6, Univ. Bonn, 53115 Bonn, Germany

The numerical solution of PDE-constrained control problems requires to repeatedly solve a system of PDEs for the involved variables (state, costate and control). Specifically constraints in form of a linear elliptic PDE with the practically most relevant case of a Dirichlet boundary control are considered, which are formulated as a saddle point problem. In addition, inequality constraints on the control may be posed.

The proposed numerical solution scheme is based on wavelet expansions. Striving for efficiency and optimal complexity, I will address preconditioning issues, the selection of appropriate norms in the control functional, and adaptive methods for the resolution of singularities. An adaptive algorithm for the system of optimality conditions is presented, together with remarks on the convergence and convergence rates which yield asymptotically optimal results when compared to wavelet-best N-term approximations of the relevant variables. Also some numerical experiments are discussed.

On multiscale simulations of polymeric fluid flows

Claude LeBris

CERMICS,
Ecole Nationale des Ponts et Chaussees,
6 & 8 avenue Blaise Pascal,
Cite Descartes,
Champs-sur-Marne,
77455 Marne La Vallee Cedex 2, FRANCE
lebris@cermics.enpc.fr

We will address various questions arising in the modelling of polymeric fluid flows. The micro-macro simulations couple a deterministic macro description of the fluid with a stochastic description of the microscopic scale.

Convergence of AFEM for General Elliptic Operators and the Laplace-Beltrami Operator

Ricardo H. Nochetto
University of Maryland
rhn@math.umd.edu

We first review the basic principles for convergence of adaptive finite element methods (AFEM) for the Laplace operator, as shown recently by P. Morin, R.H. Nochetto, and K. Siebert. Marking according to a posteriori error estimators and data oscillation, followed by refinement with an interior node property, guarantees energy and data oscillation reduction and leads to convergence of adaptive loops with a linear rate. This result hinges on an orthogonality property for the energy norm as well as the crucial fact that energy error and data oscillation decouple for the Laplace operator. This is no longer true for general elliptic operators with variable coefficients and the Laplace-Beltrami operator on surfaces. In both cases the energy error couples with either the approximation of coefficients or the surface (geometric error). We next present novel error/oscillation and error/geometry reduction estimates, based on quasi-orthogonality properties, which show that AFEM are a contraction for their collective contribution. We also illustrate the theory with numerical experiments, which lead to optimal meshes. These results are joint with K. Mekchay and P. Morin.

An a Posteriori Modeling Error Analysis for Free Surface Flows

Simona Perotto

MOX–Modeling and Scientific Computing, Dipartimento di Matematica "F. Brioschi", Politecnico di Milano, Via Bonardi 9, 20133 Milano, Italy (simona.perotto@mate.polimi.it).

Physical phenomena, taking place on large domains, are usually characterized by different space and time scales due to the coexistence of heterogeneous physical features, often localized in specific portions of the domain. This is the main difficulty involved, for instance, in the simulation of the flow field around an airfoil (where the equations for compressible and incompressible fluids are merged) [5], or of the motion of water in a complex hydrodynamic configuration, such as a channel network or a river delta [2, 3].

In view of a mathematical modeling of these phenomena, it is reasonable to resort to a suitable coupling of models rather than to a single model. The idea consists of using simultaneously different mathematical models, thus solving the most complex ones only where it is strictly necessary. With this aim, two different coupling strategies can be essentially pursued: one can decide to couple dimensionally homogeneous but physically heterogeneous models, or vice-versa, dimensionally heterogeneous but physically homogeneous models. Moreover, the matching "subdomain-to-model" can be decided *a priori*, driven, for example, by proper physical considerations, or, alternatively, can be dictated by an automatic tool provided by a suitable *a posteriori* model error analysis.

In the framework of the simulation of the free surface flows, we have analyzed the *a priori* dimensionally heterogeneous-physically homogeneous coupling of the well-known shallow water equations (2D and 1D, respectively) [2] as well as the *a posteriori* dimensionally homogeneous-physically heterogeneous coupling of Saint-Venant like equations [3, 4]. This communication essentially focus on this last strategy. In particular, we derive an *a posteriori* modeling error estimator for the shallow water equations, by extending the dual-based approach provided in [1] for steady equations to the case of a (generic) time-dependent problem. This error estimator is successfully validated on some standard hydrodynamic configurations, such as a channel with an obstacle and a river bifurcation.

The actual ambitious goal consists of extending the a posteriori approach to a dimensionally heterogeneous-physically homogeneous coupling.

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Adaptive semi-discrete central-upwind schemes for systems of conservation laws

Smadar Karni[a], Alexander Kurganov[b], Guergana Petrova[c], and Bojan Popov[c]

[a]Department of Mathematics, University of Michigan, Ann Arbor, MI 48109

[b]Mathematics Department, Tulane University, New Orleans, LA 70118

[c]Department of Mathematics, Texas A&M University, College Station, TX 77840

We present two adaptive strategies for solving hyperbolic (systems of) conservation laws.

The first one is based on a smoothness indicator that measures the weak local truncation error of the approximate solution and this way identifies the "rough" solution regions. This is a universal method that is not restricted to a particular discretization scheme. We implement it in scheme adaption and mesh adaption algorithms, using the central-upwind schemes.

The second adaptive strategy concerns nonconvex hyperbolic conservation laws. It uses the fact that the reconstruction step in Godunov type projection evolution methods is crucial in computing the unique entropy solution to nonconvex (systems of) conservation laws and may lead to numerical approximations that converge to different weak solutions. We propose a simple adaptive algorithm that simultaneously captures the entropy solution and achieves high resolution.

Optimality of a standard adaptive finite element method

Rob Stevenson
Department of Mathematics Utrecht University, The Netherlands.

We present a construction an adaptive finite element method for solving elliptic equations that has optimal computational complexity. Whenever for some $s > 0$, the solution can be approximated to accuracy $\mathcal{O}(n^{-s})$ in energy norm by a continuous piecewise linear function on some partition with n triangles, and one knows how to approximate the right-hand side in the dual norm with the same rate with piecewise constants, then the adaptive method produces approximations that converge with this rate, taking a number of operations that is of the order of the number of triangles in the output partition. The method is similar in spirit to that from [*SINUM*, 38 (2000), pp.466–488] by Morin, Nochetto, and Siebert, and so in particular it does not rely on a recurrent coarsening of the partitions. Although the Poisson equation in two dimensions with piecewise linear approximation is considered, it can be expected that the results generalize in several respects.

WORKSHOP 11

Numerical linear algebra

ORGANISERS:

Lothar Reichel & Steve Vavasis

Model-based methods for computing extreme eigenpairs of definite matrix pencils

P.-A. Absil, C. G. Baker, and K. A. Gallivan

School of Computational Science, Florida State University, Tallahassee FL 32306-4120, USA
(http://www.csit.fsu.edu/{~absil,~cbaker,~gallivan})

The generalized eigenvalue problem

$$Ax = \lambda Bx,$$

where A and B are $n \times n$ real symmetric matrices with B positive definite, arises in many scientific applications. The symmetric/positive-definite pencil (A, B) is known to admit n real eigenvalues $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_{n-1} \leq \lambda_n$ with associated B-orthonormal eigenvectors $v_1, \ldots, v_n$ (see [Ste01]). We consider the problem of computing extreme eigenpairs (λ_1, v_1) or (λ_n, v_n), which we call *leftmost* and *rightmost* eigenpair, respectively.

Most methods for computing extreme eigenpairs can be classified into two categories: (i) methods that (almost) always converge to an extreme eigenvector, but with a local linear convergence that can be very slow; (ii) methods that, in contrast, have local superlinear (usually, cubic) convergence, but without global convergence to extreme eigenvectors. It is natural to think of combining the individual advantages of these two categories and obtain an iteration for which convergence holds globally and is superlinear. In addition, with a view to dealing with large-scale problems, we require that the method be "matrix-free" in the sense that A and B are only used through matrix-vector products.

There is *a priori* evidence that such a method can come from an optimization approach; indeed, for the problem of finding a minimum of a smooth cost function on a Euclidean space, the model trust region scheme proposed by Steihaug [Ste83] and Toint [Toi81], where the trust-region subproblems are approximately solved using a truncated CG inner iteration, possesses a similar combination of advantages. In this talk, we will (i) recall how the problem of computing extreme eigenpairs of (A, B) can be rewritten in the form of an optimization problem on a smooth set (Riemannian manifold); (ii) present a recently-proposed truncated-CG scheme on Riemannian manifolds [ABG04, ABG05]; (iii) apply this scheme to the computation of extreme eigenpairs of definite pencils, to obtain a matrix-free, globally convergent, locally superlinear algorithm with minimal memory storage requirements; (iv) illustrate on numerical experiments that this new algorithm outperforms Krylov-based methods on several problems; (v) show how the particular structure of the eigenproblem makes it possible to further improve the performance of the trust-region-based algorithm. Some details follow.

It is well known that the leftmost and rightmost eigenvectors of (A, B) can be expressed as minimizers and maximizers of the Rayleigh quotient

$$\mathbb{R}_0^n \to \mathbb{R} : y \mapsto \frac{y^T A y}{y^T B y}, \tag{3}$$

where $\mathbb{R}_0^n$ denotes $\mathbb{R}^n$ without the origin. More precisely, assuming that $\lambda_1 < \lambda_2$ and $\lambda_{n-1} < \lambda_n$,

$$\frac{v_1^T A v_1}{v_1^T B v_1} < \frac{y^T A y}{y^T B y} < \frac{v_n^T A v_n}{v_n^T B v_n}$$

for all y that are collinear with neither v_1 nor v_n. The difficulty is that the optimizers of (3) are not isolated: all the points αv_1, $\alpha \in \mathbb{R}_0$, are minimizers, and all the points αv_n, $\alpha \in \mathbb{R}_0$, are maximizers. This is a cause of major difficulties of practical and theoretical nature; for example, applying the Newton method to the Rayleigh quotient (3) in $\mathbb{R}^n$ yields the iteration mapping $x \mapsto 2x$, from which no information can be drawn. A remedy to this difficulty is to impose some normalization condition on y that picks typically one or two allowed points in each (or almost each) line $\{\alpha y : \alpha \in \mathbb{R}^n\}$. This was recognized in the early work of Bradbury and Fletcher [BF66] where several normalization conditions were considered (such as $\|y\|_1 = 1$, $\|y\|_2 = 1$ and $\|y\|_\infty = 1$) and a nonlinear conjugate-gradient optimization approach was proposed. For the generalized eigenproblem, we propose to use the normalization $\|y\|_B = 1$, where $\|y\|_B := \sqrt{y^T B y}$; this particular normalization yields simplifications in the forthcoming developments. The optimization problem is thus to minimize or maximize the cost function

$$f : \{y \in \mathbb{R}^n : y^T B y = 1\} \to \mathbb{R} : y \mapsto \frac{y^T A y}{y^T B y}. \tag{4}$$

The minimizers are $\pm v_1$ and the maximizers are $\pm v_n$, i.e., the eigenvectors of (A, B) associated with the extreme eigenvalues.

The remaining issue is to adapt the classical (Euclidean) Steihaug approach to the minimization of f constrained to the appropriate manifold setting. In recent work [ABG04, ABG05], we proposed a generalization of trust-region methods, and of the truncated CG algorithm in particular, to Riemannian manifolds. The technique consists in (approximately) solving trust-region problems on a sequence of tangent planes at the successive iterates. More precisely, given a current iterate x_k on the manifold, the cost function is lifted to the tangent plane to the manifold at x_k (which is a Euclidean space) using a *retraction*; a quadratic model of the lifted function is (approximately) optimized within a trust-region using a truncated CG algorithm; finally, the computed optimizer on the tangent plane is warped back to the manifold via the retraction, to obtain a new iterate. Then the same process is repeated from the new iterate. The local and global convergence properties of this optimization scheme were analyzed in [ABG05], where it was shown that the favorable properties of the original Euclidean scheme are preserved: under mild regularity conditions, the iteration has strong global convergence properties to minimizers of the cost function; the local convergence is superlinear with an order in general bounded above by two (quadratic convergence) and selected using a parameter appearing in the inner stopping criterion.

The first algorithm we consider in this talk is nothing more than a particularization of this general algorithm to the optimization of the Rayleigh quotient (4) on the manifold $\{y : y^T B y = 1\}$. We will report on numerical experiments where the new method outperforms a recently-proposed Krylov-based method for the generalized eigenproblem [GY02]. We will also report on a block version of the algorithm;

this block version is derived as a trust-region algorithm for minimizing a generalized Rayleigh quotient defined on the Grassmann manifold of fixed-dimensional subspaces of $\mathbb{R}^n$.

We will also present modifications of the methods that take advantage of the particular structure of the eigenvalue problem. This includes the use of preconditioners; a dynamic shift strategy inspired from Sameh and Wisniewski's dynamically shifted Tracemin algorithm [SW82, ST00]; and a subspace acceleration technique *à la* Davidson [Dav75, SV96]. Interestingly, the subspace acceleration strategy is only used for speed-up purposes; global convergence holds without it. We will show how all these variants fit into a framework of model-based optimization methods.

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Solving structured Markov chains: numerical methods and open problems

D.A. Bini

Dipartimento di Matematica, Università di Pisa, Largo B. Pontecorvo, I-56127 Pisa

A wide variety of queueing models deriving from diverse applications lead to solving Markov chains where the stochastic matrix has a strong structure. In the most part of cases the matrices have infinitely many rows and columns, and the problem is ultimately reduced to computing the minimal nonnegative solution of matrix equations of the kind

$$X = \sum_{i=-1}^{+\infty} A_i X^{i+1} \tag{5}$$

where X and A_i are $m \times m$ matrices, $A_i \geq 0$, $\sum_{i=-1}^{+\infty} A_i$ is convergent and row stochastic.

We recall the main methods for numerical solving (5), point out their interplay with structured matrices, present some recent results relating (5) to canonical (Wiener-Hopf) factorizations and address some related open problems.

Darboux transformation with shift: Stability and sensitivity analysis

M.I. Bueno[a], and F.M.Dopico[b]
[a]Mathematics Department, The College of William and Mary
114 Jones Hall, P.O. Box 8795,
Williamsburg, VA 23187-8795, USA

[b]Departamento de Matemáticas, Universidad Carlos III de Madrid,
Avda. de la Universidad, 30,
28911 Leganés, Spain

In several problems of numerical analysis, given the orthogonal polynomials associated with a positive absolutely continuous measure defined in terms of a weight function $\omega(t)$ on an interval (a, b), and given a polynomial $\pi(t)$ of degree m which is nonnegative on (a, b), it is necessary to construct the orthogonal polynomials associated with $\pi(t)\omega(t)$ on the same interval. The parameters of the three-term recurrence relation that defines a sequence of orthogonal polynomials are the entries of a semi-infinite tridiagonal matrix called monic Jacobi matrix. Darboux transformation with shift is the process that produces the monic Jacobi matrix associated with $(t-\alpha)\omega(t)$ when the monic Jacobi matrix associated with $\omega(t)$ is known. Several authors as D. Galant, W. Gautschi, G. Golub and J. Kautsky have studied in the past an algorithm that implements such transformation and, based on numerical experiments, they assured that this algorithm "appears to be numerically stable". However, no formal analysis of the stability of the algorithm has ever been done before.

In this work, we present an analysis of the stability of Darboux transformation with shift. In general, when the shift $\alpha \neq 0$, numerical experiments show that the algorithm can be unstable. However, when the shift $\alpha = 0$ or when we consider polynomials orthogonal with respect to a positive measure on an interval (a, b), and $\pi(t) = (t - \alpha)$ is a strictly positive polynomial on the same interval, the algorithm is stable in a well defined way. Let us point out that the case in which the measure is positive and the polynomial $\pi(t)$ is strictly positive in the interval is the most important in terms of the applications. The study of the different kind of stability involved in our work requires a detailed analysis of the structured condition numbers of the Darboux transformation.

HSS matrices and bounded condition number discretizations of PDEs

S. Chandrasekaran[a], and M. Gu[b]

[a]Electrical & Computer Engineering Department
University of California, Santa Barbara
shiv@ece.ucsb.edu

[b]Department of Mathematics
University of California, Berkeley
mgu@math.berkeley.edu

With the advent of the Fast Multi-pole Method (FMM) of Greengard and Rokhlin and much subsequent work, it has become clear that matrices with low numerical ranks in their off-diagonal blocks play a key role in the rapid numerical solution of elliptic PDEs.

We show that any matrix-vector multiply that is amenable to FMM acceleration can be rapidly inverted by converting the whole system to a sparse matrix equation. The complexity of the resulting solver for an elliptic PDE is closely tied to the number of underlying spatial dimensions. For example solving a two-dimensional integral equation that has a kernel with a diagonal singularity now requires only $O(n^3)$ flops if the integral equation is discretized on an $n \times n$ grid.

Note that this is identical in cost to solving a two-dimensional elliptic PDE by a finite-element method on the same grid. In short, we can say that this levels the playing field between sparse discretization techniques (like finite element, finite difference and finite volume) and dense discretization techniques like volume integral formulations.

However this by itself is not very interesting since anything other than a linear-time solver is not usually practical. We show that by exploiting the low-rank structures present in the fill-in that occurs during Gaussian elimination of these sparse matrices we can achieve linear-time solvers.

A natural proposal that has been made is to use these linear-time solvers as preconditioners. That is essentially a very good idea that is being explored currently on several fronts. However, a related question is whether it is possible to come up with discretizations of elliptic PDEs that have bounded condition number and are still amenable to FMM acceleration? Of course we might have to give up sparsity, but that would not be an issue as the time and space complexity would still be linear. We explore some ideas from harmonic analysis that indicate that the answer to this question might be in the affirmative.

Contribution to the H-matrix theory and eigenvalue localization

Ljiljana Cvetković and Vladimir Kostić

Department of Mathematics and Informatics, Faculty of Science, University of Novi Sad

Eigenvalues of a given matrix A can be localized by well known Geršgorin theorem: they belong to Geršgorin set, which is the union of Geršgorin disks (each of them is a simple function of matrix entries). It is known, see [Varga, R.S.: *Geršgorin and His Circles.* Springer Series in Computational Mathematics, Vol. 36, 2004.], that Geršgorin theorem is, as a statement, equivalent to the statement that each strictly diagonally dominant matrix is nonsingular. In a similar way, starting from some class of nonsingular matrices, for example, for some subclass of H-matrices, one can obtain various Geršgorin-type localization theorems, as it was done in [Cvetković, Lj., Kostić, V., Varga, R.S.: *A new Geršgorin-type eigenvalue inclusion area.* ETNA 18(2004), 73-80].

A new subclass of H-matrices, as well as the corresponding eigenvalue localization theorem will be presented here and the relationship between new and old results will be established.

Optimal Order Parallel Algebraic Multigrid Preconditioners

Hans De Sterck[a] and Ulrike Yang[b]

[a]Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada, hdesterck@uwaterloo.ca

[b]Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, PO Box 808, Livermore, California 94551, USA, umyang@llnl.gov

Algebraic multigrid (AMG) is a very efficient iterative solver and preconditioner for sparse unstructured linear systems. Traditional coarsening schemes for AMG can, however, lead to computational complexity growth as problem sizes increase, resulting in increased memory use and execution time, and diminished scalability. We propose modified parallel AMG coarsening schemes, that are based on solely enforcing a maximum independent set property, resulting in sparser coarse grids. The modified coarsening techniques remedy memory and execution time complexity growth for various large three-dimensional (3D) problems. If used within AMG as a preconditioner for Krylov subspace methods, the resulting iterative methods tend to converge fast. For some difficult problems, however, these methods don't converge well enough, and improved interpolation is necessary in order to obtain scalable methods. Scalability is investigated on computers with up to thousands or tens of thousands of processors, including machines of Blue Gene/L type.*This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under contract number W-7405-Eng-48 and subcontract number B545391.

Accurate symmetric rank revealing factorizations of some structured matrices

Froilán M. Dopico[a], Plamen S. Koev[b], and Juan M. Molera[c]

[a]Departamento de Matemáticas. Universidad Carlos III de Madrid

[b]Department of Mathematics. M.I.T.

[c]Departamento de Matemáticas. Universidad Carlos III de Madrid

A rank revealing decomposition (RRD) of a matrix A is a factorization $A = XDY^T$ with X and Y well-conditioned matrices, and D diagonal and nonsingular. A RRD of A reveals the rank of A in exact arithmetic, and also in floating point arithmetic by revealing how "close" is the matrix A to a rank-deficient matrix. In this sense the singular value decomposition is an optimum RRD. However, the singular value decomposition is expensive to compute, and, in practice, the goal is to compute a RRD by some finite and cheaper procedure. Moreover, if the RRD of A is accurately computed with small forward errors, then it is also possible to compute the singular value decomposition with high relative accuracy. This can only be done for special classes of matrices. Our goal in this talk is to compute accurate RRD's of symmetric matrices that preserve the symmetry, i.e., $A = XDX^T$. If this is achieved then the eigenvalues and eigenvectors of A can be computed with high relative accuracy. We show how to compute accurate symmetric RRD's for the following classes of symmetric matrices: scaled Cauchy matrices, Vandermonde matrices, totally nonnegative matrices and graded matrices.

A Restarted Krylov Subspace Method for the Efficient Evaluation of Matrix Functions

Michael Eiermann and Oliver G. Ernst

Institut für Numerische Mathematik und Optimierung
Technische Universität Bergakademie Freiberg
D-09596 Freiberg, Germany

Given a matrix $A \in \mathbb{C}^{n\times n}$ and a complex-valued function f defined on some subset D of the complex plane, our objective is the evaluation of $f(A)b$ for arbitrary vectors $b \in \mathbb{C}^n$. Typical applications are the solution of the linear system of equations $Ax = b$ by $x = f(A)b$, where $f(\zeta) = \zeta^{-1}$, the solution of the ordinary initial value problem $y'(t) = Ay(t), y(0) = b$ by $y(t) = f(tA)b$, where $f(\zeta) = e^{\zeta}$, and the solution of the identification problem in stochastic semigroups, which requires the evaluation of $f(A)b$ with $f(\zeta) = \log\zeta$.

The Arnoldi approximation to $f(A)b$ ($\|b\|_2 = 1$) from the Krylov space $\mathcal{K}_m(A,b) = \operatorname{span}\{b, Ab, \ldots, A^{m-1}b\}$ is given by $V_m f(H_m)e_1$, where the columns of V_m form an orthonormal basis of $\mathcal{K}_m(A,b)$, the Hessenberg matrix H_m is the compression of A onto $\mathcal{K}_m(A,b)$ and $e_1 \in \mathbb{R}^m$ is the first unit vector. In the context of solving $Ax = b$, this approach is well known as the full orthogonalization method (FOM) which simplifies to the conjugate gradient method in the case of A being Hermitian positive definite.

In general, the storage and computational requirements make FOM impractical. It is therefore often modified to compute an approximation with respect to a space $\mathcal{K}_m(A,b)$ of affordable size, after which the algorithm is restarted using the current approximation as the initial guess. A key quantity for restarted FOM and other restarted Krylov solvers (i.e., for Krylov method approximating $f(A)b$ with $f(\zeta) = 1/\zeta$) is the residual vector $r = b - Ax$ of the current approximation x.

We show that, despite the fact that a residual is not available for general f, such a restarting scheme can indeed be constructed for arbitrary f. We further show that this scheme has a superlinear rate of convergence if $f(\zeta) = e^{\zeta}$, or more generally, if f is an entire function.

Structured Perturbations in Scalar Product Spaces

Stef Graillat

Laboratoire LP2A
Université de Perpignan Via Domitia
52, avenue Paul Alduy
F-66860 Perpignan Cedex, France
E-mail: graillat@univ-perp.fr
URL: http://gala.univ-perp.fr/~graillat

We consider divers structured matrix condition numbers and structured backward errors. We show that for most matrices belonging to Lie and Jordan algebras associated with a scalar product that is orthosymmetric and unitary, there is no or little difference between the structured case and the unstructured case. Hence, for these classes of matrices, the usual unstructured perturbation analysis is sufficient.

This is a joint work with Françoise Tisseur (University of Manchester).

Updating and downdating an upper trapezoidal sparse orthogonal factorization

Ángel Santos-Palomo[a] and Pablo Guerrero-García[b]

[a,b]Dept. Matemática Aplicada, University of Málaga (Spain), {santos,pablito}@ctima.uma.es,
Complejo Tecnológico, Campus Teatinos, 29071 Málaga (Spain)

We describe how to update and downdate an upper trapezoidal sparse orthogonal factorization, namely the sparse QR factorization of A_k^T, where A_k is a "tall and thin" full column rank matrix formed with a subset of the columns of a fixed matrix A. In order to do that, we have adapted to rectangular matrices (with fewer columns than rows) Saunders' techniques of early 70s for square matrices, by using the static data structure of George and Heath of early 80s but allowing row downdating on it. An implicitly determined column permutation allow us to dispense with computing a new ordering after each update/downdate; it fits well into the LINPACK downdating algorithm and ensures that the updated trapezoidal factor will remain sparse. We give all the necessary formulae even if the orthogonal factor is not available, and we comment on our implementation using the sparse toolbox of MATLAB 5.

GMRES Methods for Least Squares Problems

Ken HAYAMI[a] and Tokushi ITO[b]

[a]National Institute of Informatics, 2-1-2, Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan, hayami@nii.ac.jp, http://www.nii.ac.jp/

[b]Business Design Laboratory Co., Ltd., Tokyo, Japan, ito@business-design.co.jp

Consider the least squares problem : $\min_{\boldsymbol{x}\in\mathbf{R}^n} \|\boldsymbol{b} - A\boldsymbol{x}\|_2$, where $A \in \mathbf{R}^{m\times n}$, and $m \geq n$ or $m < n$, and A may be rank-deficient.

Previous iterative methods for solving this problem were mainly based on solving the normal equation: $A^{\mathrm{T}}A\boldsymbol{x} = A^{\mathrm{T}}\boldsymbol{b}$ using the (preconditioned) conjugate gradient (CG) method, i.e. the CGLS approach [1].

However, this approach has the drawback that the condition number of $A^{\mathrm{T}}A$ is square of that of A, and may result in slow convergence for very ill-conditioned problems, even with preconditioning.

Zhang and Oyanagi [5] proposed the CR-LS(k) method, which uses a $n \times m$ matrix B and essentially applies the Orthomin(k) method [4] to $\min_{\boldsymbol{z}\in\mathbf{R}^m} \|\boldsymbol{b} - AB\boldsymbol{z}\|_2$.

In this talk, we will follow up this idea and introduce an appropriate $B \in \mathbf{R}^{n\times m}$ and apply the more robust GMRES (Generalized Minimal Residual) method [3] to $\min_{\boldsymbol{z}\in\mathbf{R}^m} \|\boldsymbol{b} - AB\boldsymbol{z}\|_2$ as well as $\min_{\boldsymbol{x}\in\mathbf{R}^n} \|B\boldsymbol{b} - BA\boldsymbol{x}\|_2$ [2].

First, we give the necessary and sufficient condition that B should satisfy in order that the proposed methods give a least squares solution of the original problem.

Next, for implementations for B, we propose an incomplete QR decomposition IMGS(l). Numerical experiments for full rank problems with $m \geq n$ show that the simplest case $l = 0$, which is equivalent to $B = (\mathrm{diag}(A^{\top}A))^{-1}A^{\top}$, gives best results, and converges faster than previous methods based on CGLS for severely ill-conditioned problems. Similar results are shown for the case $m < n$.

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Computing Reducing Subspaces for the Discretized Navier-Stokes Model

G. Hechme

Département de Mathématiques, 6, Av. Le Gorgeu, CS 93837. 29238 BREST Cedex 3. FRANCE ; email : grace.hechme@univ-brest.fr

In this work we focus on the computation of reducing subspaces corresponding to the eigenvalues of largest real parts of a matrix pencil. This task arises from the stability analysis for the discrete analog of a $3D$ hydrodynamics system of equations linearized at a steady state. Investigating the stability around this steady state leads to an algebraic generalized eigenvalue problem with the characteristic of having a singular right hand-side. A deflation procedure along with the Jacobi-Davidson method for generalized eigenvalue problems are investigated to compute the required reducing subspaces. Several numerical results are reported.

Accurate Eigenvalue and QR Decompositions of Totally Positive Matrices

Plamen Koev

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139

A matrix all of whose minors are positive is called *totally positive* (TP).

For computational purposes, the TP matrices are best represented not by their entries, but as products of positive bidiagonals. This representation allows highly accurate computations: eigenvalues, SVD, inverses, etc., can be computed to high relative accuracy. Using this representation one can perform accurate computations with derivative TP matrices, such as products, Schur complements, and submatrices of TP matrices.

In this talk we present our latest results: An accurate algorithm that computes eigenvectors of a TP matrix with the correct *combinatorial properties*: the jth computed eigenvector will have exactly $j-1$ sign changes.

We also demonstrate that the matrix Q of the QR decomposition of a TP matrix has the same variation of sign properties as the eigenvector matrix, and present an algorithm that would guarantee these in floating point arithmetic as well.

Interpolation in the bivariate tensor-product Bernstein basis and applications to CAGD

José-Javier Martínez and Ana Marco

Departamento de Matemáticas, Universidad de Alcalá

Campus Universitario

28871 - Alcalá de Henares (Madrid), SPAIN

The approach to curve implicitization through bivariate interpolation presented in [2] for the case of Sylvester and Bézout resultant matrices in the power basis is extended to the case of Bernstein-Bezoutian matrices [1] constructed when the polynomials are given in the bivariate tensor-product Bernstein basis. The computation of the implicit equation (i.e. of the interpolating polynomial) involves the bidiagonal factorization of the inverses of certain totally positive matrices.

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Eigenvalue patterned condition numbers: Toeplitz and Hankel cases

S. Noschese[a] and L. Pasquini[b]

[a]Dipartimento di Matematica "Guido Castelnuovo", Università di Roma "La Sapienza", P.le A. Moro, 2, 00185 Roma, Italy. noschese@mat.uniroma1.it

[b]Dipartimento di Matematica "Guido Castelnuovo", Università di Roma "La Sapienza", P.le A. Moro, 2, 00185 Roma, Italy. lionello.pasquini@uniroma1.it

We study the sensitivity of a selected set of simple eigenvalues of a matrix A to matrix perturbations belonging to a chosen subspace. In [1] the zero-structured perturbations have been considered. Here the focus is on patterned perturbations and the cases of the Toeplitz and of the Hankel matrices are investigated in detail. Useful expressions of the absolute and relative patterned condition numbers of the eigenvalue λ and of the analogue of the matrix $W_\lambda := y\,x^H$ which leads to the traditional condition number of λ, are given. MATLAB codes are defined to compare traditional, zero-structured and patterned condition numbers. An accurate report on a significant collection of numerical tests is included.

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Structured matrices in Control, Filtering and Eigenvalue Problems

Vadim Olshevsky

Department of Mathematics
University of Connecticut
196 Auditorium Road, U-9
Storrs, CT 06269, USA

In this talk several recent results will be described. The results concern with quite different problems and applications, but the common denominator is that structured matrices methods were useful in obtaining them.

- I will start with the Bezoutain interpretation of the Kharitonov theorem on stability of interval polynomials. A new matrix proof will be presented for the both continuous-time and discrete-time settings.
- Then a generalization of the Kharitonov theorem to quasi-polynomials (combination of powers and exponentials) and more generally, to entire functions. The quasi-polynomial case occurs when considering systems with multiple delays, and the more general entire functions case is related to the systems with distributed delays.
- The next topic will be filtering for generalized stochastic processes and the role played by the Gohberg-Semencul formula.
- It will be shown that the so-called Pseudo-noise matrices can be converted into Hadamard-Sylvester matrices via row/column permutations.
- Order-one quasiseparable matrices, their relation to generalizations of orthogonal polynomials and their role in the eigenvalue algorithms will be described.

The results were obtained in several joint works with I. Gohberg, Y. Eidelman, T. Bella, E. Tyrtyshnikov, I. Oseledets and L. Sakhnovich.

Accurate solution of the classical and polynomial eigenproblems

Victor Pan

Department of Mathematics and Computer Science Lehman College, City University of New York Bronx, New York 10468. E-Mail: vpan@lehman.cuny.edu

We propose new techniques which substantially improve the known methods for accurate solution of the classical and polynomial eigenproblems.

Tikhonov regularization and orthogonal polynomials

Lothar Reichel[a]

[a]Department of Mathematical Sciences, Kent State University, Kent, OH 44242, USA

Linear ill-posed problems arise in many applications in science and engineering when one seeks to determine the unknown input for a linear system from known output. The available output is generally contaminated by measurement or discretization errors. Ill-posed problems have to be replaced by problems that are less sensitive to the error in the data before solution. Tikhonov regularization is one of the most popular replacement techniques. A regularization parameter, which determines how close the regularized problem is to the original one, and how sensitve the regularized problem is to perturbations, has to be specified. We discuss how the connection between partial Lanczos bidiagonalization, orthogonal polynomials, and Gauss quadrature can be exploited to determine a suitable value of the regularization parameter.

Symplectic Householder Transformations, a geometric and algebraic approach

Ahmed SALAM[a], Anas ELFAROUK[a], and Eman Al-Aidarous[b]

[a]*Laboratoire de Mathématiques Pures et Appliquées, Université du Littoral-Côte d'Opale. C.U. de la Mi-Voix, 50 rue F. Buisson, B.P. 699, 62228 Calais, Cedex, France.*
Ahmed.Salam@lmpa.univ-littoral.fr

[b]*Institute of Mathematical and Physical Sciences, University of Wales Aberystwyth Ceredigion SY23 3BZ Wales UK*

The aim of this paper is to introduce and to study symplectic Householder transformations, based on an algebraic and geometric approach. The construction is so that the parallel between orthogonal Householder and symplectic Householder becomes easy and natural. A block form is given. An algorithm for computing a SR factorization via symplectic Householder transformations is obtained following a similar scheme as for QR factorization via Householder transformation. Numerical tests are given.

Approximation of Largest Eigenpairs of Matrices and Applications to PageRank Computation

Avram Sidi

Computer Science Department, Technion - Israel Institute of Technology, Haifa 32000, Israel

E-mail: asidi@cs.technion.ac.il

http://www.cs.technion.ac.il/~asidi/

In this work, we propose different approaches, for the treatment of the following problems: (i) computation of the largest eigenvalue of a matrix and the corresponding eigenvector when neither is known, (ii) computation of the eigenvector of a matrix corresponding to its largest eigenvalue when this eigenvalue is known. The matrix is arbitrary, large, and sparse. We treat the first problem by Krylov subspace methods for eigenvalue problems, and discuss the use of the method of Arnoldi in detail. We treat the second problem by vector extrapolation methods, emphasizing the use of minimal polynomial extrapolation (MPE) and reduced rank extrapolation (RRE); we also discuss the use of Krylov subspace methods for linear systems. We propose effective modes of applying the methods considered, and devise ways of improving their performance further. With these improved approaches, the rates of convergence of the methods are increased substantially; as a result, both the computing time and the storage requirements can be reduced considerably, after proper tuning of some parameters. We also provide the mathematical theory that is relevant to the different modes of use. Finally, we consider the computation of the PageRank of the Google matrix, a problem that has attracted much attention recently; we show how the approaches discussed here can be used effectively in PageRank computations. We also study an extrapolation method for PageRank computation that was proposed recently by Kamvar et al. We generalize this method, and prove that the resulting generalization is very closely related to MPE, and also provide a convergence theorem for it.

Optimal semi-iterative methods for complex SOR with tools from potential theory

Apostolos Hadjidimos[a] and Nikos Stylianopoulos[b]

[a]Department of Mathematics, University of Crete, P.O. Box 2208, GR-714 09 Heraklion, Greece. e-mail: hadjidim@math.uoc.gr

[b]Department of Mathematics and Statistics, University of Cyprus, P.O. Box 20537, CY-1678 Nicosia, Cyprus. e-mail: nikos@ucy.ac.cy

We consider the application of semi-iterative methods (SIM) to the standard SOR method with complex relaxation parameter ω, under the following two assumptions: (i) the associated Jacobi matrix J is consistently ordered and weakly cyclic of index 2 (ii) the spectrum $\sigma(J)$ of J belongs to a compact subset Σ of the complex plane $\mathbb{C}$, which is symmetric with respect to the origin. By using results from potential theory, we determine the region of optimal choice of $\omega \in \mathbb{C}$ for the combination SIM-SOR and settle, for a large class of compact sets Σ, the classical problem of characterizing completely all the cases for which the use of the SIM-SOR is advantageous over the sole use of SOR, in view of the hypothesis that $\sigma(J) \subset \Sigma$. In particular, our results show that, unless the outer boundary of Σ is an ellipse, SIM-SOR is always better and, furthermore, one of the best possible choices is an asymptotically optimal SIM applied to the Gauss-Seidel method. In addition, we derive the optimal complex SOR parameters for all ellipses which are symmetric with respect to the origin.

How Hyper is your Graph? On the Factor-Width of Matrices

Sivan Toledo
School of Computer Science, Tel-Aviv University

The correspondence between weighted undirected graphs and diagonally-dominant symmetric matrices is well known and has been exploited in many ways in the past. The main theme of the talk will be a generalization of this relationship to hypergraphs and to more complex classes of matrices.

A real diagonally-dominant symmetric matrix A can always be decomposed into a symmetric factorization $A = UU^T$, where U has at most two nonzeros per column (and in particular, all the nonzeros in a column of U have the same absolute value). The nonzero structure of U is that of a vertex-edge incidence matrix of a graph G_A; the nonzero structure of A itself is that of a vertex-vertex adjacency matrix of the same graph. This relationship has been exploited in the past, among other things, to

- characterize graph properties, such as the size of balanced vertex separators, by eigenvalues of the corresponding matrix;
- to bound the smallest nonzero eigenvalue of A using structures in the graph;
- to bound the generalized spectral condition number of a matrix pencil in terms of embeddings of the corresponding graphs;
- to design preconditioners by sparsifying the corresponding graph;
- to obtain combinatorial graph algorithms for computing the null space of diagonally-dominant matrices.

Recently, Boman, Chen, Parekh and I began to generalize this relationship to all symmetric positive-definite matrices. We have defined the *factor width* of a matrix A as the smallest integer k such that A has a symmetric factorization $A = UU^T$ with at most k nonzeros in each column of U. We have characterized factor-width-2 matrices exactly (this class is slightly larger than diagonally-dominant matrices), we devised algorithms for deciding whether a matrix has factor-width 1 or 2, and we have found ways to compute upper and lower bounds on the factor width of a matrix. We have also shown that for all $k = 1, 2, \ldots, n$, there are n-by-n factor-width-k matrices.

We argue that the factor width is a good measure of the "combinatorial complexity" of a matrix. A diagonally-dominant matrix is combinatorially simple in the sense that it has an incidence factor U that corresponds to a weighted undirected graph; this graph can tell us a lot about the matrix. A factor-width-3 matrix does not have such a factor, but it does have an incidence factor that corresponds to a hypergraph with at most 3 vertices per edge. Perhaps this hypergraph can tell us something useful about the matrix.

The talk will explain the notion of factor width and how it arose from support preconditioning. The talk will also emphasize many open problems related to factor widths: how to efficiently determine the factor width of a matrix, how to factor it into its width-k incidence factor, how to determine its null space given such a factorization, what is the spectral relationship between factor-width-k and $k-1$ matrices, and more.

Method of Block-Triangular Partition of Band Matrix in Constructing Best Estimators

A. Torokhti[a], P. Howlett[a], and I. Yamada[b]

[a]School of Mathematics and Statistics, University of South Australia, SA 5095, Australia

[b]Department of Communications and Integrated Systems, Tokyo Institute of Technology, Ookayama, Meguro-ku, Tokyo 152-8552, Japan

We propose and justify a new technique for the solution of the best approximation problem arising in the optimal estimation of random vectors. The technique is based on finding a lower band matrix which represents an optimal filter. The motivation is as follows.

In many experiments and measurements, noisy data is only partially observed and it may be necessary to process the data immediately. Moreover, the filter may process data from only the most recent trials while preceding data is forgotten. This type of the filter is said to be *causal* and to have *a preassigned information capacity.*

A *linear* causal filter with a preassigned information capacity is interpreted as a lower band matrix A. An *optimal* filter is determined from minimization of the associated error. The problem is formulated in terms of the lower band matrix A. Due to the special structure of the matrix under consideration, its direct determination from such a problem is difficult. To avoid associated difficulties, we apply a block-triangular partition of the band of matrix A in lower triangular and upper triangular non-zero blocks, L_{ij} and U_{ij} with $i = 1, \ldots, l, j = 1, \ldots, s_i$ where l and s_i are given. Next, we show that the original minimization problem in terms of the lower band matrix A is reduced to l minimization problems in terms of the blocks L_{ij} and U_{ij}. The solution to each of such problem is based on special decompositions of the triangular block. In particular, we apply the Cholesky factorization.

Such a device works under quite restrictive assumptions on positive definiteness for some matrices associated with the blocks of A. An extension of the solution to a more general case when the assumption on matrix positive definiteness is withdrawn, can be obtained with a technique which exploits a modified Cholesky factorization. Therefore, we reformulate the problems in terms of the modified Cholesky factorization.

We obtain the solution of such a reformulated problem. As a result, the optimal filter with a preassigned information capacity is presented in terms of blocks L_{ij} and U_{ij}. A rigorous analysis of the associated error is given.

The described approach also implies associated computational advantages. Since blocks L_{ij} and U_{ij} are much smaller then the original matrix A, the proposed technique implies a considerable reduction in computational cost.

Related results can be found in [1, 2].

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Structures preserved by the QR algorithm

Steven Delvaux and Marc Van Barel

Department of Computer Science
Katholieke Universiteit Leuven
Celestijnenlaan 200A
B-3001 Leuven (Heverlee)
Belgium
{Steven.Delvaux,Marc.VanBarel}@cs.kuleuven.ac.be

It is a classical result in numerical linear algebra that, when applying the QR algorithm on a Hessenberg matrix, the resulting matrix is again of Hessenberg type. A similar statement can be made for Hermitian matrices, thus the property of being Hermitian is also preserved by the QR algorithm. Combining these two properties, one is led to the classical $\mathcal{O}(n)$ algorithm for applying a QR step on a Hermitian, tridiagonal matrix.

Besides the classical theory, in recent years a new interest has been brought to the QR algorithm when applied to various other types of matrices. Examples of these are arrowhead matrices, companion matrices and lower semiseparable matrices (the latter being matrices A for which every submatrix that can be taken out of the lower triangular part of A, has rank at most one). It turned out that for each of these cases, a fast QR variant can be developed, with underlying reason being the preservation of certain structures.

The aim of this talk is to investigate in a unified way, and from a theoretical point of view, some general kinds of structures that are preserved by applying the QR algorithm on a given matrix. We will handle two types of them: the first we call polynomial structures, for example a matrix being Hermitian or Hermitian up to a rank one correction. The second we call rank structures, for example a matrix being Hessenberg, or lower semiseparable.

An advantage of our approach is that we define a structure by decomposing it into a collection of 'building stones' which we call structure blocks. This allows us to state the results in their natural, most general context.

In this talk we investigate two classes of structures that are preserved by applying a QR step on a matrix A. The first we call polynomial structures, for example a matrix being Hermitian or Hermitian up to a rank one correction, and the second we call rank structures, which are encountered for example in all kinds of what we could call Hessenberg-like and semiseparable-like matrices.
A *polynomial structure* on $\mathbb{C}^{n\times n}$ is defined as a collection $\mathcal{P} = \{p_k\}_k$, where each p_k is a polynomial in 7 variables. A matrix $A \in \mathbb{C}^{n\times n}$ satisfies the structure $\mathcal{P} = \{p_k\}_k$ if for every k, $p_k(A, A^H, A^{-1}, A^{-H}, \text{Herm}_k, \text{Uni}_k, (\text{Rk } r)_k) = 0$, for certain Herm_k Hermitian, Uni_k unitary, $(\text{Rk } r)_k$ of rank at most r.
We define a *rank structure* on $\mathbb{C}^{n\times n}$ as a collection of structure blocks $\mathcal{R} = \{\mathcal{B}_k\}_k$ where each structure block $\mathcal{B}_k$ is a 4-tuple of numbers $\mathcal{B}_k = (i_k, j_k, r_k, \lambda_k)$ with i_k a row index, j_k a column index, r_k a rank upper bound and $\lambda_k \in \mathbb{C}$ a shift element. A matrix $A \in \mathbb{C}^{n\times n}$ satisfies the rank structure $\mathcal{R}$ if for every k, $\text{Rank } A_k(i_k : n, 1 : j_k) \leq r_k$, where $A_k := A - \lambda_k I$.
We show how these results lead to efficient QR-algorithms for different classes of rank structured matrices.

Solving Elliptic Finite Element Systems in Near-Linear Time with Support Preconditioners

Erik Boman[a], Bruce Hendrickson[b], and Stephen Vavasis[c]

[a]Sandia National Labs, Albuquerque, NM 87185-1111, USA, egboman@sandia.gov

[b]Sandia National Labs, Albuquerque, NM 87185-1111, USA, bah@cs.sandia.gov

[c]Department of Computer Science, Cornell University, Ithaca, NY 14853, USA, vavasis@cs.cornell.edu

We show how support preconditioners can be applied to a class of linear systems arising from use of the finite element method to solve linear elliptic problems. Our technique reduces the problem, which is symmetric and positive definite, to a symmetric positive definite diagonally dominant problem. Significant theory has already been developed for preconditioners in the diagonally dominant case. We show that the degradation in the quality of the preconditioner using our technique is only a small constant factor.

A Multigrid Method for Structured Matrices

Elena Zhebel

Institut für Numerische Mathematik und Optimierung
Technische Universität Bergakademie Freiberg
D-09596 Freiberg, Germany

Given is a system of linear equations $A\mathbf{u} = \mathbf{f}$ with coefficient matrix $A \in \mathbb{R}^{n\times n}$ possessing a block tridiagonal structure, however each of the block matrices is again block tridiagonal. Linear systems of this type arise by discretizing three-dimensional elliptic boundary value problems

$$\begin{aligned} -\nabla \cdot (a(x)\nabla u(x)) = f(x),\ x \in \Omega := (0,\alpha) \times (0,\beta) \times (0,\gamma), \\ u(x) = g(x),\ x \in \partial\Omega, \end{aligned} \tag{6}$$

where $a \in \mathbb{R}^{3\times 3}$ is diagonal, by a standard 9-point or 27-point difference scheme on a uniform but not necessarily equidistant grid Ω_h.

We are particularly interested in problems arising from geophysical applications, namely the electrical imaging of the subsurface, where u denotes an electric potential, a stands for the electric conductivity of the subsurface and f represents information about the current source and its amplitude. Here, the mesh Ω_h is usually far from equidistant. Moreover, the coefficient a is in general discontinuous across internal boundaries of Ω; it actually may show jumps of several orders of magnitude.

It is well known that the rate of convergence of a standard multigrid algorithm deteriorates due to this discontinuity (and/or due to the irregularity of the mesh). Algebraic multigrid methods may suffer from the lack of preserving the structure of A, i.e., on coarser levels the regular structure of the coefficient matrix is no longer guaranteed. In two spatial dimensions, there are matrix-dependent blackbox multigrid solvers (see, e.g., Alcouffe et al. (SIAM J. Sci. Stat. Comput., 1981, 430–454) and de Zeeuw (Jornal of Computational and Applied Mathematics, 1990, 33:1—27)) which overcome these difficulties. We present a similar approach for three-dimensional problems.

We describe the matrix-dependent grid transfer operations as well as the incomplete block LU smoothing procedure which are the basic ingredients of our algorithm. We further consider numerical examples which show the efficiency of the presented method.

WORKSHOP 12

Relations with computer science:

Algorithmic game theory and metric embeddings

ORGANISERS:

Avrim Blum & Allan Borodin

The Price of Routing Unsplittable Flow

Baruch Awerbuch[a], Yossi Azar[b], and Amir Epstein[c]

[a]Department of Computer Science, Johns Hopkins University, Baltimore, MD 21218, USA

[b]School of Computer Science, Tel Aviv University, Tel-Aviv 69978, Israel

[c]School of Computer Science, Tel Aviv University, Tel-Aviv 69978, Israel

The essence of the routing problem in real networks is that the traffic demand from a source to destination must be satisfied by choosing a single path between source and destination. The splittable version of this problem is when demand can be satisfied by many paths, namely a flow from source to destination. The unsplittable, or discrete version of the problem is more realistic yet is more complex from the algorithmic point of view; in some settings optimizing such unsplittable traffic flow is computationally intractable.

In this paper, we assume this more realistic unsplittable model, and investigate the "price of anarchy", or deterioration of network performance measured in total traffic latency under the selfish user behavior. We show that for linear edge latency functions the price of anarchy is exactly 2.618 for weighted demand and exactly 2.5 for unweighted demand. These results are easily extended to (weighted or unweighted) atomic "congestion games", where paths are replaced by general subsets. We also show that for polynomials of degree d edge latency functions the price of anarchy is $d^{\Theta(d)}$. Our results hold also for mixed strategies.

Previous results of Roughgarden and Tardos showed that for linear edge latency functions the price of anarchy is exactly $\frac{4}{3}$ under the assumption that each user controls only a negligible fraction of the overall traffic (this result also holds for the splittable case). Note that under the assumption of negligible traffic pure and mixed strategies are equivalent and also splittable and unsplittable models are equivalent.

On Routing without Regret

Avrim Blum

Carnegie Mellon University, Pittsburgh PA 15213-3891, USA.

There has been substantial work in learning theory and game theory on adaptive "no-regret" algorithms for problems of repeated decision-making. For example, one could use such an algorithm to choose a path to drive to work each day so that no matter what the sequence of traffic patterns (and even given only feedback for the route actually taken on any given day) the average time spent per day approaches that of the best fixed route in hindsight. Recently, several (simple) efficient algorithms have been shown to have this property in a wide variety of settings, including online routing, where the set of decisions (e.g., the set of all possible paths) if listed explicitly could be exponential in the natural parameters of the problem.

In this talk I will discuss new (and old) work on such no-regret algorithms, and then turn to the question: suppose we are in the setting of routing, and delays on each edge are actually caused by congestion due to other players in the system (as in the work of Roughgarden-Tardos). If all players use no-regret algorithms, under what conditions can we expect overall behavior to quickly approach an approximate Nash equilibrium?

Portions of this talk are joint work with Eyal Even-Dar, Katrina Ligett, and Brendan McMahan.

Can there be a theory of algorithms?

Allan Borodin

Department of Computer Science, University of Toronto, Toronto, OntarioM5S 3G4, Canada

One of the most fundamental ideas in computing is that there is an almost universal agreement on what it means to be "computable" (i.e. the Church-Turing thesis) and a common agreement on the meaning of "efficiently computable" (i.e. polynomial time computation). We are interested in developing a theory of "basic algorithms", say in the context of Combinatorial Search and Optimization Problems. In particular, we would like to have precise definitions for concepts such as greedy algorithms, dynamic programming, local search, primal dual algorithms, etc. Can we ever hope to have agreement on the meaning of such concepts? What are the limitations of such approaches? Can such definitions lead to new algorithms? In particular, can such a theory be helpful in developing general methods for constructing (say) truthful mechanisms such as in the work of Lehmann, O'Callaghan and Shoham, Mu'alem and Nisan, and Briest, Krysta and Voecking?

Metric Embeddings into ℓ_1 and the Sparsest Cut Problem

Anupam Gupta

Carnegie Mellon University, Pittsburgh PA 15232, USA.

We will talk about two problems: (a) that of finding good embeddings of finite metric spaces into ℓ_1, the "Manhattan" metric and (b) finding small-expansion cuts in graphs. We will show how the two problems are related to each other, and how results for the former give us approximation algorithms for the latter.

The talk will outline some of the recent results in metric embeddings, including the ideas behind embeddings of so-called "negative-type" metric spaces on n points into ℓ_1, and of n-point subsets of ℓ_1 into Euclidean space—whilst ensuring, in both cases, that distances between points are distorted by a multiplicative factor of at most $O(\log^{3/4} n)$, thus improving on the long-standing previous bounds of Bourgain (1985).

The talk is partly based on results with Shuchi Chawla and Harald Räcke, and will be self contained.

Derandomization of Auctions

Gagan Aggarwal[a], Amos Fiat[b], Andrew Goldberg[c],
Jason Hartline[c], Nicole Immorlica[d], and Madhu Sudan[e]

[a]Stanford U., Stanford, CA. gagan@cs.stanford.edu

[b]Tel Aviv U., Tel Aviv, Israel. fiat@tau.ac.il

[c]Microsoft Research, Mountain View, CA. {goldberg,hartline}@microsoft.com

[d]Microsoft Research, Redmond, WA. nickle@mit.edu

[e]M.I.T., Cambridge, MA. madhu@mit.edu

We study the problem of designing seller-optimal auctions, i.e. auctions where the objective is to maximize revenue. Prior to this work, the only auctions known to be approximately optimal in the worst case employed randomization. Our main result is the existence of deterministic auctions that approximately match the performance guarantees of these randomized auctions. We give a fairly general derandomization technique for turning any randomized mechanism into an *asymmetric* deterministic one with approximately the same revenue. In doing so, we bypass the impossibility result for symmetric deterministic auctions and show that asymmetry is nearly as powerful as randomization for solving optimal mechanism design problems. Our general construction involves solving an exponential-sized flow problem and thus is not polynomial-time computable. To complete the picture, we give an explicit polynomial-time construction for derandomizing a specific auction with good worst-case revenue. Our results are based on toy problems that have a flavor similar to the hat problem of Ebert (1998).

Approximations and streaming algorithms for geometric data

Piotr Indyk
EECS, MIT
32 Vassar St, Cambridge, MA

The topic of this talk is the design of algorithms for *data streams*. Such algorithms can use only a limited storage (much smaller than the size of the input data) and can access the input elements using only a single pass over the data. Algorithms using a small number of passes are also of interest, but we are not going to consider them in this talk. Data stream algorithms are of interest in several areas, including network traffic analysis and databases.

The design of data stream algorithms is often equivalent to constructing small "approximations" of the input data. In this talk I will present an overview of methods for constructing such approximations, for streams that represent sets of low-dimensional points. Specifically I will focus on two methods: one based on the notion of a *core-set*, and another one based on randomized linear embeddings. I will also present open problems in the area.

An application of market equilibrium in distributed load balancing in wireless networking

Kamal Jain

Microsoft Research

The principles of market equilibria have been used in distributed load balancing and congestion control. In this talk, I will focus on one natural case that arises in wireless networking.

Suppose there are cellular phone towers who have capacities to serve a certain number of cellular phones. These towers communicate with the cellular phones at variable powers. A cellular phone chooses to communicate through that particular tower from which it is getting the best reception. I will discuss various algorithms how cellular towers can choose the powers level to balance the loads. These algorithms use market equilibrium and the theory of bipartite graph matching.

The Effects of Network Structure on Equilibria and Computation

Michael Kearns

Computer and Information Science, University of Pennsylvania

I will survey a number of recent network or graph-theoretic models of games and markets, and describe results that relate topological properties of the graph to properties of various equilibrium concepts, including Nash, correlated, price, and evolutionarily stable equilibria. In many cases the relationship between graph structure and outcome leads to efficient algorithms for (approximate) equilibrium computation.

A Metric Notion of Dimension and its Algorithmic Applications

Robert Krauthgamer

IBM Almaden Research Center, San Jose, CA, USA.

Let us define the dimension of a metric space is as the minimum $k > 0$ such that every ball in the metric space can be covered by 2^k balls of half the radius. This definition has several attractive features besides being applicable to every metric space. For instance, it coincides with the standard notion of dimension in Euclidean spaces, but captures also nonlinear structures such as manifolds.

I will survey some recent results dealing with metric spaces of low dimension (under the above definition). This includes embeddings into low-dimensional Euclidean spaces, as well as algorithms for computational problems involving metric spaces, such as Nearest Neighbor Search and the Traveling Salesmen Problem.

Metric Variance

Avner Magen
University of Toronto

We introduce the concept of *metric variance*, which is the variance of a random distance in the space. We show a connection between this parameter and nonembeddability results and in particular show that metrics that "behave poorly" must have a small metric variance.

On Earthmover Distance, Metric Labeling, and 0-Extension

Yuval Rabani

Technion - Israel Institute of Technology, Technion City, Haifa 32000, Israel.

We study metric labeling and its special case, 0-extension, in the context of transportation metrics (a.k.a. earthmover distance). Researchers recently proposed using transportation metrics to derive polynomial time-solvable relaxations for these problems. Until now, however, it was not known if the integrality ratio is constant or not. We show several results, mostly lower bounds, concerning these problems and relaxations.

This is joint work with Howard Karloff, Subhash Khot, and Aranyak Mehta.

Computing (Correlated) Equilibria in Multi-Player Games

Tim Roughgarden

Stanford University

This talk will survey recent research (joint with Christos Papadimitriou) on the complexity of computing equilibria—particularly correlated equilibria—in games with a large number of players. Such games, in order to be computationally meaningful, must be presented in some succinct, game-specific way. I will discuss a general algorithmic framework for identifying the complexity of optimizing over the correlated equilibria of a compactly represented multi-player game, and applications of this paradigm to symmetric games, graphical games, and congestion games. Consequences include (on the positive side) the first polynomial-time algorithm for this problem in symmetric games and (on the negative side) the first complexity-theoretic justification of the standard restrictions on the topology of a graphical game.

Nash Equilibria in Random Games

Imre Bárány[a], Santosh Vempala[b] and Adrian Vetta[c]

[a]Rényi Institute of Mathematics, Hungarian Academy of Sciences, and Department of Mathematics, University College, London, UK. barany@renyi.hu

[b]MIT, Cambridge MA, USA. vempala@math.mit.edu

[c]Department of Mathematics and Statistics, and School of Computer Science, McGill University, Montreal, Quebec, Canada. vetta@math.mcgill.ca

We consider Nash equilibria in random 2-player games and analyze a simple Las Vegas algorithm for finding an equilibrium. The algorithm is combinatorial and always finds an equilibrium; on $m \times n$ payoff matrices, it runs in time $O(m^2 n \log n + n^2 m \log m)$ with high probability. Our main tool is a polytope formulation of equilibria.

Approximation Techniques for Utilitarian Mechanism Design

Berthold Vöcking

RWTH Aachen, Department of Computer Science, 52076 Aachen, Germany

The talk is about the design of efficiently computable incentive compatible, or truthful, mechanisms for combinatorial optimization problems with multi-parameter agents. We focus on approximation algorithms for NP-hard mechanism design problems. Such algorithms need to satisfy certain monotonicity properties to ensure truthfulness. Since most of the known approximation techniques do not fulfill these properties, we study alternative techniques.

We give a method to transform a pseudopolynomial algorithm into a monotone fully polynomial time approximation scheme (FPTAS). This method can be applied to various problems like, e.g., *knapsack*, *constrained shortest path*, or *job scheduling with deadlines*. The monotone FPTAS for the *knapsack problem* gives a very efficient mechanism for *multi-unit auctions*. The best previous result for such auctions was a 2-approximation. We also derive a monotone polynomial time approximation scheme for the *generalized assignment problem* with any bounded number of parameters per agent.

The most efficient way to solve packing integer programs (PIPs) is LP-based randomized rounding, which is also not monotone. We show that monotone primal-dual greedy algorithms achieve almost the same approximation ratios for PIPs as randomized rounding. This significantly improves on the known approximation ratios of truthful mechanisms for two famous mechanism design problems–*combinatorial auctions* and *unsplittable flow routing*.

Joint work with Patrick Briest and Piotr Krysta.

WORKSHOP 13

Real-number complexity

ORGANISERS:

Peter Buergisser & Gregorio Malajovich

Efficient Algorithms for Computing the Betti Numbers of Semi-algebraic Sets.

Saugata Basu

School of Mathematics, Georgia Tech

Computing homological information of semi-algebraic sets (or more generally constructible sets) is an important problem for several reasons. From the point of view of computational complexity theory, it is the next logical step after the problem of deciding emptiness of such sets, which is the signature NP-complete problem in appropriate models of computation.

In this talk I will describe some recent progress in designing efficient algorithms for computing the Betti numbers of semi-algebraic sets in several different settings. I will describe a single exponential time algorithm for computing the first few Betti numbers in the general case and polynomial time algorithms in case the set is defined in terms of quadratic inequalities. One common theme underlying these algorithms is the use of certain spectral sequences – namely, the Mayer-Vietoris spectral sequence and the "cohomological descent" spectral sequence first introduced by Deligne. Certain parts of this work is joint with R. Pollack, M-F. Roy and (separately) with T. Zell.

On efficient computation of parabolic Julia sets

Mark Braverman

Dept. of Computer Science, University of Toronto

In this talk we describe a new interpolation technique that allows us to obtain a poly-time algorithm for computing Julia sets with parabolic points. The complexity of these sets was previously thought to be exponential.

Counting Complexity and Euler Characteristic of Semialgebraic Sets

Peter Bürgisser[a], Felipe Cucker[b], and Martin Lotz[a]

[a]Dept. of Mathematics, University of Paderborn, D-33095 Paderborn, Germany

[b] Dept. of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong

We define counting classes $\#\mathrm{P}_{\mathbb{R}}$ and $\#\mathrm{P}_{\mathbb{C}}$ in the Blum-Shub-Smale setting of computations over the real or complex numbers, respectively. The problem $\#\textsc{Feas}_{\mathbb{R}}$ of counting the number of real roots of a given real multivariate polynomial (returning ∞ if this number is not finite) turns out to be complete for $\#\mathrm{P}_{\mathbb{R}}$. Similarly, the problem $\#\mathrm{HN}_{\mathbb{C}}$ of counting the number of complex zeros of a system of complex multivariate polynomials turns out to be complete for $\#\mathrm{P}_{\mathbb{C}}$.

Over $\mathbb{C}$, we show that the problem of computing the geometric degree of a complex affine variety is polynomial time equivalent to $\#\mathrm{HN}_{\mathbb{C}}$. Moreover, we prove that the problem of computing the Euler characteristic of a complex affine (or projective) variety is polynomial time equivalent to $\#\mathrm{HN}_{\mathbb{C}}$. Over $\mathbb{R}$, we prove that the computation of the (modified) Euler characteristic of a given semialgebraic set is polynomial time equivalent to the counting problem $\#\textsc{Feas}_{\mathbb{R}}$.

It is remarkable that these statements of polynomial time equivalence carry over to the Turing model of computation, when restricting to input polynomials with integer coefficients. The corresponding problem of counting complex zeros lies in FPSPACE and it is #P-hard. It is a fascinating open question to find out how close this counting problem is to #P: can it be reduced in polynomial time to the computation of the permanent?

We complement our results on the Euler characteristic by showing that for fixed k, the problem of computing the kth Betti number of the set of real zeros of a given integer polynomial is PSPACE-hard.

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Exotic quantifiers, complexity classes, and complete problems

Peter Bürgisser[a] and Felipe Cucker[b]

[a]Dept. of Mathematics, University of Paderborn, D-33095 Paderborn, Germany,
e-mail: pbuerg@upb.de

[b]Dept. of Mathematics, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong,
e-mail: macucker@math.cityu.edu.hk

We introduce some operators defining new complexity classes from existing ones in the Blum-Shub-Smale theory of computation over the reals. Each one of these operators is defined with the help of a quantifier differing from the usual ones, $\forall$ and $\exists$, and yet having a precise geometric meaning. Our agenda in doing so is twofold. On the one hand, we show that a number of problems whose precise complexity was previously unknown are complete in some of the newly defined classes. This substantially expands the catalog of complete problems in the BSS theory over the reals thus adding evidence to its appropriateness as a tool for understanding numeric computations. On the other hand, we show that some of our newly defined quantifiers have a natural meaning in complexity theoretical terms. An additional profit of our development is given by the relationship of the new complexity classes with some complexity classes in the Turing model of computation. This relationship naturally leads to a new notion in complexity over the reals (we call it "gap narrowness") and to a series of completeness results in the discrete, classical setting.

Some connections between the Blum-Shub-Smale and generalizations of Valiant's model

Pascal Koiran

LIP, École Normale Supérieure de Lyon

In Valiant's framework VP and VNP are the main complexity classes, and they are almost the only classes that have been defined and studied. By contrast, there are literally hundreds of complexity classes in discrete complexity theory. In this talk I will argue that it is fruitful to study new complexity classes within (generalizations of) Valiant's framework. The main motivation is to obtain connections with the Blum-Shub-Smale model of computation.

On the Curvature of the Central Path of Linear Programming Theory

Jean-Pierre Dedieu[a], **Gregorio Malajovich**[b], and Mike Shub[c]

[a] MIP. Département de Mathématique, Université Paul Sabatier, 31062 Toulouse cedex 04, France (dedieu@mip.ups-tlse.fr).

[b] Departamento de Matemática Aplicada, Universidade Federal de Rio de Janeiro, Caixa Postal 68530, CEP 21945-970, Rio de Janeiro, RJ, Brazil (gregorio@ufrj.br)

[c] Department of Mathematics, University of Toronto, 100 St. George Street, Toronto, Ontario M5S 3G3, Canada (shub@math.toronto.edu)

We give a bound for the total curvature of the central path of Linear Programming. This bound was obtained through integral geometry, elimination and a bound of the number of solutions of a multi-homogeneous polynomial system. Similar arguments may work for other real algebraic curves.

We consider linear programming problems in primal-dual form

$$\begin{array}{c} \min \quad \langle c, x\rangle \\ Ax - s = b \\ s \geq 0 \end{array} \qquad \text{and} \qquad \begin{array}{c} \max \quad \langle b, y\rangle \\ A^T y = c \\ y \geq 0 \end{array} .$$

Here $m > n \geq 1$ and A is an $m \times n$ real matrix assumed to have rank n, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ are given vectors, b is not in the range of A and c is non-zero, $y, s \in \mathbb{R}^m$ and $x \in \mathbb{R}^n$ are unknown vectors (s is the vector of slack variables).

Interior point methods for linear programming proceed by numerically following the *central path*, usually defined in terms of a logarithmic barrier.

More precisely, The *primal central path* is the curve $(x(\mu), s(\mu))$, $0 < \mu < \infty$, defined as the curve of minimizers of the function $-\mu \sum_1^m \ln(s_i) + c^T x$ restricted to the primal polyhedron. Similarly, the *dual central path* is the curve $y(\mu)$, $0 < \mu < \infty$, defined as the curve of maximizers of the function $\mu \sum_1^m \ln(y_i) + b^T y$ restricted to the dual polyhedron. The *primal/dual central path* is the curve $(x(\mu), s(\mu), y(\mu))$. It is given by a system of polynomial equations:

$$\begin{cases} Ax - s = b \\ A^T y = c \\ sy = \mu \mathrm{e} \\ y > 0, \ s > 0 \end{cases}$$

where e denotes the vector in $\mathbb{R}^m$ of all 1's.

Current 'long step' interior point algorithms work at a speed better than available complexity estimates. In order to gain some understanding on why this happens, we proved that the curvature of the central path is bounded in the average, independently of the number m of restrictions.

Main Theorem: *Let $m > n \geq 1$. Let A be an $m \times n$ matrix of rank n, and let $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$, c non-zero such that (A,b) is in general position. Then the*

average total curvature of the primal/dual central paths (resp. primal central paths, dual central paths) of the strictly feasible polytopes defined by (A,b) is less than or equal to $2\pi n$ *(resp.* $2\pi(n-1)$, $2\pi n$.*)*

The average is taken among all possible *sign conditions* that keep the primal polytope compact and strictly feasible. A sign condition $\epsilon \in \{-1,+1\}^m$ acts on the data (A,b,c) by multiplying the i-th row of A and the i-th coordinate of b by ϵ_i. There are 2^m sign conditions, of which $\binom{m-1}{n}$ keep the primal polytope compact and strictly feasible.

The idea of the proof is to interpret the length of the Gauss curve in terms of the number of real zeros of a certain polynomial system, using an argument of integral geometry.

Then, the number of zeros of that polynomial system was estimated through elimination theory and a multi-homogeneous version of Bézout's theorem.

What we actually obtain is a bound on the number of complex solutions. This includes solutions that are not real and solutions corresponding to other possible sign conditions. This is the reason why we need to average among all sign conditions defining a meaningful (compact strictly feasible primal) linear programming problem.

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A uniform version of Valiant's algebraic complexity classes

Guillaume Malod

Laboratory of Prof. SATO, Masahiko, Dept. of Intelligence Science and Technology, Kyoto University
malod@kuis.kyoto-u.ac.jp

Valiant's classes VP and VNP give an interesting insight into the difficulty of computing certain polynomials. The permanent polynomial, for instance, is complete for the class VNP, a result which is similar to its completeness for $\sharp$P.

A uniform version of these complexity classes is given here. The uniformity condition helps to prove a characterization of uniform VNP by circuits of polynomial size and depth, in the spirit of the circuit characterization of $\sharp$P by Venkateswaran (1992).

On the approximation of the stationary solutions of certain parabolic semilinear PDEs

Guillermo Matera

Universidad Nacional de General Sarmiento, Argentina

We consider a family of polynomial systems which arises in the approximation of the stationary solutions of the semilinear heat equation with nonlinear boundary conditions of Neumann type. We deform the input problem into a well–conditioned family of discrete stationary problems of same kind and exhibit a polynomial–time homotopy continuation method solving any member of this family, significantly improving the exponential behaviour of all known algorithms solving a generic instance of this family.

This is joint work with Ezequiel Dratman (Universidad de Buenos Aires, Argentina).

Probabilistically checkable proofs over the reals

Klaus Meer

University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark

Probabilistically checkable proofs (PCPs) represent one of the most important recent areas in theoretical computer science. The idea is to make verification proofs of NP-problems more stable: Reading only a *constant* number of data units in such a potential proof will be sufficient to detect faults with high probability.
In this talk we introduce the PCP notion for the real number model of Blum, Shub, and Smale. We then study the existence of certain such "transparent" proofs for the class $\mathrm{NP}_{\mathbb{R}}$ of real number decision problems verifiable in polynomial time.

On the number of random digits required in the Monte Carlo integration of definable functions

C. L. Alonso[a], J. L. Montaña[b], and L. M. Pardo[c]

[a]Universidad de Oviedo (calonso@uniovi.es)

[b]Universidad de Cantabria (montanjl@unican.es)

[c]Universidad de Cantabria (pardo@matesco.unican.es)

We study the convergence of stochastic processes predicting the mean value of a semi-algebraic function. We take as starting point Chebyshev and Hoeffding inequalities. We provide sharp estimates not only for the number of examples but also for the minimal precision with which examples must be generated to ensure an approximation of a given error with high confidence. Both lower bounds, on the number of examples and the precision, are given as a function of some suitable parameters: dimension, degree and number of polynomials involved in the description of the semi-algebraic function.

As a main conclusion our results suggest the idea that even if learning processes involving semi-algebraic objects can require an exponential number of examples in the desired learning precision, these examples can be drawn within a polynomial (linear) precision in the error and confidence precision.

Polynomial root-finding with matrix eigen-solving

Victor Pan

Lehman College, City University of New York

Numerical matrix methods are increasingly popular for polynomial root-finding. This approach usually amounts to the application of the QR algorithm to the highly structured Frobenius companion matrix of the input polynomial. The structure, however, is routinely destroyed already in the first iteration steps. To accelerate this approach, we exploit the matrix structure of the Frobenius and generalized companion matrices, employ various known and novel techniques for eigen-solving and polynomial root-finding, and in addition to the Frobenius input allow other highly structured generalized companion matrices. Employing polynomial root-finders for eigen-solving is a harder task because of the potential numerical stability problems, but we found some new promising directions, particularly for sparse and/or structured input matrices.

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Parallels between Real and p-adic Complexity

J. Maurice Rojas

Texas A&M University, TAMU 3368, College Station, Texas 77843-3368, USA,
rojas@math.tamu.edu

Two fundamental theorems on sparse polynomial systems — Khovanski's Theorem on Real Fewnomials and Rojas' Theorem on p-adic Fewnomials — show that the number of solutions (over certain complete fields) can be much smaller than, and independent from, the underlying geometric degree.

Whether this independence from geometric degree extends, even partially, to the complexity of deciding the **existence** of roots is a tantalizing possibility. We reveal new cases — over the reals and p-adics — under which such speed-ups are possible, and we classify the threshold where such speed-ups are not possible. Quantum complexity, and results related to Carmichael numbers, make a surprise appearance in the p-adic case.

Some of these results are recent joint work with Frederic Bihan and Casey Stella.

Software for Symbolic-Numeric Solutions of Polynomial Systems

Jan Verschelde

University of Illinois at Chicago
Department of Mathematics, Statistics, and Computer Science
851 S. Morgan St. (M/C 249), Chicago, IL 60607-7045, USA
jan@math.uic.edu, http://www.math.uic.edu/~jan

We use numerical homotopy continuation methods to obtain approximate solutions to systems of polynomial equations. A solution is called singular when the Jacobian matrix of the polynomial system is singular at the solution. Because Newton's method (as used in continuation methods) fails when the Jacobian matrix is singular, singular solutions are challenging to numerical solvers.

In this talk we will survey recent progress on dealing with singular solutions, as they occur as isolated roots of high multiplicity (joint work with Anton Leykin and Ailing Zhao) or as they occur as nonisolated roots belonging to positive dimensional solution sets (joint work with Andrew J. Sommese and Charles W. Wampler). In both instances, the solutions are "symbolic-numeric": they are not merely approximate numbers, but satisfy extra equations, which are part of the description of the solution sets.

As software is the practical foundation of computational mathematics, we will emphasize the recent updates in PHCpack, a software package to solve polynomial systems using homotopy continuation methods.

Random systems of equations and the Rice formula

Mario Wschebor

Centro de Matemática. Facultad de Ciencias Universidad de la República. Montevideo. Uruguay.
wschebor@cmat.edu.uy

The purpose of the talk is the application of the Rice formula to compute the moments of the number of roots of a random fields, to a certain number of problems arising in the study of polynomial systems.

We start with a new proof of Shub & Smale's theorem (1992) on the expectation of the number of roots of a system of m random polynomial equations in m real variables, having a special isotropic Gaussian distribution. The method permits also a certain number of extensions to other probability distributions on the coefficients.

We also consider second moments of the number of roots and obtain some results on the asymptotic behaviour of the random number of roots, as the size of the system tends to infinity.

Kantorovitch analysis for multiple zeros of univariate analytic functions

Jean-Claude Yakoubsohn

MIP, Université Paul Sabatier - Toulouse

Kantorovich analysis [1], [2] is well known to give an existence result of a regular solution of the equation $f(x) = 0$. This result works in the context of a Banach space and requires the map f is C^2 and its derivative admits a inverse bounded on a certain domain. In the eighties Smale give specific results when f is assumed be n analytic map: the existence of a solution is only deduced from data at one point. Several papers reflected the ideas of Smale of this period which constitute the background of the α-theory [4, 3]. In the introduction of [4], Smale suggests to use Kantorovitch theory to derive similar theorems. Wang and Han realizes this program in [5] and gives the better point estimates to prove the existence of a root.

Here, we propose a Kantorovitch analysis to prove the existence of cluster of zeros for a univariate analytic function from data at one point.

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Analytic reparametrizaton of algebraic sets as a tool for their computer modelling

Yosef Yomdin

Department of Mathematics, Weizmann Institute

A finite smoothness reparametrizaton of an algebraic set is its partition into simple pieces, each parametrized by a unit cell, with the uniform bound on the derivatives, up to a prescribed order, of the parametrization mappings. In a similar way one can define an analytic reparametrizaton. Such reparametrizations have been first applied to the control of the complexity growth (entropy, volume growth) in smooth dynamical systems. Some other application in Analysis and Number Theory have recently appear. We outline an application of a reparametrization with controlled high order derivatives to the problem of an efficient computer modelling of algebraic sets. The main application considered is to singularities and "near-singularities".

WORKSHOP 14

Computational dynamics

ORGANISERS:

Jean-Pierre Ramis, Carles Simo & Warwick Tucker

On Exponential Parts of Solutions of a Linear Differential System

M. A. Barkatou
Université de Limoges, LACO, 123, Av. A. Thomas, 87060 Limoges cedex, France

In this talk we shall study the relationship between the exponential parts of formal solutions of a differential system $dY/dz = A(z)Y$ having a singularity of pole type at $z = 0$ and the singular parts of the eigenvalues of the matrix $A(z)$ considered as Puiseux series in z. In particular, we will show that the exponential parts of a given differential system and the eigenvalues of its matrix do agree up to a certain order and that with suitable conditions on the matrix $A(z)$ some formal invariants of the differential system $dY/dz = A(z)Y$ can be computed from the characteristic polynomial of $A(z)$.

Validated Integration of ODEs and PDEs with Taylor Model-based Tools

Martin Berz, Kyoko Makino, and Shashikant Manikonda

Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

ODE Integration schemes based on Taylor model methods establish high order relationships between initial conditions and final conditions in a simple one-stage approach. For integration domains of sufficient size, the one-step error is usually seen to be substantially less than for other methods. In practice, for extended integration times, it is important to restrict the growth of these one-step errors, and various methods to assure this will be discussed.

Besides the so-called shrink wrapping and preconditioning approaches, we focus our attention on a new method based on validated iterative approximation and the solution of successively more and more simplified systems, until in the last step, a simple Euler step is sufficient to achieve the requested accuracy.

We also address methods for the validated solution of PDEs, in particular those for 3D Laplace and Poisson equations. For various practical problems, very precise and validated solutions of this PDE are required; but with conventional finite element or finite difference codes this is difficult to achieve because of the need for an exceedingly fine mesh which leads to often prohibitive CPU time.

We present an alternative approach based on high-order quadrature and a high-order finite element method. Both of the ingredients become possible through the use of Taylor model methods. The solution in space is first represented as a Helmholtz integral over the two-dimensional surface. The latter is executed by evaluating the kernel of the integral as a Taylor model of both the two surface variables and the three volume variables inside the cell of interest. Finally, the integration over the surface variables is executed as a mere polynomial integration, resulting in a local Taylor model of the solution within one cell. Examples of the practical performance of the method are given.

Invariant manifolds in quasi-periodic systems: theory, computation and applications

Àlex Haro[a] and Rafael de la Llave[b]

[a]Dept. Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, 08007 Barcelona, Spain

[b]Dept. Mathematics, University of Texas, Austin TX 78712, USA

We explain the parameterization method to prove the existence of invariant tori and their associated invariant manifolds (whiskers) in quasi-periodic systems. The method provides an effective algorithm to compute these manifolds.

We apply the computer methods to two examples, that are quasi-periodic perturbations of the Hénon map (dissipative) and the standard map (conservative).

On the fractalization of invariant curves in quasi-periodically forced 1-D maps

Àngel Jorba and Joan Carles Tatjer

Dept. Matemàtica Aplicada i Anàlisi, Universitat de Barcelona, 08007 Barcelona, Spain

In this talk we will present some results concerning the destruction of smooth and attracting invariant curves in quasi-periodically forced 1-D dynamical systems. More concretely, we will focus on the case in which the length of the curve goes to infinity when a parameter approaches some critical value. We will first discuss some connections between this behaviour and the lack of reducibility of the curve. We will show that in some cases the curve keeps being a smooth curve as long as it is attracting, although the numerical simulations seem to show that it is not longer a regular curve, but a strange nonchaotic attractor (SNA). We will also show some numerical simulations to question the existence of SNAs in some concrete situations.

On the macroscopic dynamics of randomly perturbed conservative maps

Oliver Junge[a], Jerrold E. Marsden[b], and Igor Mezic[c]

[a]Faculty of Computer Science, Electrical Engineering and Mathematics, University of Paderborn, D-33095 Paderborn, Germany

[b]Control and Dynamical Systems, California Institute of Technology, MC 107-81, Pasadena, CA 91125, USA

[c]Department of Mechanical and Environmental Engineering, UCSB, Santa Barbara, CA 93106, USA

We study the macroscopic dynamics of randomly perturbed area preserving maps of $\mathbb{R}^2$ to itself, focusing on the standard map and a discrete Duffing oscillator as specific examples. We relate the level of uncertainty to the large scale features in the dynamics in a precise way. We also study the effect of such perturbations on bifurcations in such maps. The main tools used for these investigations are a study of the eigenfunction and eigenvalue structure of the associated Perron-Frobenius operator along with set oriented methods for the numerical computations.

Non-symmetric choreographies in the N-body problem

Tomasz Kapela

Pedagogical University, Kraków, Poland

A choreography is a solution of the N-body problem in which all bodies move on the same curve following each other with a constant phase shift.

We present a general method of the computer assisted proofs of the existence of choreographies. Having approximated initial conditions for choreography, e.g. from some numerical simulations, using this method we can prove that in some neighbourhood there exist true initial conditions for choreography. We also obtain the local uniqueness of that solution up to natural symmetries of the N-body problem.

In comparison with the previous method we do not use any symmetry constraints to isolate a solution. It allowed us to prove the existence of a non-symmetric choreography with 7 bodies. This confirmed fact, till now known only numerically, that non-symmetric choreographies really exist.

This is a joint work with Carles Simó.

Methods of harmonic analysis applied to critical functions from Dynamical Systems

Amit Apte[a], Rafael de la Llave(presenter) [b], and Nikola Petrov[c]

[a]Univ. of North Carolina. Chapel Hill NC

[b]Univ. of Texas, Austin TX/ ICREA, Barcelona

[c]Univ. of Oklahoma, Norman OK

In harmonic analysis there are several characterizations of the regularity of functions based on the behavior of their Fourier coefficients (Littlewood-Paley theory) or Wavelet transforms.

We discuss numerical implementations of these methods.

We also apply them to compute the regularity of some functions that appear in dynamical systems as critical functions for the persistence of invariant circles in "non-twist" map cases and in circle maps. These functions are associated to phenomena which admit a Renormalization Group description.

In each case, we have several million Fourier coefficients.

We observe that for these functions many of the bounds from harmonic analysis are saturated. We also obtain further information on the self-similar structure of the functions.

Computable conditions for the verification of chaos in one-dimensional dynamics

S. Luzzatto
Dept. of Mathematics, Imperial College, London

I will present some recent work concerning the verification of the occurrence of stochastic dynamics in families of dynamical systems. This problem is generally difficult because stochastic dynamics only occurs for topologically nowhere dense set of parameters. However, it is also interesting because this set often has positive probability.

I will discuss a combination of numerical, geometrical, and probabilistic techniques which make it possible to obtain explicit bounds for the measure of such a set. As an application of these results we obtain a first ever lower bound for the measure of the set of parameters corresponding to stochastic behaviour in the quadratic logistic family.

Validated Global Optimization and Stability Estimates

Kyoko Makino, Martin Berz, and Youn-Kyung Kim

Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA

We assess local stability of fixed points by constructive normal form methods. Depending on the character of the problem, they lead either to locally valid Lyapunov functions, or allow the use of Nekhoroshev-type stability estimates for long but finite times. In both cases, these methods require to determine a validated maximum of a very complicated function, which is an iteratively generated polynomial describing a Lyapunov- or KAM surface to higher and higher accuracy. The quality of the upper bounds of this near-invariant directly relates to quality of the stability estimates.

However, the resulting functions are exceedingly complex and have a very substantial cancellation problems as well as a large numbers of local extrema, and thus represent a formidable challenge for any global optimization tool. We utilize the new Taylor model-based validated optimizer COSY-GO for this task, and compare with the performance of the well-known code Globsol, which can solve the problem only for the case of low-dimensional subspaces. We also compare with a non-validated stochastic optimizer for the purpose of benchmarking. Specifically, we utilize the method for the study of the stability of the Tevatron at Fermilab near Chicago, which is currently the highest energy particle accelerator in the world.

Transversal Connecting Orbits from Shadowing

Brian A. Coomes[a], Huseyin Koçak[b], and Kenneth J. Palmer[c]

[a]Department of Mathematics, University of Miami, Coral Gables, Florida 33124, USA

[b]Departments of Mathematics and Computer Science, University of Miami, Coral Gables, Florida 33124, USA

[c]Department of Mathematics, National Taiwan University, Taipei 106, Taiwan

A numerical method for rigorously establishing the existence of a transversal connection orbit from one periodic orbit to another periodic orbit of a differential equation in R^n is presented. As the first component of this method, a general shadowing theorem that guarantees the existence of such a connection orbit near a suitable pseudo connection orbit is proved. The second component consists of a refinement procedure for numerically computing a pseudo connection orbit between two pseudo periodic orbits with sufficiently small local errors so as to satisfy the hypothesis of the theorem.

Hypergeometric Solutions of Linear Difference Equations with Hypergeometric Coefficients

S. A. Abramov[a], M. Bronstein[b], M. Petkovšek[c]

[a]Russian Academy of Sciences, Dorodnicyn Computing Centre, Vavilova 40, 119991, Moscow GSP-1, Russia (sabramov@ccas.ru)

[b]INRIA – Projet CAFÉ, 2004, Route des Lucioles, B.P. 93, F-06902 Sophia Antipolis Cedex, France (Manuel.Bronstein@sophia.inria.fr)

[c]Department of Mathematics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia (marko.petkovsek@fmf.uni-lj.si)

Let t be a $\Pi\Sigma$ monomial over a difference field k. We show that if we can compute all the hypergeometric solutions of equations with coefficients in k, and if we can compute a basis for the solutions in $k[t]$ of equations with coefficients in $k(t)$, then we can compute all the hypergeometric solutions of equations with coefficients in $k(t)$. Together with known algorithms for solving pseudo-orbit problems and for computing dispersions and spreads, this gives an algorithm for computing all the hypergeometric solutions of equations with coefficients in $k(t)$.

Transversal homoclinic points of the Hénon map

Daniel Stoffer and Urs Kirchgraber

Department of Mathematics, ETH-Zürich,CH-8092 Zürich, Switzerland

Using shadowing techniques we prove that the Hénon map $H_{a,b}(x,y) = (a - x^2 + by, x)$ admits a transversal homoclinic point for a set of Parameters which is not very small. For the area and orientation preserving Hénon map (corresponding to $b = -1$) we prove that a transversal homoclinic point exists for $a \geq 0.265625$. Applying a computer–assisted version of our scheme we show that the result holds for a much closer to -1. This supports an old conjecture due to Devaney and Nitecki dating back to 1979, claiming that in the case $b = -1$ the Hénon map admits a transversal homoclinic point for $a \geq -1$.

Discussion of a parametrized first order linear system of Lame type

Felix Ulmer

Université de Rennes 1
UFR de Mathematiques
Campus de Beaulieu
35042 Rennes cedex, France.

In this talk I will present some results from a joint work with M. van der Put and F. Loray. We consider a first order linear system over $C(z)$, $\frac{d}{dz}$ of Lame type depending on 3 parameters with one relation. The exponents of the system do not depend on the parameters, which allows a complete study of this system along the lines of the "Morales-Ramis" theorem. Our aim is to study properties of the differential Galois group according to the values of the parameters. We first compute polynomial equations in the parameters describing the reducible systems, i.e. whose differential Galois group is a reducible linear group. For each possible finite differential Galois group we present a method to construct the finitely many parameter values which give this finite differential Galois group.

The PDE approach for approximating invariant manifolds

Alex Vladimirsky

Dept. of Mathematics, Cornell University, Ithaca, NY 14853, USA.

The problem of constructing an invariant manifold (of co-dimension k) can be locally reduced to solving a (system of k) quasi-linear PDE(s), which can be efficiently solved in Eulerian framework using Ordered Upwind Methods. These methods rely on a careful use of the direction of information propagation to systematically advance the computed "boundary" and to de-couple the discretized system. We illustrate our approach by constructing invariant manifolds for the test problems in optimal control, geometric optics, and seismic imaging. Some of the presented results are a joint work with J. Guckenheimer (Cornell University, NY) and J.A. Sethian (UC Berkeley, CA).

About the Galoisian approach to the non-integrability of (Hamiltonian) differential systems

Jacques-Arthur Weil

LACO, Université de Limoges, 123 avenue Albert Thomas, 87060 Limoges Cedex, France

The results of Ziglin and Morales-Ramis, connecting the non-integrability of Hamiltonian systems with the "non-integrabilit" of the differential Galois (or monodromy) group of a variational equation, have been the center of intense activity in the last years. The aim of several authors, including the speaker, was to study numerous families of systems to develop a methodology in order to make these criteria more effective (or easier to use). In this talk, I will report on several of these works and show directions to apply these ideas concretely, including the (yet unpublished) result of Morales, Ramis and Simo concerning the use of higher variational equations to prove non-integrability.

Non-computable quadratic Julia sets

Mark Braverman[a], Michael Yampolsky[b] (presenting)

[a]Department of Computer Science, University of Toronto

[b]Department of Mathematics, University of Toronto

A planar set is called computable if there exists an algorithm for drawing this set on a computer screen with an arbitrarily fine magnification. In a joint work with M. Braverman we have recently demonstrated the existence of non-computable Julia sets. We will discuss this result, as well as some related new results obtained jointly with I. Binder and M. Braverman.

Rigorous Numerics for Dissipative PDEs

Piotr Zgliczyński

Jagiellonian University, Institute of Computer Science,
Nawojki 11, 30–072 Kraków, Poland
umzglicz@cyf-kr.edu.pl

We describe an algorithm for rigorous computation of evolution of dissipative PDEs with periodic boundary conditions.

The algorithm has been implemented for Kuramoto-Sivashinsky PDE with odd and periodic boundary conditions. For this equation using this algorithm we are able to verify assumptions of some topological fixed point theorems to obtain the proof of the existence of multiple periodic orbits, both stable and unstable ones.

Main idea of the algorithm: we use Fourier series, the phase space is split into main modes (a finite dimensional subspace) and the tail (of an infinite dimension). The evolution of main modes is governed by differential inclusion obtained from Galerkin projection plus the influence of the tail. The evolution of the tail is governed by linear differential inequalities. In one time step we evolve both main modes and the tail, and we check suitable consistency conditions, which guarantee that we produce valid bounds for the solution of the dissipative PDE under consideration.

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WORKSHOP 15

Geometric modelling and animation

ORGANISERS:

Tom Lyche & Larry Schumaker

Geometry Processing Algorithms Based on Volume Discrete Models

Pere Brunet, Alvar Vinacua

Universitat Politecnica de Catalunya
Barcelona, Spain

The talk will present some results of our recent work on Geometry Processing Algorithms Based on Discrete Volume Representations. The algorithms are based on the concept of discrete membrane, a face-connected set of voxels that contain the surface of the object. A discrete membrane represents a fat surface approximation of the object boundary. The first part of the talk will present the main concepts and definitions together with a number of properties of these discrete representations. In the second part of the talk, four problems will be presented and discussed: model reconstruction and model repair, the optimization of topological properties of isosurfaces, the computation of the largest planar region of an object and the generation of polyhedral approximations. Optimal or quasi-optimal error-bounded solutions based on these discrete volume representations will be presented for each of them, and applications will be discussed.

Towards Automatic Segmentation in Reverse Engineering

Guido Brunnett

Chemnitz University of Technology, Department of Computer Science, Computer Graphics and Visualization, Strasse der Nationen 62, D-09107 Chemnitz, Germany

Reverse Engineering addresses the problem of semi-automatic creation of a CAD model for an existing 3D-object. Typically, this reconstruction is based on an unstructured set of data points measured from the objects boundary. In the segmentation phase these points are grouped into subsets to facilitate subsequent steps as surface fitting. In order to estimate local surface properties an initial data structure is required to support elementary geometric operations. Very often a triangulation of the point set is used as the basic data structure. This approach is advantageous because the triangulation may also be used to define an initial parametrization in the surface fitting step. However, the computation of a global triangulation especially for noisy data sets involves major difficulties with respect to efficiency, geometric correctness and robustness. Therefore direct (or point based) methods have been proposed as alternatives that avoid the computation of a global triangulation.

In this talk we review some of the most critical aspects of the reverse engineering process. Especially, we summarize important approaches to compute a triangulation. In the main part of the talk we report on our direct segmentation method that uses a neighborhood graph as the basic data structure. One of the main features of our method is to proceed by alternating the steps of segmentation and local geometry approximation. Our method is especially well suited for the reconstruction of piecewise algebraic models. If the boundary of the original object contains such surfaces the segmentation is optimized based on the result of a surface fitting procedure.

Parameterizing the Interiors of Solid Models Bounded by B-splines

Elaine Cohen and William Martin

University of Utah

The boundary model, or *b-rep model* as it has come to be popularly known, is by far the most widely employed general form for representing geometric models. A solid model is one that partitions space in a well defined manner, specifically, into a disjoint union of 3 regions that unambiguously delineate the interior, exterior and boundary of the object. Continuing the pioneering traditions of Coons Patches and Bźierer Surfaces, Computer Aided Design (CAD) models have focused nearly exclusively on boundary representations. However, as modern design environments are increasingly expected to provide simultaneous, interactive model support for geometric form, simulated physical behavior, and visualization of complex phenomena there is a demand for more sophisticated representations to meet this requirements. Additional impetus for more spatially complete representations comes from the progress, use, and intriguing promise of new controllable materials that allow us to design and fabrication products from heterogeneous (graded) materials.

Stimulated by this exciting trend we present algorithms to parameterize the interior of a closed geometric model bounded by B-splines boundaries. This approach leads to a gridless volumetric model, which can have significant analytical uses as well.

Geometrical and Algorithmic Foundations for the Minkowski Algebra of Complex Sets

Rida T. Farouki

Department of Mechanical and Aeronautical Engineering, University of California, Davis, CA 95616, USA

Exact algebraic operations on complex–number sets yield rich geometrical and algorithmic structures, with diverse applications in science and engineering. For "simple" operands, such as circular disks, precise descriptions of Minkowski sums, products, powers, and roots can be expressed in terms of Cartesian and Cassini ovals, and higher-order generalizations, and are extensible to accommodate Minkowski values of polynomials over given sets and the solution of simple algebraic equations. For more general complex sets, algorithms can be formulated to approximate Minkowski combinations by general (piecewise-smooth) boundaries to any desired precision. The Minkowski algebra of complex sets is a natural generalization of (real) interval arithmetic to sets of complex numbers. Its two–dimensional nature, however, endows it with much richer geometrical content — it provides a versatile "shape operator" language, with connections to mathematical morphology, geometrical optics, and the stability analysis of dynamic systems.

How Parameterization Affects the Approximation Order of Parametric Curve Fitting

Michael S. Floater

Centre of Mathematics for Applications, Department of Informatics, University of Oslo, 0316 Oslo, Norway

We study the problem of fitting parametric curves, such as polynomials, splines, and subdivision curves, to 'geometric data', i.e., sequences of points, possibly with 'geometric derivatives', i.e., tangent directions, curvature vectors etc. In all these problems we must choose the parameter values at which the fitted curves pass through (or close to) the data. There are several well-known methods for choosing these parameter values, such as the uniform and chord length parameterizations, but until recently little seemed to be known about how they affect the approximation order of the curve-fitting method (when the data is sampled from a smooth curve whose parameterization is not available).

In this talk we give a summary of some recent results on this topic, one of which is that chord length parameter values give full, fourth order approximation for cubic polynomial interpolation and and complete cubic spline interpolation, but that uniform parameter values give second accuracy at best.

Medial Axis as Distance Functions in Level Set Methods

Hans Hagen and Marco Schneider

Computer Graphics and Visualization Group Department of Computer Science University of Kaiserslautern

Distance functions play a major role in many research domains like Nearest Neighbor Interpolation, Voronoi Tessellations or Shortest Path Finding. The Euclidean distance approach is too restrictive in many cases. We present a very general method to define distance functions. Since a unit circle is topologically equivalent to a Jordan curve we first proof that a distance function can be described by a Jordan curve and a center-point.

The choice of this center-point is essential considering a good representation for the site defined by the Jordan curve. Centers of mass, so-called centroids, may be located out of the site e.g. using non-convex sites. From the theory of Level Set Methods we derive a shrinking process of the Jordan curve which leads to its medial axis. Extending this approach by defining a time-dependent scalar field on the medial axis the original Jordan curve can be rebuilt. Hence, only a medial axis itself defines a distance function. Consequently medial axes provide more flexible representatives than seed points, and the center of the unit circle constitutes a degenerated case of the medial axis approach.

Shape from Shaders

Charles Loop and Jim Blinn

Microsoft Research

The programs that run on programmable Graphics Processing Units, often used to implement sophisticated lighting models, are known as *shaders*. In this this talk, I will describe how shader programs have enabled a new class of algorithms for the display of curves and surfaces in real-time. In particular, I will describe rendering algorithms for quadratic and cubic curves, as well as quadric surfaces defined by Bézier tetrahedra. The advantages of this approach are high performance, compact geometric form, and resolution independence. The high performance comes from GPU paralellism. Geometric data is reduced since only the coefficients of the underlying polynomials are used for rendering, and not a much larger piecewise linear approximation. As a result, no faceting artifacts are present, meaning that shapes are resolution independent.

Hermite Subdivision with Shape Constraints

Tom Lyche

University of Oslo
Oslo, Norway

Subdivision is a technique for creating a smooth curve or surface out of a sequence of successive refinements of polygons, or grids. Subdivision has found applications in geometric design, computer games and animation. We consider here Hermite interpolatory subdivision where we start with values and derivatives at some points and use this information to compute values and derivatives at intermediate points. Repeating this process we obtain a function and some of its derivatives defined on a dense set of points. The scheme has smoothness C^r if the function has a C^r extension to the whole domain. We give algorithms for constructing subdivision curves that locally preserve positivity, monotonicity, and convexity and for surfaces preserving directional monotonicity. An advantage of using Hermite subdivision is that it is easier to combine smoothness and shape due to their local nature. A disadvantage is that derivatives or tangents have to be specified. However, we obtain a desired shape even with poor estimates for the derivatives. In this talk we consider a geometric formulation of Hermite some subdivision schemes which can also be used in the parametric case.

Geometric Properties of Parametric Tension Schemes

Carla Manni

Dipartimento di Matematica, Università di Roma "Tor Vergata", ITALY

Parametric techniques were introduced some years ago in the context of tension methods tailored for shape-preserving interpolation of functions and curves. They consist in considering and constructing the required function as a particular parametric curve whose components are usually chosen in simple polynomial or spline spaces. The related interpolating schemes inherit from this construction a clear geometric interpretation of the shape/tension parameters and other useful geometric and computational properties.

In particular, properties rising from classical geometric constructions of geometric B-splines and from subdivision algorithms of B-spline and Bézier curves seem of interest.

We present and we analyse some of these properties of parametric interpolating and/or approximating schemes and we discuss some possible connections with some subdivision schemes recently proposed and investigated in the literature.

Computations with the Control Polygon

Knut Mørken

Department of Informatics and CMA, University of Oslo
P.O. box 1053, Blindern, 0316 Oslo, Norway

One of the most distinctive features of splines is that when they are represented as linear combinations of B-splines, they are well approximated by their coefficients, the so-called control polygon. This property is usually at the core of systems for designing with splines and is also commonly exploited for plotting and for bounding the extent of a spline. In other contexts this property is seldom used. The aim of this talk is to show that many computations with splines can be done by repeatedly performing the same computation with the control polygon relative to different knot vectors (grids). This approach is usually both more effective and more robust than more traditional methods. An added bonus is that many methods that work directly on the control polygon generalises to subdivision schemes. Another added bonus is that any new algorithms that are developed for splines automatically also work for polynomials. In this talk I will consider (among others) methods for computing zeros of splines, various approximation methods and an algorithm for computing the Chebyshev spline (a generalisation of the Chebyshev polynomial).

This presentation draws on joint work in the past with Tom Lyche, Martin Reimers and Karl Scherer.

Some Challenges in the Problem of Finding Shape Preserving Representations

J. M. Peña

Departamento de Matematica Aplicada, Universidad de Zaragoza (Spain)

Recent alternatives for the Bézier model for representing polynomial curves and surfaces and for the rational Bézier model are discussed. On one hand, the evaluation algorithm used in the Bézier model is the de Casteljau algorithm, which has a quadratic complexity. More efficient algorithms have been found and the stability and shape preserving properties of the corresponding representations are presented. On the other hand, remarkable curves useful in many applications only can be approximated in the rational Bézier model but can be represented exactly in other alternative shape preserving representations. Recent advances and open problems for these alternatives are considered.

Morphing as Shadow Metamorphosis

Hartmut Prautzsch

Universität Karlsruhe (TH), D-76128 Karlsruhe, Germany
prau@ira.uka.de

Any two objects A and B can be viewed as two different projections of their cross product $A \times B$. Rotating and projecting $A \times B$ results in a continuous transformation of A into B. During the rotation, the contour on the cross product remains the same although its projection changes. Based on this result, a fast and simple morphing algorithm has been implemented. Its characteristics are presented in the talk.

The Analysis of Subdivision Curves

Malcolm Sabin

Numerical Geometry Ltd., 26 Abbey Lane, Lode, Cambridge, CB5 9EP, UK

A subdivision scheme is a set of rules whereby a polygon is refined, creating a new polygon, whose vertices are specific linear combinations of the original. When these rules are applied repeatedly, the polygon converges towards a curve.

Since subdivision schemes were first described by de Rham and then reinvented by Chaikin we have learnt how to analyse them. It is now possible to take a description of the coefficients used in a stationary scheme and determine by algorithm

(i) the region of the curve influenced by a given original polygon vertex,

(ii) whether indeed the process converges to give a limit curve,

(iii) whether, for some degree of polynomial, the original vertices lying on such a polynomial implies that the limit curve is a polynomial of the same degree,

(iv) what level of continuity of derivative the limit curve has,

(v) what the values of the convergent derivatives are at a dense set of points on the curve,

(vi) whether the limit curve has polynomial pieces, and if so whether the number of such pieces is finite or not,

(vii) the relative size of the spatial frequency components of the limit curve above the Shannon limit.

While time will not permit detailed description of all of these analyses, they are built on just a couple of linked foundations, the z-transform and eigenanalysis, and the talk will show how these two give all the above. We now have sufficient understanding not only to analyse any given stationary curve scheme, but also to design such schemes systematically for required properties. Further, it is now possible to go beyond the limit of stationarity and design geometrically sensitive schemes which are much more likely to be acceptable for demanding applications in design of, for example, automobiles.

Surface theory is as yet much less complete, but it is expected that the principles from curve theory will be equally applicable.

Recent Results on Trivariate Macro-Elements

Larry L. Schumaker

Vanderbilt University, Nashville, TN 37240

Given a tetrahedral partition $\triangle$ of a domain Ω, a C^r trivariate spline space is the space of piecewise polynomials functions defined on $\triangle$ that have r continuous derivatives on Ω. Such spaces are useful for dealing with a variety of problems in approximation, CAGD, data fitting, visualization, and the numerical solution of PDE's. The most useful of these spaces are defined via macro-elements, which provide a way of constructing a trivariate spline locally. But trivariate macro-element spaces are not easy to find, and except for the classical polynomial element, had been constructed only for the C^1 case. Here we report on recent work with Peter Alfeld in which we construct three new C^2 trivariate macro-element spaces.

On Geometric Interpolation of the Circle Like Curves

Gašper Jaklič[a], Jernej Kozak[b], Marjeta Krajnc[a], and Emil Žagar[b]

[a]IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

[b]FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

In [1] the problem of geometric interpolation of the planar data by parametric polynomial curves has been revisited. In particular, the conjecture that a parametric polynomial curve of degree $\leq n$ can interpolate $2\,n$ given points in $\mathbb{R}^2$ has been confirmed for $n \leq 5$ under certain natural restrictions. Furthermore, the optimal asymptotic approximation order $2\,n$ has been confirmed provided the polynomial interpolating curve exists. But the existence of the interpolating curve for the general n has been quite a while an open challenge since the pioneer work on the geometric interpolation [2]. In this work the problem of geometric interpolation of circle like curves is solved. It is proved that any convex planar curve close to the circle arc can be interpolated in $2\,n$ points by a polynomial parametric curve of degree $\leq n$ with the maximal approximation order, i.e., $2\,n$.

As a partial result, the fact that one can always find two nonconstant polynomials $x_n,\ y_n \in \mathbb{R}[t]$ of degree $\leq n$ such that

$$x_n^2(t) + y_n^2(t) = 1 + t^{2n} \tag{7}$$

is confirmed.

The importance of the equation (7) has already been noted in [3] considering a slightly different approximation problem, and the solution has been established for the odd n. However, it turns out that the equation (7) has at least one solution for all $n \in \mathbb{N}$. It is based upon a particular rational parametrization of a circle arc. The coefficients of the polynomials x_n and y_n are explicitly given by the values of Chebyshev polynomials of the first and the second kind.

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WORKSHOP 16

Image and signal processing

ORGANISERS:

Albert Cohen & Guillermo Sapiro

Bregman iteration and the dual of BV in inverse problems in imaging and elsewhere

Stanley Osher

We shall review some new results obtained with: M. Burger, S. Kindermann, D. Goldfarb, O. Scherzer, W. Yin and JJ Xu in image processing and general inverse problems. State-of-the-art image restoration methods result from easy to explain nonlinear optimization and functional analysis considerations.

Real-time, Accurate Depth of Field using Anisotropic Diffusion and Programmable Graphics Cards

Marcelo Bertalmio

Computer graphics cameras lack the finite Depth of Field (DOF) present in real world ones. This results in all objects being rendered sharp regardless of their depth, reducing the realism of the scene. On top of that, real-world DOF provides a depth cue, that helps the human visual system decode the elements of a scene. Several methods have been proposed to render images with finite DOF, but these have always implied an important trade-off between speed and accuracy. In this talk, we introduce a novel anisotropic diffusion Partial Differential Equation (PDE) that is applied to the 2D image of the scene rendered with a pin-hole camera. In this PDE, the amount of blurring on the 2D image depends on the depth information of the 3D scene, present in the Z-buffer. This equation is well posed, has existence and uniqueness results, and it is a good approximation of the optical phenomenon, without the visual artifacts and depth inconsistencies present in other approaches. Because both inputs to our algorithm are present at the graphics card at every moment, we can run the processing entirely in the GPU. This fact, coupled with the particular numerical scheme chosen for our PDE, allows for real-time rendering using a programmable graphics card.

On Geometric Variational Models for Inpainting Surface Holes

Vicent Caselles

We shall present several geometric approaches for filling-in holes in surfaces. The basic idea is to represent the surface of interest in implicit form, and fill-in the holes with a scalar, or systems of, geometric partial differential equations, often derived from optimization principles. These equations include a system for the joint interpolation of scalar and vector fields, a Laplacian-based minimization, a mean curvature diffusion flow, and an absolutely minimizing Lipschitz extension. We shall discuss the theoretical and computational framework, as well as examples with synthetic and real data.

Robust Uncertainty Principles and Signal Recovery from Highly Incomplete Information

Emmanuel Candes[a] and Terence Tao[b]

[a]Applied and Computational Mathematics, Caltech, Pasadena, CA 91125

[b]Department of Mathematics, University of California, Los Angeles, CA 90095

In many fields of science and technology, one is often able to make only a limited number of measurements about an object of interest, i.e. a digital signal, a digital image and so on. Many important problems in medicine or astrophysics such as Magnetic Resonance Imaging (MRI) or Computed Tomography (CT) of course all come to mind. This situation raises several fundamental questions: can we recover a digital signal from a small number of linear measurements? how many measurements do we need to make about an object to be able to reconstruct this object to within fixed accuracy?

This talk introduces mathematical results showing that it is possible to reconstruct (1) sparse signals exactly and (2)"compressible" signals accurately from limited measurements. Although our methodology extends to a variety of setups and signals in any dimension, consider a special instance of the first claim. We show how one can reconstruct a piecewise constant image from highly incomplete frequency samples—provided that the number of discontinuity point be roughly of the order of the number of observed data points. Further, we demonstrate that from the knowledge of about $K \log N$ nonadaptive random measurements, one can essentially reconstruct the K largest coefficients of a signal in any fixed basis. That is, one has an error of approximation which is nearly the same as that one would achieve by knowing ALL the information about the object and selecting the most relevant. Our methods and algorithms are very concrete, stable (in the sense that they degrade smoothly as the noise level increases) and practical; in fact, they only involve solving simple convex optimization programs, which in most cases, are just linear programs.

This work interacts significantly with the agenda of information theory and especially coding theory, theoretical computer science, signal processing and with the field of random matrix theory. If time allows, I will discuss some of these connections.

(Parts of this work are also joint with Justin Romberg.)

Sparse Nonnegative Solutions of Underdetermined Linear Equations by Linear Programming

David L. Donoho and Jared Tanner

Stanford University

Department of Statistics

Consider an underdetermined system of linear equations $y = Ax$ with known y and $d \times n$, matrix A with $d = \delta n$ for $\delta \in (0, 1)$. We seek the sparsest nonnegative solution, i.e., the nonnegative x with fewest nonzeros satisfying $y = Ax$. In general this problem is NP-hard. However, for many matrices A there is a threshold phenomenon: if the sparsest solution is sufficiently sparse, it can be found by linear programming. Quantitative values for a strong and weak threshold will be presented. The smaller threshold guarantees the recovery of the sparsest solution x_o, whereas a weaker - larger - sparsity constraint ensures the recovery of the sparsest solution for most x_o. These thresholds are surprisingly large, indicating the utility of l^1 convex relaxation in recovering the sparsest solution. For instance, with aspect ratio $n/d = 2$ the strong and weak thresholds are approximately $.13d$ and $.55d$ respectively as $n \to \infty$. Interesting properties used in the proof of these results include the neighborliness of polytopes in high dimensions.

Morphological Component Analysis and Applications

Jean-Luc Starck

The Morphological Component Analysis (MCA) is a a new method which allows us to separate features contained in an image when these features present different morphological aspects. We show that MCA can be very useful for decomposing images into texture and piecewise smooth (cartoon) parts or for inpainting applications. We extend MCA to a multichannel MCA (MMCA) for analyzing multispectral data and present a range of examples which illustrates the results.

A Novel Concept for Image Compression by Linear Splines over Adaptive Triangulations

Armin Iske

Department of Mathematics, University of Leicester, Leicester LE1 7RH, UK

This talk proposes a novel concept for image compression. The resulting compression method combines a recent image approximation algorithm with a customized scattered data coding scheme. The image approximation is given by a linear spline over an anisotropic triangulation, being the Delaunay triangulation of a small set of significant pixels. The significant pixels are selected by an adaptive thinning algorithm, which recursively removes less significant pixels in a greedy way, using a sophisticated criterion for measuring the significance of a pixel. The required coding of the scattered set of significant pixels relies on a hierarchical splitting of the pixel domain, which leads to a non-redundant representation of the significant pixels by a binary tree. This talk first explains selected theoretical and computational aspects of the image compression method, before very recent improvements concerning a postprocessing local optimization procedure are discussed. Finally, our compression method is compared with the standard JPEG2000 by using test cases of geometric images and popular real images. Large parts of the presented material are based on current joint work with Laurent Demaret and Nira Dyn.

Lossy, lossless and near-lossless image compression based on multiresolution analysis

Francesc Aràndiga

Departament de Matemàtica Aplicada. Universitat de València (Spain)

Being able to control the quality of the decoded data instead of the compression rate is suitable for applications where quality control is most important, yet we would like to be as economical as possible with respect to storage and speed of computation. In this paper we present a compression algorithm based on Harten's framework for multiresolution that guarantees a specific estimate of the error between the original and the decoded image measured in different norms.

Numerically solving PDEs on Implicit Surfaces

John B. Greer[a], Andrea L. Bertozzi[b], and Guillermo Sapiro[c]

[a]Courant Institute for Mathematical Sciences

[b]Department of Mathematics, UCLA

[c]Electrical and Computer Engineering, University of Minnesota

We will examine recent methods for solving PDEs defined on codimension-one surfaces in R^N. By using geometric tools from level set methods (see Sethian, 2000, or Osher and Fedkiw, 2003), we compute PDEs on these curved domains using only finite difference schemes and standard Cartesian grids in the embedding space. In addition to being easy to implement, the methods are readily adaptable to problems where the surface evolves by some external velocity field. These methods suffer the disadvantage of being defined on a larger domain – a small band around the curve or surface. In addition, well-behaved PDEs like the heat equation become degenerate diffusions when computed in the ambient space. We will discuss current work in this area, including an application of this method to fourth order equations on surfaces, including the Cahn-Hilliard equation and a model for a thin film of fluid driven down a sphere by gravity. We also present a newer formulation of these level set methods that avoids the degeneracy problems mentioned above.

Information-theoretic models in image processing

Gadiel Seroussi

Information Theory Research, Hewlett-Packard Laboratories

We describe some of the successful modeling techniques that have been used in information-theoretic approaches to image processing tasks such as lossless image compression, denoising, and simulation. We discuss the relations between various commonly used modeling tools, show how they are used in practice, and explain their effectiveness in terms of a hierarchical organization of "learning" tasks. We draw parallels to natural phenomena, including some of the modern theories of language learning, and speculate that our techniques are just primitive analogues of sophisticated mechanisms that Nature has built through the evolutionary process.

Comparing Manifolds Given as Point Clouds

Facundo Memoli

Stanford University

Point clouds are one of the most primitive and fundamental manifold data representations. A popular source of point clouds are three dimensional shape acquisition devices such as laser range scanners. Another important field where point clouds are found is in the representation of high-dimensional manifolds by samples. With the increasing popularity and very broad applications of this source of data, it is natural and important to work directly with this representation, without having to go through the intermediate and sometimes impossible and distorting steps of manifold reconstruction. A geometric framework for comparing manifolds given by point clouds is discussed in this talk. The underlying theory is based on Gromov-Hausdorff distances, leading to isometry invariant and completely geometric comparisons. This theory is embedded in a probabilistic setting as derived from random sampling of manifolds, and then combined with results on matrices of pairwise geodesic distances to lead to a computational implementation of the framework. The theoretical and computational results here presented are complemented with experiments for real three dimensional shapes.

Regression Level Set Estimation

Rebecca Willett and Robert Nowak

University of Wisconsin-Madison
1415 Engineering Hall
Madison, WI 53706

In many applications, the location of contours or level sets are of principal importance, while the amplitude of the function away from any contour is secondary, if not irrelevant. For example, doctors may wish to identify regions where uptake of a pharmaceutical exceeds some critical level, or mapping agencies may want to extract networks of roads from satellite images. However, plug-in estimators based on signal estimates with very low mean amplitude errors over the entire function may be unsuitable for accurate regression level set extraction. In this talk, I will propose an explicit method for recovering regression level sets from noisy multi-dimensional data and demonstrate its theoretical optimality. This method minimizes a novel error metric proportional to the symmetric difference between the true set of interest and its estimate weighted by the integral of the distance between the function and the critical level of interest. The minimization can be computed quickly and accurately by pruning a binary tree representation of the observations. Theoretical analysis reveals that the tree pruning criterion must be distinctly different from that typically used for signal reconstruction to achieve optimal error decay rates; specifically, the criterion used for regression level set estimation must facilitate a degree of spatial adaptivity not possible using more traditional, tree size-based criteria. These theoretical results supported by several simulations and numerical studies.

WORKSHOP 17

Stochastic Computation

ORGANISERS:

Brad Baxter & Thomas Müller-Gronbach

Exponential Brownian Motion and Divided Differences

Brad Baxter and Raymond Brummelhuis

School of Economics, Mathematics and Statistics, Birkbeck College, University of London

We calculate an analytic value for the correlation coefficient between a geometric, or exponential, Brownian motion and its time-average, a novelty being our use of divided differences to elucidate formulae. This provides a simple approximation for the value of certain Asian options regarding them as exchange options. We also illustrate that the higher moments of the time-average can be expressed neatly as divided differences of the exponential function via the Hermite–Genocchi integral relation, as well as demonstrating that these expressions agree with those obtained by Oshanin and Yor when the drift term vanishes.

Linear multi-step methods for SDEs

E. Buckwar[a], R. Horváth-Bokor[b], and R. Winkler[a]

[a]Dept. of Mathematics, Humboldt University Berlin, Unter den Linden 6, 10099 Berlin, Germany

[b]Faculty of Information and Technology, Catholic University, Prater utca 50/a, 1083 Budapest, Hungary

In this talk we deal with linear multi-step methods for SDEs and study when the numerical approximation shares asymptotic properties of the exact solution in the mean-square sense. As in deterministic numerical analysis we use a linear time-invariant test equation and perform a linear stability analysis. Standard approaches used either to analyse deterministic multi-step methods or stochastic one-step methods do not carry over to stochastic multi-step schemes. In order to obtain sufficient conditions for asymptotic mean-square stability of stochastic linear two-step-Maruyama methods we construct and apply Lyapunov-type functionals.

A Physicist's Perspective on Optimization Problems

David S. Dean

Laboratoire de Physique Théorique, UMR CNRS 5152, IRSAMC, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 04, France

The statistical mechanical approach to the study of optimization problems has lead to progress in a number of ways. The approach is based on identifying the cost function, which needs to be minimized, with the energy of a physical system whose phase space is equivalent to the free adjustable parameters in the optimization problem. The zero temperature energy of the resulting physical system thus corresponds to the optimal solution. This formulation can be exploited in two ways. First, physically motivated minimization techniques such as simulated annealing can be applied to optimization problems, often leading to near optimal solutions. The statistical mechanical approach can also be used to carry out computations of average or typical values of optimal solutions, where the non adjustable parameters (describing the realization of the instance) in the system are taken to be quenched random variables. I will review the basis of this physical approach and highlight some of the problems which have been successfully tackled with this approach. Finally I will show how one can analyze a class of problems whose phase space is the set of permutations of an ensemble of quenched random positions, an example being the Traveling Salesman Problem. In particular it will be shown how this method leads to conjectures about the average length of the optimal path for the Maximal Traveling Salesman Problem.

High resolution coding of stochastic processes

Steffen Dereich

Institute of Mathematics, Berlin University of Technology, 10623 Berlin, Germany

Let X denote a stochastic process (the *original*) and n a natural number. We consider the finding of a good approximation $\hat{X}$ to X (*reconstruction*) that attains at most n distinct values (the *quantization problem*). As measure for the discrepancy between the original and the reconstruction (*distortion*) we consider a finite moment of a Banach space distance $\|X - \hat{X}\|$.

We summarize some new results for the asymptotic quantization problem (the *high-resolution quantization problem*) for Brownian motion. Next, we analyze optimal quantization strategies and compare the results with a particular random strategy in quantization.

In the second part we will treat the quantization problem for a solution X of a stochastic partial differential equation. As distortion we consider again an arbitrary moment of the $L^p[0,1]$- and supremum-norm distance. We shall see that the problem is closely related to the quantization problem for Brownian motion. Based on this link we derive the strong asymptotics of the best-achievable distortion when n tends to infinity.

Approximation to the Mean Curve of Longest Common Subsequences

Raphael Hauser

Oxford University Computing Laboratory

The *longest common subsequence* (LCS) problem concerns a question from discrete probability that appears naturally in bioinformatics, speech recognition and other areas where hidden Markov models play a role: given two random sequences of length n, let L_n be the expected length of the longest subsequence that appears in both sequences. When n tends to infinity, L_n/n converges to the so-called *Chvatal-Sankoff constant*. This setup also has generalisations in which the sequences are not of equal length or where m sequences are compared, giving rise to the *mean curve*. The precise value of these objects is not known, but upper and lower bounds to less than one correct digit have been described in the literature. We present and analyse a new large deviations based method for the computation of the best known upper bounds yet and give numerical results that are precise enough to reject a classical conjecture.

This talk is pieced together from papers that are joint work with Clement Dürringer (Toulouse), Servet Martinez (Santiago de Chile) and Heinrich Matzinger (Bielefeld and Georgia Tech).

Stochastic Differential Equations with Additive Fractional Noise: Approximation at a Single Point

A. Neuenkirch

Technical University of Darmstadt, Department of Mathematics,
Schlossgartenstrasse 7, 64289 Darmstadt, Germany
e-mail: neuenkirch@mathematik.tu-darmstadt.de

We study pathwise approximation of scalar stochastic differential equations with additive fractional Brownian noise of Hurst parameter $H > 1/2$, considering the mean square error at a single point. For a Wagner-Platen type approximation scheme, which uses only point evaluations of the driving fractional Brownian motion, we derive the exact rate of convergence, also for non-equidistant discretizations.

Moreover, for a subclass of equations we establish a lower error bound that holds for arbitrary methods, which use a fixed number of evaluations of the driving fractional Brownian motion. For this subclass of equations, the introduced Wagner-Platen type scheme provides the matching upper error bound and hence performs asymptotically optimal.

Monte Carlo Methods with Few Random Bits for Integration and Integral Equations

Harald Pfeiffer

Universität Jena, Mathematisches Institut, Ernst-Abbe-Platz 2, D-07737 Jena, Germany
pfeiffha@minet.uni-jena.de

Monte Carlo methods are algorithms that involve randomness. They are widely applied in multivariate integration and integral equations. As source of randomness a method typically uses random numbers from $[0,1]$, and many classical methods use about $d\,n$ of those random numbers where d is the dimension and n the number of function values.

Here, we replace random numbers from $[0,1]$ with random bits from $\{0,1\}$, and we want to use only a small number of them. We consider problems for which (deterministic and) general Monte Carlo methods are well studied: the approximate integration of functions from Hölder and Sobolev classes and the local solution of Fredholm integral equations with smooth data. We are also interested in how the constants that appear in the complexity bounds depend on the dimension. Usually, the complexity bounds are for a fixed class of problem elements in a fixed dimension so that the dependence of the constants on the dimension is unknown.

We construct methods with random bits that have the optimal order of complexity, which is the same as that for Monte Carlo methods with random numbers from $[0,1]$. The methods for integration use only about $d\,\log n$, those for integral equations only about $d\,\log 2n$ random bits. Concerning the problem of the integration of Hölder classes and that of the solution of integral equations on classes C^r we show upper bounds with constants explicitly depending on the dimension. For example, for integral equations with continuously differentiable kernels and right-hand sides the optimal cost of deterministic methods with error ε is of order $(1/\varepsilon)^{2d}$. We obtain methods with a cost of $c\,((1/\varepsilon)^2 + d\log 2(1/\varepsilon))$ or $c\,d5\,(1/\varepsilon)^{2d/(d+1)}$. Our approach to the problems is to reduce them to the approximation of means.

On the complexity of solving stochastic differential equations

Klaus Ritter

Technische Universität Darmstadt, Germany

We consider strong and weak approximation problems for stochastic differential equations, and we are interested in the following basic question: what is the minimal computational cost needed to solve a problem with error at most ϵ? This minimal cost is called the ϵ-complexity, and it quantifies the intrinsic difficulty of a problem.

An answer to this questions usually includes the construction of an (almost) optimal algorithm: its error is at most ϵ and its computational cost is close to the ϵ-complexity. On the other hand, a lower bound is needed for the computational cost of all algorithm that solve the problem with error at most ϵ.

In this talk we focus on lower bounds. They are derived from the fact that every algorithm may only use a finite number of function or derivative values of the drift and diffusion coefficient and, in case of strong approximation, a finite number of evaluations of a trajectory of the driving Brownian motion.

Linear multi-step methods for SDEs

Renate Winkler and Evelyn Buckwar

Dept. of Mathematics, Humboldt University Berlin, Unter den Linden 6, 10099 Berlin, Germany

In this talk the numerical approximation of solutions of Itô stochastic differential equations is considered. We construct stochastic linear multi-step methods and develop the fundamental numerical analysis concerning their mean-square consistency, numerical stability in the mean-square sense and mean-square convergence. For the special case of two-step Maruyama schemes we derive conditions guaranteeing their mean-square consistency. Further, for the case of small noise SDEs, we obtain expansions of the local error in terms of the stepsize and a parameter ϵ, where the latter quantifies the smallness of the noise. Simulation results using several explicit and implicit stochastic linear k-step schemes, $k = 1,\ 2$, illustrate the theoretical findings.

WORKSHOP 18

Symbolic analysis

ORGANISERS:

Elizabeth Mansfield, Peter Olver & Mike Singer

homalg: An abstract MAPLE-package for homological algebra

Mohamed Barakat and Daniel Robertz

Lehrstuhl B für Mathematik, Technical University of Aachen, 52062 Germany

Homological algebra is a natural extension of the theory of modules over rings. The category of modules and their homomorphisms is replaced by the category of chain complexes and their chain maps. A module is represented by any of its resolutions. The module is then recovered as the only non-trivial homology of the resolution. All resolutions of a module are equivalent in a well-defined sense. The role of kernels and cokernels in the abelian category of modules is played by exact triangles in the category of complexes. The notions of derived functors and their homologies, connecting homomorphism and the resulting long exact homology sequences play a central role in homological algebra.

The MAPLE-package `homalg` provides a way to deal with these powerful notions. The package is abstract in the sense that it is independent of any specific ring arithmetic. If one specifies a ring in which one can solve the ideal membership problem and compute syzygies, the above homological algebra constructions over that ring become accessible using `homalg`.

The talk will also demonstrate applications in algebraic topology and algebraic control theory.

Cohomological questions in Computational D-module theory

Francisco J. Castro-Jiménez[a] and José María Ucha[b]

[a,b]Facultad de Matemáticas, University of Seville (Spain)

We use computational D-module theory to answer some questions on the comparison of the meromorphic de Rham complex to another complex of differential forms associated to a complex hypersurface singularity. Moreover, following previous works of F.J. Calderón and L. Narváez we consider the so called logarithmic D-modules associated to the given hypersurface and we prove a duality formula between them. To this end we use explicit syzygies and free resolutions of the considered logarithmic D-modules when the hypersurface f is of free Spencer type. The notion of free hypersurface we use here was introduced by K. Saito. We also solve an special case of a conjecture by T. Torrelli about the D-annihilator of the meromorphic function of $1/f$.

Dividing Fairly - Division of polynomials and power series by linear differential operators

Sebastian Gann[a] and Herwig Hauser[b]

[a]University of Innsbruck, Austria, sebastian.gann@uibk.ac.at

[b]University of Innsbruck, Austria, herwig.hauser@uibk.ac.at

We will discuss the division of polynomials and power series by linear partial differential operators with polynomial coefficients. This concept can be applied to algorithmically construct polynomial and power series solutions of the corresponding PDEs. After giving an idea of the algorithmic treatment we will focus on questions of finiteness and efficiency.

Rational and Replacement Invariants of a Group Action

E. Hubert[a] and I. Kogan[b]

[a] INRIA Sophia Antipolis, France

[b] North Carolina State University, Raleigh, USA

Group actions are ubiquitous in mathematics. They arise in diverse areas of applications, from classical mechanics to computer vision. A classical but central problem is to compute a generating set of invariants.

We consider a rational group action on the affine space. We propose a construction of a finite set of rational invariants and a simple algorithm to rewrite any rational invariant in terms of those generators. The construction comes into two variants both consisting in computing a reduced Gröbner basis of a polynomial ideal. That polynomial ideal is of dimension zero in the second variant that relies on the choice of a cross section, a variety that intersects generic orbits in a finite number of points. A generic linear space of complementary dimension to the orbits can be chosen for cross-section.

When the intersection of a generic orbit with the cross section consist of a single point, the rewriting of any rational invariant in terms of the computed generating set becomes a trivial replacement. For the general case we introduce a finite set of *replacement invariants* that are algebraic functions of the rational invariants. Any rational invariant can be trivially rewritten in terms of those by substitution.

We thus hold here an algebraic version of the moving frame method of Fels and Olver.

The present algebraic construction is easily implemented in specialized or general purpose computer algebra systems.

Symmetries of initial-value problems

Peter E Hydon and Richard Watts

Dept. Mathematics & Statistics, University of Surrey, Guildford GU2 7XH, UK.

Symmetry analysis is a powerful tool that enables the user to construct exact solutions of a given differential equation in a fairly systematic way. Symmetry-finding packages have been implemented in the major computer algebra systems. It is widely believed that symmetries of an initial-value problem are symmetries of the unconstrained differential equation. We demonstrate that this is untrue; the initial conditions can produce new symmetries. Consequently, it is sometimes possible to solve an initial-value problem exactly, even though the differential equation cannot be fully-solved. We present a constructive method for finding the symmetries of a given ordinary differential equation that is subject to an initial condition. Symbolic algebra is an essential part of the method, but there remain some obstacles to the development of routines that will find symmetries without intervention from the user; these obstacles are discussed.

Group-Invariant Moving Frames: Symbolic Computation and Applications.

Ian Anderson[a] and Irina Kogan[b]

[a]Utah State University, anderson@math.usu.edu

[b]North Carolina State University, iakogan@ncsu.edu

New symbolic algorithms for performing computations on a jet bundle relative to a group-invariant moving frame will be presented. This includes: prolongation of vector fields relative to an invariant moving frame, integration by parts, group-invariant Euler-Lagrange and Helmholtz operators, Noether correspondence. Applications to calculus of variations will be considered. This is a preliminary report.

Deflation of polynomial systems at isolated solutions

Anton Leykin[a], Jan Verschelde[b], and Ailing Zhao[c]

[a]Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 South Morgan (M/C 249), Chicago, IL 60607-7045, USA. leykin@math.uic.edu

[b](same) jan@math.uic.edu

[c](same) azhao1@uic.edu

Given a multiple isolated solution $\mathbf{x}^*$ of a polynomial system of equations $\mathbf{f}(\mathbf{x}) = 0$, we provide a symbolic algorithm, called *deflation*, to construct an augmented system of equations with the solution $\mathbf{x}^*$ having lower multiplicity.

The algorithm is arranged in a way that makes it possible to apply numerical methods to deflate the system provided only an approximation to $\mathbf{x}^*$. This means that, in a numerically stable way, we can produce a sequence of deflations ending in a new polynomial system which has the original multiple solution as a regular root.

We also explore several symbolic-numeric approaches to computing the multiplicity structure of an isolated solution, which provide an idea for improving our original deflation algorithm.

Solution Spaces of Hypergeometric Systems and the Structure of Hypergeometric Terms

S. A. Abramov[a], M. Petkovšek[b]

[a]Russian Academy of Sciences, Dorodnicyn Computing Centre, Vavilova 40, 119991, Moscow GSP-1, Russia (sabramov@ccas.ru)

[b]Department of Mathematics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia (marko.petkovsek@fmf.uni-lj.si)

A *discrete hypergeometric system* is a system of first-order linear homogeneous partial difference equations with polynomial coefficients, containing a single unknown. Algebraically, the solution space of a consistent hypergeometric system has dimension one. Here we consider the unknowns as discrete functions which are defined either everywhere on $\mathbb{Z}^d$, or at the nonsingular points of the system. Contrary to the algebraic framework, in our setting the dimension of the corresponding solution spaces can be, under some conditions, anything between 1 and ∞. This has some interesting consequences for structure theorems, as well as for summation algorithms that deal with hypergeometric terms.

Progress on Methods for Symbolic and Numeric Elimination Methods for Differential Systems

Greg Reid

Applied Mathematics Dept, University of Western Ontario, Canada

I will describe developments in the field of treatment of over-determined systems of equations and differential equations in the area of numerical and non-commutative geometry of differential equations.

Recent work with Moreno Maza and my students Wenyuan Wu and Robin Scott on using continuation methods to introduce approximate triangular decompositions is discussed. Comparisons with exact methods are given. I will discuss some new developments which apply homotopy continuation methods for determining missing constraints in systems of PDE. This is joint work with Jan Verschelde, Wenyuan Wu and Allan Wittkopf. Specifically by using so-called diagonal homotopies, which allow incremental processing of constraints, one at a time, and mixed volumes, the number of continuation paths can be dramatically decreased. I will also discuss developments in non-commutative completion methods for analytic PDE, with applications to group classification problems.

Linear Two-Point Boundary Value Problems in Symbolic Computation: A New Approach

Markus Rosenkranz

Radon Institute for Computational and Applied Mathematics
Austrian Academy of Sciences, A-4040 Linz, Austria

A linear two-point boundary value problem is given by a linear ordinary differential equation together with a set of linear boundary conditions set up at the two boundary points of a finite real interval. The given differential equation is inhomogeneous, and its right-hand side is regarded as a symbolic parameter, so solving it can be seen as finding a linear operator (the so-called "Green's operator") that maps the symbolic parameter to the symbolic solution of the boundary value problem (assuming existence and uniqueness).

Up now, boundary value problems have not received much attention in symbolic computation since they may be subsumed under the heading of differential equations. However, standard methods from there will often not be effective due to the presence of the symbolic parameter. Moreover, it seems more natural to work directly on the level of operators since boundary value problems are, as described above, operator problems in disguise. We will present a new solution approach that takes these issues into account and sheds some light on the essence of integral operators in symbolic computation.

Explicit invariants for linear q-difference equations

Jacques SAULOY

Laboratoire Emile Picard, UMR 5580. UFR MIG. Université Paul Sabatier, 118, route de Narbonne 31062 - Toulouse Cedex - France

Following suggestions by Birkhoff or resting on theories developed during the last decade, many algebraic and analytic invariants for complex linear q-difference equations are presently known and well understood. Many are q-analogs of corresponding objects linked to differential equations. We shall give definitions and a few theoretical properties, then hint at possible explicit ways of computing and using them, with the hope to attract experts in symbolic computation to this field.

Vessiot Theory of Differential Equations

Werner M. Seiler

Interdisciplinary Centre for Scientific Computing
Heidelberg University

The Vessiot theory is a dual version of the more familiar Cartan-Kähler theory based on vector fields. It can be directly applied to differential equations without the need to transform to an exterior differential system. After an introduction to the basic ideas, we present some elementary applications like the method of characteristics, internal symmetries or the analysis of singularities.

Superintegrability of the Calogero-Moser Model via Invariant Theory and Symbolic Analysis

Roman Smirnov[a], Ray McLenaghan[b], and Josh Horwood[c]

[a]Department of Mathematics and Statistics, Dalhousie University, Halifax, Nova Scotia, Canada B3H 3J5
[b]Department of Applied Mathematics, Universtiy of Waterloo, Waterloo, Ontario, Canada, N2L 3N6

[c]Department of Applied Mathematics and Theoretical Physics, Cambridge, CB3 0WA, UK

I will discuss how the invariant theory of Killing tensors can be effectively employed in the study of integrable Hamiltonian systems. The Calogero-Moser system defined in the Euclidean space will be thoroughly investigated from this viewpoint with the aid of the KillingTensor package designed by Josh Horwood.

Elliptic boundary problems and Shapiro–Lopatinskij condition

Katya Krupchyk[a] and Jukka Tuomela[b]

[a] University of Joensuu, Finland

[b] University of Joensuu, Finland

The boundary value problems for elliptic PDEs are well-posed, if the boundary conditions satisfy the Shapiro–Lopatinskij condition. This condition in case of overdetermined systems is given in terms of exactness of the complex of symbol maps of a boundary problem operator and its compatibility operator. We first propose a constructive method to compute this compatibility operator based on Gröbner bases techniques. Unfortunately, the existing criteria for the Shapiro–Lopatinskij condition (even in square case) are not satisfactory from computational point of view. Then we obtain a criterion for checking this condition which is more effective than previous ones. In case of two independent variables we further show how to check this criterion using symbolic computation.

Some Symmetry Classifications of Hyperbolic Vector Evolution Equations

Thomas Wolf[a], Stephen Anco[b]

[a]Department of Mathematics, Brock University, Canada L2S 3A1, email: twolf@brocku.ca

[b]Department of Mathematics, Brock University, Canada L2S 3A1, email: sanco@brocku.ca

From the many different aspects of integrability theory of nonlinear partial differential equations (PDEs), to date the most computationally direct and effective test has proved to be the condition that a PDE system should possess sufficiently many higher symmetries. In particular, for all currently known examples of nonlinear scalar PDEs, the existence of one higher symmetry implies the existence of infinitely many, i.e. a symmetry hierarchy.

In the talk, new results are reported on symmetry classifications of integrable $O(N)$-invariant classes of hyperbolic equations $U_{tx} = f(U, U_t, U_x)$ for an N-component vector $U(t, x)$ are considered. In each class all scaling-homogeneous equations admitting a higher symmetry of least possible scaling weight have been determined. Compared with previous symmetry investigations, the novelty in dealing with hyperbolic equations lies in the occurrence of zero and negative weights in an homogeneous ansatz. The method of undetermined coefficients consequently leads to non-linear PDE systems as determining equations with non-polynomial solutions, instead of algebraic systems.

WORKSHOP 19

Computational geometry and topology

ORGANISERS:

Rich Schwartz & Abigail Thompson

The A-Polynomial and the FFT

Marc Culler (UIC)

The A-polynomial is an invariant of the fundamental group of a knot manifold M. The plane curve defined by the A-polynomial is very closely related to the image of the PSL(2,C) character variety of M under the restriction map to the PSL(2,C) character variety of the boundary torus of M. The coefficients of the polynomial, and its Newton polygon, directly reveal topological properties of the knot manifold M, while the curve defined by the polynomial carries geometric information in the generic case of a hyperbolic knot manifold. A-polynomials also have fascinating connections to number theory via the Mahler measure. And they are notoriously difficult to compute.

Previous attempts at computing A-polynomials have been based on elimination theory, using resultants or Grobner bases. In this talk I will describe new computational methods that use techniques from numerical algebraic geometry, such as homotopy continuation. A new ingredient is the computation of the defining equation of a plane curve by interpolation, using the FFT. The homotopy computations that arise here coincidentally provide fascinating pictures of the main component of the PSL(2,C) character variety that shed light on its relationship to the results of Kronheimer and Mrowka on SU(2) representations.

Dirichlet Domains in the Complex Hyperbolic Plane

Martin Dereux (Grenoble)

I will present computer techniques to study Dirichlet domains for groups of isometries of the complex hyperbolic plane given by a set of generators, and discuss the strengths and limitations of the method. The original motivation is to study Mostow's construction of non-arithmetic lattices in PU(2,1) (Pacific J. of Math. 1980). As another application, I will explain how to prove that certain triangle groups admit surjective homomorphisms to cocompact lattices.

Does a Random 3-Manifold Fiber over the Circle?

Nathan Dunfield (Caltech)

I'll discuss the question of when a tunnel number one 3-manifold fibers over the circle — the motivation is here is to try to get some handle on the Virtual Fibration Conjecture for hyperbolic 3-manifolds. In particular, I will discuss a criterion of Brown which answers this question from a presentation of the fundamental group. I will describe how techniques of Agol, Hass, and W. Thurston can be adapted to calculate this very efficiently by using that the relator comes from an embedded curve on the boundary of a genus 2 handlebody. I will then describe some experiments which strongly suggest the answer to the question: Does a random tunnel-number one 3-manifold fiber over the circle? I will end by explaining how to prove that the the observed answer is indeed correct in one of the two cases. (joint work with Dylan Thurston, Harvard)

Steps into Computational Algebraic Topology

Herbert Edelsbrunner (Duke)

A nested sequence of progressively larger topological spaces implies a sequence of homology groups connected by maps induced by the inclusions of the corresponding spaces. For each pair of groups, we call the image of the earlier in the later group a persistent homology group. Given a function on a topological space, the sublevel sets form such a nested sequence of spaces. The corresponding persistent homology groups can be encoded using a finite multiset in the extended plane. Assuming a triangulation of the space and a piecewise linear function, we have an algorithm that computes his encoding in worst-case time cubic in the size of the triangulation. However, its observed running time is vastly better so that even triangulations with a few million simplices can be processed in a matter of seconds. We have proved that for two continuous functions on a common space, the bottleneck distance between the two encodings is bounded from above by the maximum norm of the difference function.

Homological Sensor Networks

Rob Ghrist (UIUC)

As engineers design smaller and smaller sensors for robotic and security systems, their capabilities change from global to local. For example, you can't put a GPS unit on a micro-scale sensor. The question thus arises: what types of global problems can a swarm of local sensors solve? We show how a little bit of homological algebra goes a long way in allowing sensor swarms to solve global problems, such as coverage, pursuit-evasion, and target counting. We argue from this that computational homology is a crucial tool for sensor networks, communication, and security problems.

Computing Surface Intersections

Joel Hass (UC Davis)

Finding the intersection of two surfaces in 3-space is a commonly occurring problem in computational geometry, geometric design, CAD etc. For common representations of smooth surfaces (such as bicubic patches) it is surprisingly difficult to achieve. Exact computation of the intersection is essential for for applications such as automatic mesh generation. We describe a new approach based on some topological ideas. This is joint work with R. Farouki, C. Y. Han, T. W. Sederberg and X. Song.

Triangulated Manifolds

Frank H. Lutz (TU Berlin/ZIB)

I will discuss several aspects of triangulated surfaces and higher-dimensional manifolds:

1. Enumeration (of triangulations with few vertices).
2. Recognition (with bistellar flips).
3. Combinatorics (of minimal triangulations).
4. Constructions (of small non-shellable/non-PL spheres; of Seifert manifolds).
5. Realization (of surfaces of small genus).

Computation and Billiards in Triangles

Pat Hooper (Yale)

Surprisingly, there are many open problems about billiards in triangles. The most notable question is "Does every triangle have a periodic billiard path?" This question has been resolved in the acute and right triangle cases, and also when a triangle has rational angles. The resolution of the rational case involves Teichmuller theory of compact surfaces, but this resolution gives rise to further open questions (e.g. "Classify the Veech triangles.") Recently, Rich Schwartz and I have approached triangular billiards from a computational direction, yielding new results. I will demonstrate our program, McBilliards, and discuss some of these new results. I will also discuss some of the computational problems faced in constructing such a program.

Fundamental Domains in Three Dimensional Anti DeSitter Space

Anna Pratoussevitch (Bonn)

We consider finite level discrete subgroups G of the simply connected Lie group U(1,1). The Lie group has the structure of a 3-dimensional Lorenz manifold, and G acts by left translation. The quotient is the spaceform for associated to G. To understand these spaceforms we want to construct a fundamental domain F for the action of G, so that F has totally geodesic faces. We will show how to do this when G has an elliptic element whose order exceeds the level. For the co-compact examples, the associated spaceform is topologically the link of a quasi-homogeneous Gorenstein singularity. The quasi-homogeneous singularities of Arnold's series E,Z,Q are of this type. We compute the combinatorics of the fundamental domains of the groups corresponding to the links in Arnold's series and show how they relate to the classical uniform polyhedra.

Alternate Heegaard genus bounds distance

Martin Scharlemann (UC Santa Barbara)

Given a surface P, the curve complex of P is a graph in which the vertices correspond to isotopy classes of essential curves on P. Two vertices are connected by an edge if the corresponding isotopy classes of curves have disjoint representatives. If the surface P is a Heegaard splitting of a 3-manifold M, then d(P) is the minimum distance in this graph between vertices corresponding to curves that bound compressing disks on opposite sides of P.

By a theorem of Hartshorn, it is known that if M is an irreducible compact orientable 3-manifold and P is a Heegaard surface for M, then d(P) is bounded above by the genus of any properly embedded essential surface. We prove that d(P) is similarly bounded by the genus of alternate Heegaard surface Q as long as Q is not isotopic to a stabilization of P.

Geometric Knot Theory

John Sullivan (Berlin)

Geometric knot theory is the study of geometric properties of space curves that derive from their topological knottedness. Perhaps the most famous result is the Fary/Milnor theorem relating total curvature to bridge number, but the field can be dated back to work of Pannwitz on quadrisecant lines for knots. There has been a surge of interest in geometric knot theory over the past decade, partly due to biophysical applications to the shapes of knotted polymers like DNA molecules. One interesting problem with some surprising answers asks for the shapes of knots and links tied tight in rope of fixed thickness. We will survey recent results on this so-called ropelength problem, as well as some new strengthened versions of the theorems of Pannwitz and Fary/Milnor.

Computing with Simple Curves on a Surface

Dylan Thurston (Harvard)

The abstract of the talk was unavailable at the time of production.

WORKSHOP 20

Mathematical control theory and applications

ORGANISERS:

Eduardo Casas, Uwe Helmke,

Jean-Pierre Raymond & Enrique Zuazua

Recent Advances in the Error Estimates for the Numerical Approximation of Semilinear Elliptic Optimal Control Problems

Eduardo Casas

University of Cantabria, Spain

We study the numerical approximation of optimal control problems governed by semilinear elliptic equations under the presence of pointwise control constraints. We distinguish three cases: distributed, Neumann and Dirichlet controls. We discretize the control problems by using finite element methods and we compare the solutions of the discrete control problems with the solutions of the continuous control problems. For distributed or Neumann controls we can consider piecewise constant approximations or piecewise linear and continuous approximations. We will see the improvement of the second type of approximations with respect to the first ones. In the case of Dirichlet controls, the only reasonable discretization is defined by piecewise linear and continuous approximations.

By doing some extra assumptions on the saturation set of points of the control constraints, we can improve the error estimates for piecewise linear controls. Under this extra assumption, in the case of piecewise constant control constraints there is an idea due to Meyer and Rösch who improves the order of approximation by doing a postprocessing step. This idea will be compared with that proposed by Hinze which suggests to discretize the state equation but not the control space.

The results presented in this talk correspond to some previous research in collaboration with N. Arada, M. Mateos, J.-P. Raymond and F. Tröltzsch.

Using the continuous SVD decomposition to approximate spectra of dynamical systems

L. Dieci[a] and C. Elia[b]

[a]School of Mathematics, Georgia Institute of Technology, Atlanta, GA, 30332 U.S.A.

[b]Dipartimento di matematica, University of Bari, I-70125 Bari, Italy.

We explore numerical techniques based on the continuous SVD of the fundamental matrix solution to approximate the Lyapunov spectrum and the exponential dichotomy spectrum of an n-th dimensional system of ordinary differential equations. Theoretical justifications and numerical examples are given.

Local exact controllability of the Boussinesq system in dimension 3

S. Guerrero
Dpto. E.D.A.N., University of Sevilla, Aptdo. 1160, 41080 Sevilla, Spain. E-mail: sguerrero@us.es. This work has been partially supported by D.G.E.S. (Spain), Grant BFM2000–1317.

We present a result concerning the *local exact controllability* of the Boussinesq system in dimension 3. We consider a smooth fixed trajectory $(\overline{y}, \overline{p}, \overline{\vartheta})$ of the Boussinesq system; here, $\overline{y}$ represents the velocity vector field, $\overline{p}$ is the pressure and $\overline{\vartheta}$ is the heat variable. They all evolve from time $t = 0$ to time $t = T$.

Our goal is to control the couple $(\overline{y}, \overline{p})$. More precisely, by means of a control (of the distributed or of the boundary kind) we try to drive the associated solution (y, p) to $(\overline{y}, \overline{p})$ at final time $t = T$. This will be possible provided that $y|_{t=0}$ is close enough to $\overline{y}|_{t=0}$.

The main tool in this paper is a global Carleman inequality for the solutions of an associated linear system.

Interpolation curves on S^n and SO_n by rolling and wrapping

K. Hüper

National ICT Australia Ltd
Canberra Research Laboratory
Systems Engineering and Complex Systems Program
Locked Bag 8001, Canberra ACT 2601
Australia

A procedure is presented to produce smooth interpolating curves on manifolds, given in closed form. This makes the corresponding algorithm easy to implement. The method is a combination of the pull back/push forward technique with unrolling data from the manifold into some vector space, solving there the interpolation problem, and then wrapping the resulting interpolation curve back to the manifold. The standard sphere and the rotation group are studied in detail. A numerical experiment is also presented. We relate our results to the concept of parallel transport in differential geometry. Moreover, connections to certain optimal control problems are presented.
This is joint work with Fatima Silva Leite, Coimbra and Yueshi Shen, Canberra.

Optimal Control of Spin Systems

Navin Khaneja

Harvard University, navin@eecs.harvard.edu

In this talk I will discuss some optimal control problems that arise in manipulation of coupled spin dynamics in NMR spectroscopy. Experiments have shown that ideas from optimal control can lead to substantial improvement in sensitivity of NMR experiments. I will discuss some recent results and open problems in control of spin systems with applications to coherent spectroscopy and quantum computing.

Semi-smooth Newton Methods in Function Spaces

Karl Kunisch

University of Graz, Austria

Semi-smooth Newton Methods provide superlinearly convergent algorithms for problems which are not C^1-smooth. These situation arises in optimal control with control- or state constraints, in contact and friction problems, and in BV-regularized problems, for example. We provide a general framework for the applicability of these methods.- For certain classes a smoothing of the problem formulation is required and the question of the choice of a smoothing parameter arises. It is answered by means of path-following techniques.

A Novel Algorithm for Joint Diagonalisation with a Geometric Interpretation

Jonathan H. Manton

Queen Elizabeth II Fellow Department of Information Engineering Research School of Information Sciences and Engineering (Building 115) The Australian National University Canberra ACT 0200

The algebraic eigenvalue problem arises in myriad signal processing and control applications. The joint diagonalisation problem is a generalisation of the algebraic eigenvalue problem; given real symmetric matrices $A_1, \cdots, A_k$ it is required to find an orthogonal matrix X such that $X^T A_1 X, \cdots, X^T A_k X$ are all diagonal or, if this is not possible, such that the $X^T A_i X$ are all approximately diagonal in some suitable sense. Joint diagonalisation algorithms have been used to solve blind source separation problems, notably in the work by Cardoso. Although empirical evidence suggests existing algorithms work well most of the time, sometimes the algorithms return erroneous results. This talk will present a very different approach to the joint diagonalisation problem. The approach is characterised by its geometric interpretation, and indeed, leads to a new joint diagonalisation algorithm based on the minimisation of a convex cost function on a Riemannian manifold. Since the cost function is convex, the algorithm is the first to guarantee global convergence.

Optimal control and numerical algorithms for ODEs

Clyde Martin

Department of Mathematics, Texas Tech University, Lubbock, TX 79409, USA,
martin@math.ttu.edu

In this talk we will explore some of the potential applications of control theory to the numerical solution of differential equations. In the first method we will consider an optimal control method for solving ODEs that are defined on the surface of a sphere. In the second method we will consider a method where the corrector is based on control theoretic smoothing splines. In this case the model and the regulator are removed from the system. We see that numerical algorithms for solving ODEs are inherently control theoretic. It remains to be seen if control theoretic methods can add to the numerical theory.

Stability and controllability of an abstract evolution equation of hyperbolic type and some applications

S. Nicaise

Université de Valenciennes et du Hainaut Cambrésis, Le Mont Houy, 59313 Valenciennes Cedex 9, France

We consider the stability of an abstract evolution equation using Liu's principle based on the exponential stability of the inverse problem with a linear feedback and on an integral inequality. Russell's principle also yields some exact controllability results. Some concrete examples with new stability and controllability results illustrate the interest of our approach.

Recent results on exact controllability for Navier Stokes equations

Jean-Pierre Puel

Laboratoire de Mathématiques Appliquées, Université de Versailles St Quentin, 45 avenue des Etats Unis 78035 Versailles France.

We will present the recent results of exact controllability to trajectories for the Navier Stokes equations. We will give the strategy and the different mathematical ingredients of the proof, in particular the underlying global Carleman estimate which is needed. We will discuss also some variants, for example if we want to decrease the number of controls. An alternative strategy in the proof will also be presented, starting with a supplementary control which can be eliminated in a second step.

Controlling Quantum Dynamics Phenomena with Shaped Laser Pulses Acting as Photonic Reagents

Herschel Rabitz

Department of Chemistry, Princeton University

Since the development of the laser some 40 years ago, a longstanding dream has been to utilize this special source of radiation to manipulate dynamical events at the atomic and molecular scales. Hints that this goal may become a reality began to emerge in the 1990's, due to a confluence of concepts and technologies involving (a) control theory, (b) ultrafast laser sources, (c) laser pulse shaping techniques, and (d) fast pattern recognition algorithms. These concepts and tools have resulted in a high speed instrument configuration capable of adaptively changing the driving laser pulse shapes, approaching the performance of thousands of independent experiments in a matter of minutes. Each particular shaped laser pulse acts as a "Photonic Reagent" much as an ordinary reagent would at the molecular scale. Although a Photonic Reagent has a fleeting existence, it can leave a permanent impact. Current demonstrations have ranged from manipulating simple systems (atoms) out to the highly complex (biomolecules), and applications to quantum information sciences are being pursued. In all cases, the fundamental concept is one of adaptively manipulating quantum systems. The principles involved will be discussed, along with the presentation of the state of the field.

Incompressible Fluid: Controllability of the 2D Euler and Navier-Stokes Equation by Few Low Modes Forcing

Andrey Sarychev

DiMaD, University of Florence, Italy; asarychev@unifi.it

Presented are results of joint work with A.Agrachev, SISSA, Trieste, Italy.

We study controllability issues for the 2D Euler and Navier-Stokes (NS) systems under periodic boundary conditions. These systems describe motion of either ideal or viscous homogeneous incompressible fluid on a two-dimensional torus $\mathbb{T}^2$. We assume the system to be controlled by a degenerate forcing applied to few (the number is fixed) low modes.

In our previous work we studied global controllability of 2D and 3D Navier-Stokes (NS) systems with nonvanishing viscosity ($\nu > 0$). Methods of differential geometric/Lie algebraic control theory have been used for that study. We managed to establish criteria for global controllability of finite-dimensional Galerkin approximations of 2D and 3D NS systems. It is almost immediate to see that these criteria are also valid for the Galerkin approximations of Euler systems; in fact the Lie brackets involved into the corresponding Lie rank controllability condition do not depend on the viscous term. More intricate was establishing for 2D NS system controlled by few low modes a much more intricate sufficient criteria for global controllability in finite-dimensional observed component and for L_2-approximate controllability. The justification of these criteria has been based on a Lyapunov-Schmidt reduction of the finite-dimensional system. Such a reduction makes use of dissipativity of the NS system, and hence is not applicable to Euler system.

In the present contribution we improve and extend the controllability results in several aspects: 1) we obtain a stronger sufficient condition for controllability of 2D NS system in an observed component and for L_2-approximate controllability; 2) we prove that these criteria are valid for the case of ideal incompressible fluid ($\nu = 0$); 3) we study controllability in projection on any finite-dimensional subspace and establish a sufficient criterion for such controllability.

The heart: a controlled multi-scale distributed system

Michel Sorine

Inria Rocquencourt, 78153 Le Chesnay cedex, France

Ideas originating from the kinetic equation theory are used to model the controlled collective behaviour of actin-myosin molecular motors at the root of cardiac muscle contraction. This provides a ground model for the controlled production of mechanical energy on the cell scale. The "sliding-filament model" of the sarcomere proposed by Huxley appears as a consequence of the classical method of moments of the kinetic theory applied to the ground model. A controlled constitutive law on the tissue scale is obtained using again the method of moments as a scaling technique. Special choices of actin-myosin interaction potential, myosin binding and unbinding rates are done in order to recover the collective motor property and experimentally observed contraction and relaxation behaviours. The choice of the "controller" for this multi-scale system (the electrical models from subcellular calcium dynamics to action potential travelling waves) is discussed having in mind some direct and inverse problems associated with patient-specific cardiac simulators.

Numerical techniques for state-constrained elliptic optimal control problems

Fredi Tröltzsch

Technische Universität Berlin, Institut für Mathematik, Straße des 17. Juni 136, D-10623 Berlin,
troeltzsch@math.tu-berlin.de

The numerical solution of elliptic control problems with pointwise state-constraints is discussed. The constraints are transformed to mixed pointwise control-state constraints by a Lavrentiev type regularization. In this way, primal-dual active set strategies can be set up for the infinite-dimensional setting of the problem and are comparable with interior point techniques. The methods are analyzed and their performance is compared by several numerical examples.

Observers for linear systems

Jochen Trumpf

Department of Information Engineering
The Australian National University
Canberra ACT 0200, Australia
and
National ICT Australia Ltd.
Jochen.Trumpf@anu.edu.au

In this lecture I will attempt to summarize the state of the art in observer theory for linear finite-dimensional time-invariant systems. This will include the contributions made by the author in collaboration with Uwe Helmke and Paul Fuhrmann.

Roughly speaking, the observation problem is to calculate present values of one set of time signals given information about the past values of another set of time signals, where both signal sets are interconnected by the action of a dynamical system.

Rigorous definitions of various types of (partial) observability in the language of behaviors as well as definitions of the corresponding observers will be given. These definitions will be specialised to (strictly causal) state-space systems. I will formulate observability tests, necessary and sufficient criteria for the existence of observers as well as characterizations of observers. Some parametrization results will be reported that hook in with other areas of Mathematics such as optimization, polynomial matrix theory and singularity theory.

Some recent results on the analysis and the control of fluid-structure interactions

Marius Tucsnak

Institut Elie Cartan de Nancy and INRIA Lorraine

The aim of this presentation is to highlight some recent advances on the numerical analysis and the control of fluid structure interactions. The motion of the fluid is governed by the incompressible Navier-Stokes equations and the standard conservation's laws of linear and angular momentum rule the dynamics of the structure. The time variation of the fluid domain (due to the motion of the structure) is not known a priori, so we deal with a free boundary value problem. We consider two main cases.

The first one concerns the situation when the structure is formed by several independent rigid bodies. This situation is by now well understood. We describe some recent results on wellposedness (see, for instance, [2]) with emphasis on global existence results. The global character of the solutions we obtain is due to the fact that we don't need any assumption concerning the lack of collisions between several

rigid bodies or between a rigid body and the boundary. We give estimates of the velocity of the bodies when their mutual distance or the distance to the boundary tends to zero. We next describe and discuss the convergence of a numerical method introduced in [3]. One of the main features of this method is that it is based on a fixed mesh.

The second par of this work is devoted to the case of several articulated bodies actuated by torques acting at the various joints. This subject is motivated by the famous paper of Triantafyllou [4] and the paper which followed it (see, for instance [1]) which showed that biomimetic aquatic propulsors (or simply fish-like robots) have arguable advantages in terms of speed, maneuverability and stealth. All these works are based on the assumption that the fluid is perfect, incompressible and that it has an irrotational motion. Our main contribution lies in the fact that we tackle the case of a viscous fluid. Our theoretical and numerical approach, based on the methods we have developed for the rigid case, is aimed to tackle a wide range of problems from swimming of microorganisms to motion of large self-propelled bodies.

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A Unified Approach to the Controllability Theory of Stochastic Parabolic Equations, and Deterministic Parabolic and Hyperbolic Equations

Xu Zhang

Departamento de Matemáticas, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain; and School of Mathematics, Sichuan University, Chengdu 610064, China,
xu.zhang@uam.es

We present in this talk a unified approach to the controllability theory of stochastic forward and backward parabolic equations, and deterministic parabolic and hyperbolic equations, developed by the author and his collaborators. It is found that the controllability properties of these equations can be derived by means of a universal approach, which is based on the duality argument and the global Carleman estimate via an identity for a stochastic parabolic-like operator. Further applications will be also explored.

WORKSHOP 21

Random matrices

ORGANISERS:

Iona Dumitriu, Alan Edelman & Raj Rao

Asymptotics of rectangular random matrices: a general approach

Florent Benaych-Georges

We shall model asymptotic joint behaviour of independent rectangular $n \times p$ random matrices, when the dimensions n and p tend to infinity in such a way that n/p has a finite non-negative limit. What does *modeling of asymptotic joint behaviour* of a family $(M_i)_{i \in I}$ random matrices means? In our work, like in the one of Voiculescu, Biane, ..., this means to give a way to compute asymptotics of traces of sums and products of the M_i's. Since the trace of the k-th power of a matrix is the k-th moment of the empirical measure on its spectrum, one can derive some spectral properties from this model. An important progress, from previous works to this one, is that the rectangular random matrices we consider are very general : they are only required to have distributions which are invariant under left and right multiplications by unitary matrices. So not only Wishart matrices (i.e. rectangular matrices with Gaussian entries) are considered.

As an example, we shall give a way, via integral transforms or combinatorial methods, to derive the singular distribution of $M+N$ (i.e. the uniform distribution on the spectrum of $|M+N| := \sqrt{(M+N)(M+N)^*}$) from the singular distributions of M and N, when M, N are independent large rectangular random matrices.

Free probability and random matrices

Philippe Biane

DMA - Ecole Normale Superiure 45, rue d'Ulm - 75230 Paris Cedex 05 - France

We shall explain why free probability is the right model for explaining spectral properties of large random matrices, when the spectrum is considered globally, i.e. through the empirical measure on the eigenvalue. In particular free probability allows to make predictions for spectra of sums or products of independent large random matrices, with many explicit computations.

Strong asymptotic freeness for Wigner and Wishart matrices

Mireille Capitaine

Laboratoire de Statistique et Probabilité, Université Paul Sabatier, 118, Route de Narbonne, 31062 Toulouse Cedex 4, France

For each n in $\mathbb{N}$, let $X_n = [(X_n)_{jk}]_{j,k=1}^n$ be a random Hermitian matrix such that the n^2 random variables $\sqrt{n}(X_n)_{ii}$, $\sqrt{2n}\,\mathrm{Re}((X_n)_{ij})_{i<j}$, $\sqrt{2n}\,\mathrm{Im}((X_n)_{ij})_{i<j}$ are independent identically distributed with common distribution μ on $\mathbb{R}$. Let $X_n^{(1)}, \ldots, X_n^{(r)}$ be r independent copies of X_n and $(x_1, \ldots x_r)$ be a semicircular system in a $\mathcal{C}^*$-probability space. Assuming that μ is symmetric and satisfies a Poincaré inequality, we show that, almost everywhere, for any non commutative polynomial p in r variables,

$$\lim_{n\to+\infty} ||p(X_n^{(1)}, \ldots, X_n^{(r)})|| = ||p(x_1, \ldots x_r)|| \ . \tag{8}$$

We follow the method of Haagerup, Thorbjørnsen and Schultz which gave (8) in the Gaussian (complex, real or symplectic) case. We also get that (8) remains true when the $X_n^{(i)}$ are Wishart matrices while the x_i are Marchenko-Pastur distributed.

A β future for the classical ensembles of random matrices

Ioana Dumitriu

Dept. of Mathematics, U.C. Berkeley,
Berkeley, CA 94720, USA

Classical ensembles of random matrix theory (Gaussian, Wishart, MANOVA) have been around since the birth of the field. Their eigen-properties have been studied in depth for three values of a β parameter ($\beta = 1, 2, 4$), which can be thought of as a counting/labeling tool for their real, complex, or quaternion entries.

Different methods have been applied to different β in the study of the classical ensembles, with great success, but without providing a cohesive view. The idea to unify these studies has been introduced only recently, with the advent in the late '80s and '90s of the more general β-ensembles of arbitrary positive β, which subsume the classical cases and for which the Bernoulli β parameter acts as an inverse temperature. The work of Aomoto (1987), Kaneko (1993), Forrester (1997), Johansson (1998), and others, paved the way for a new perspective on the classical ensembles, which looks at a continuum of ensembles, rather than a few isolated cases.

With the very recent discovery of matrix models for the general β-ensembles (D. and Edelman (2002), Nenciu and Killip (2004), Sutton and Edelman (2005)), we have entered a new era in the study of the β-ensembles. The matrix models allow us to duplicate old results, prove new (and surprising) ones, and enlarge our understanding of the complex phenomena behind the three isolated instances that have started it all.

The Gaussian, Wishart, and MANOVA ensembles are and will always be special, but the future of the classical ensembles seems to have a generic β label.

This talk is based on work done by the few people mentioned above and by many others.

Advances in Stochastic Eigen-Analysis

Alan Edelman

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139.

Stochastic Eigen-Analysis is now a big subject with applications in many fields of science, finance and engineering. This talk surveys some of the important mathematics that is a very modern development as well as the computational software that is transforming theory into useful practice. Some intriguing connections and open problems will be mentioned.

Learning eigenvectors and eigenvalues from limited high-dimensional data.

David Hoyle

School of Engineering, Computer Science & Mathematics
University of Exeter,
Harrison Building, North Park Rd.,
Exeter, EX4 4QF, UK.
d.c.hoyle@exeter.ac.uk www.dcs.ex.ac.uk/dchoyle

Bioinformatics is the application of Mathematics, Statistics and Computer Science to the study of biological problems and data. Application of covariance based statistical learning algorithms, such as Principal Component Analysis (PCA) and kernel PCA, is common in Bioinformatics. Such bioinformatic data sets typically consist of a small number of sample points of very high dimension. Use of techniques from statistical physics can yield insight into the behaviour of covariance and Gram matrices in this regime, but perhaps at the expense of mathematical rigour. In this talk I will give a brief introduction to some of the problems bioinformaticians study, as well as the replica technique from statistical physics. I will present recent replica-technique results on the expected sample covariance and Gram matrix eigenvalue spectra for these high-dimensional data sets. Results for the expected accuracy of sample covariance eigenvectors will also be presented.

Large Covariance Matrices: sparsity and estimation of principal eigenvectors

Iain M. Johnstone

Department of Statistics, Stanford University

The sample covariance matrix of an $n \times p$ matrix of Gaussian white noise entries follows a Wishart distribution, and the law of its eigenvalues is an instance of the Laguerre ensemble of random matrix theory. In statistical application, the unknown population covariance matrix is typically not isotropic. If the number of variables p is comparable to sample size n and both are large, the theory for estimation of the largest eigenvalues and associated eigenvectors is still under development. Some recent results, also of Arthur Lu and Debashis Paul and others, will be surveyed, with an emphasis on settings in which the principal eigenvectors have a sparse representation.

Moments of spectral determinants of complex random matrices

Boris Khoruzhenko[a] and Yan Fyodorov[b]

[a]School of Mathematical Sciences, Queen Mary, University of London, London E1 4NS, U.K.

[b] Department of Mathematics, Brunel University, Uxbridge, UB8 3PH, U.K.

Let A be a fixed $n \times n$ matrix and U be a unitary matrix picked up at random from the unitary group $U(n)$. We express the integer moments of the spectral determinant $|\det(zI_n - AU)|^2$ in terms of the characteristic polynomial of the matrix AA^*. This result provides a useful tool for studying the eigenvalue distributions of complex random matrices. Links between this problem, Kaneko's generalization of the Selberg integral and Zirnbauer's color-flavor transformation will be discussed.

The Efficient Computation of Multivariate Statistics though the Hypergeometric Function of a Matrix Argument

Plamen Koev, Alan Edelman

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139

Various statistics of random matrices are expressible in terms of the *hypergeometric function of a matrix argument* X:

$$ {}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;X) \equiv \sum_{k=0}^{\infty}\sum_{\kappa\vdash k}\frac{(a_1)_\kappa\cdots(a_p)_\kappa}{k!(b_1)_\kappa\cdots(b_q)_\kappa}\cdot C_\kappa(X), $$

where $C_\kappa(X)$ is the zonal polynomial and $(a)_\kappa$ is the Pochhammer symbol.

For example, the cumulative density function of the largest eigenvalue of an $n\times n$ Wishart matrix with l degrees of freedom and covariance matrix Σ, $A = W_n(l,\Sigma)$ is:

$$ P(\lambda_A < x) = \frac{\Gamma_n\left(\frac{n+1}{2}\right)}{\Gamma_n\left(\frac{l+n+1}{2}\right)}\cdot\det\left(\tfrac{1}{2}x\Sigma^{-1}\right)^{l/2}\cdot{}_1F_1\left(\tfrac{l}{2};\tfrac{n+l+1}{2};-\tfrac{1}{2}x\Sigma^{-1}\right), \qquad (9) $$

where Γ_n is the multivariate Gamma function. The distributions of the trace, the smallest eigenvalue, etc., of Wishart, Laguerre, etc., random matrices are also expressible in terms of ${}_pF_q$.

In this talk we present a new, efficient algorithm for evaluating ${}_pF_q$, which makes it possible to evaluate expressions like (9) very quickly and efficiently.

We briefly discuss the combinatorial aspects of the design of our algorithm as well as various examples and applications.

Universality for eigenvalue spacings of random matrices

Arno Kuijlaars

Department of Mathematics, Katholieke Universiteit Leuven, Celestijnenlaan 200 B, 3001 Leuven, Belgium

It is a remarkable fact that the local eigenvalue correlations of random matrices become independent of the particular ensemble, but only depend on its symmetry properties, as the size of the matrix tends to infinity. This universality conjecture was formulated by Dyson in the sixties, and was verified for Gaussian and Wishart ensembles, and other special ensembles. For unitary invariant ensembles, the universality conjecture can be expressed in terms of orthogonal polynomials. Recent advances in the asymptotic theory of orthogonal polynomials have led to a proof of the universality conjecture for unitary ensembles with exponential weights. The new techniques come from integrable systems, in particular the steepest descent analysis of Riemann-Hilbert problems.

In the talk I will present an overview of these results and techniques. I also discuss what can happen at special points in the spectrum where universality fails.

Design of iterative multiuser decoders by means of random matrix theory

Ralf Müller

Department of Electronics and Telecommunications, NTNU, 7491 Trondheim, Norway

In wireless multiuser communication systems, many users share the same physical channel. The input-output relation of the channel is well-approximated by $\mathbf{y}_n = \mathbf{S}\mathbf{x}_n$ where $\mathbf{S}$ is a random matrix whose size grows linearly with the number of users. In order to improve resistance of digital communication against channel impairments, the transmitted signal $\mathbf{x}_n$ is forward error-correction encoded. The optimum estimation procedure to re-construct the transmitted data at the receiver is known to be np-complete and out of practical reach. Instead, receiver processing is performed by non-linear iterative algorithms. The convergence of these algorithms can be ensured utilizing random matrix theory to predict which design of the free parameters of the system, that is the transmit powers assigned to the users for instance, will make the iterative algorithm converge to a near-optimum solution.

The fact that the eigenvalue distribution of a large random matrix converges to a deterministic limit as its size grows large translates in this context to the fact that the (potentially chaotic) evolution of a nonlinear multi-dimensional system collapses to the evolution to a one-dimensional non-linear system as the the number of users approaches infinity.

Deterministic equivalents for certain functionals of large random matrices

W. Hachem[a], P. Loubaton[b], and J. Najim[c]

[a]Supélec, Paris. walid.hachem@supelec.fr

[b]Université Marne-La-Vallée, Paris. loubaton@univ-mlv.fr

[c]Télécom-Paris and CNRS. najim@tsi.enst.fr

Consider a $N \times n$ random matrix $Y_n = (Y_{ij}^n)$ where the entries are given by $Y_{ij}^n = \frac{\sigma_{ij}(n)}{\sqrt{n}} X_{ij}^n$, the X_{ij}^n being independent and identically distributed, centered with unit variance and satisfying some mild moment assumption. Consider now a deterministic $N \times n$ matrix A_n whose columns $(\mathbf{a}_k^n)_{1 \leq k \leq n}$ and rows $(\tilde{\mathbf{a}}_\ell^n)_{1 \leq \ell \leq N}$ satisfy

$$\sup_{n \geq 1} \max_{k,l} (\|\mathbf{a}_k^n\|, \|\tilde{\mathbf{a}}_\ell^n\|) < +\infty,$$

where $\|\cdot\|$ denotes the Euclidean norm. Let $\Sigma_n = Y_n + A_n$ then, it is shown that there exists a deterministic $N \times N$ matrix valued function $T_n(z)$, analytic in $\mathbb{C} - \mathbb{R}^+$, such that, almost surely,

$$\lim_{n \to +\infty, \frac{N}{n} \to c} \frac{1}{N} \text{Trace} \left((\Sigma_n \Sigma_n^T - z I_N)^{-1} - T_n(z) \right) = 0$$

Otherwise stated, there exists a deterministic equivalent to the empirical Stieltjes transform of the distribution of the eigenvalues of $\Sigma_n \Sigma_n^T$. For each n, the entries of matrix $T_n(z)$ are defined as the unique solutions of a certain system of non linear functional equations. This work is motivated by the context of performance evaluation of Multiple Inputs / Multiple Output (MIMO) wireless digital communication channels.

Financial applications of random matrix theory: Risk control and portfolio optimization

Marc Potters

Capital Fund Management, 6 boul Haussmann, 75009 Paris, France

In this seminar, I will discuss the use of the empirical correlation matrix (ECM) in the context of times series of returns of correlated assets (typically stocks). I will argue that the ECM can be used as is to measure the risk of portfolios constructed independently of it but should never be used for finding the optimal portfolio (minimum risk or maximum return) for a large set of assets. Using RMT, one can find the eigenvalue spectrum of the ECM in the null hypothesis of absence of true correlations. By expressing the Markowitz optimization problem in terms of eigenvalues, I will explore two noise reduction techniques: shrinkage estimator and eigenvalue clipping. I will also discuss two more specialized topics; the spectrum of ECM measured using exponentially weighted moving averages and the time structure of inter-stock correlations. The seminar will be geared to an audience with no specific knowledge of finance.

The polynomial method: From theory to the random matrix calculator

N. Raj Rao

Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139.

In infinite random matrix theory (RMT), the limiting level density of a large class of random matrices can, in principle, be obtained by applying either resolvent or free probability based theorems. These theorems are often formulated *explicitly* in terms of either the limiting level density (Marčenko and Pastur 1967, Dozier and Silverstein 2004) or the R and S transforms (Voiculescu 1986, 1987, Nica and Speicher 1998) of free probability. In practical situations, however, this explicit reliance on the level density or the R and S transforms hinders the ability to tangibly compute this limiting spectral measure in all but a few special cases. We propose a method that overcomes these hurdles.

We introduce bivariate polynomials of the form $L_{\rm uv} = \sum_j \sum_k c_{jk} u^j v^k$, where u nd v are an appropriately chosen pair of variables *implicitly* defined such that $L_{\rm uv}(u, v) = 0$. We demonstrate that resolvent and free probability based theorems can be interpreted as simple transformations of these bivariate polynomials. We use this observation to argue that these polynomials are a more natural mathematical object to work with than the explicit transforms or densities.

We then combine known theorems with new random matrix transformations, derived using the bivariate polynomial framework, to extend the class of random matrices for which the limiting level density and the limiting moments can be determined analytically. We also present a surprisingly simple computational realization based on the mathematical principles outlined so that researchers can truly begin to harness the power of infinite RMT and obtain concrete answers to their random matrix questions.

The software and documentation for this 'random matrix calculator' will be made publicly available at the URL: (`http://www.mit.edu/~raj/rmtool`).

This is joint work with Alan Edelman.

Topics on the Eigenvalues of Large Dimensional Sample Covariance Matrices

Jack Silverstein

Department of Mathematics, North Carolina State University, USA.

The talk will outline recent work on spectral properties of random matrices of sample covariance type. One topic to be covered concerns the extreme eigenvalues of a subclass of matrices which have been extensively studied. The general class is defined as $B_p = (1/n)T_p^{1/2}X_pX_p^*T_p^{1/2}$ where $X_p = (X_{ij})$ is $p \times n$ with i.i.d. complex standardized entries, and $T_p^{1/2}$ is a Hermitian square root of the nonnegative definite Hermitian matrix T_p. This matrix can be viewed as the sample covariance matrix of n i.i.d. samples of the p dimensional random vector $T_p^{1/2}(X_p)_{\cdot 1}$. It is known that if $p/n \to c > 0$ and the empirical distribution function (e.d.f.) of the eigenvalues of T_p converge as $p \to \infty$, then the e.d.f. of the eigenvalues of B_p converges a.s. to a nonrandom limit. This result is relevant in situations in multivariate analysis where the vector dimension is large, but the number of samples to adequately approximate the population matrix (required in standard statistical procedures) cannot be attained.

The subclass consists of matrices B_p where all but a finite number of eigenvalues of T_p are 1, which has been called the "spike population model". Results are obtained for the limiting behavior of those eigenvalues of B_p which correspond to the population ones which deviate from 1 (joint work with Jinho Baik at University of Michigan).

Another topic is on the class of matrices of the form $C_p = (1/n)(R_p+\sigma X_p)(R_p+\sigma X_p)^*$ where X_p is as in B_p, $\sigma > 0$, and R_p is $p \times n$ random, independent of X_p with the spectral e.d.f. of $(1/n)R_pR_p$ converging to a nonrandom limit. These matrices model situations where information is contained in the sampling of the vectors $R_{\cdot 1} \cdots R_{\cdot n}$, but the received vector is contaminated by additive noise (the columns of σX_p). The e.d.f. of the eigenvalues of C_p also converges a.s. as $p \to \infty$ (with $p/n \to c > 0$). Properties of the limiting distribution will be outlined. An example of where C_p can be used will be discussed. It is the detection problem in array signal processing: determining the number of sources (presumed large) impinging on a bank of sensors in a noise filled environment (joint work with Brent Dozier at NCSU).

Statistical-mechanical analysis on the eigenvalue distribution of random matrices

Toshiyuki Tanaka

Faculty of System Design, Tokyo Metropolitan University,
1-1 Minami-Osawa, Hachioji-shi, Tokyo, 192-0397 Japan

We review an approach to the study of limiting eigenvalue distributions of random matrices. The approach is based on statistical mechanics, and uses the replica method, which has originally been developed in the field of spin glasses (magnetic materials with randomness in spin-spin interactions), in order to study their macroscopic properties in the "thermodynamic" (=large-system) limit.

We consider ensembles of real symmetric random matrices. The key observation to the application of the replica method to the evaluation of eigenvalue distributions in the large-system limit is that the Stieltjes transform of the eigenvalue distribution of a matrix A, which is equal to $\mathrm{tr}(A - zI)^{-1}$, is obtained by differentiating $\log Z(z)$ with z, where $Z(z) = |A - zI|^{-1/2}$. In order to evaluate the average eigenvalue distribution, one therefore has to calculate $\langle \log Z(z) \rangle$, where $\langle \cdot \rangle$ denotes average over randomness of A. The replica method provides a way of evaluating $\langle \log Z(z) \rangle$ analytically, by noting the identity relation $\langle \log Z \rangle = \lim_{n \to 0} \partial \langle Z^n \rangle / \partial n$. One then applies the *replica trick*, in which one evaluates $\langle Z^n \rangle$ only for $n = 1, 2, 3, \ldots$, and the result is continued to real n to evaluate the partial derivative as well as the limit $n \to 0$.

For explanation purpose, we focus on the ensembles of random matrices of the form $A = \Xi^T \Xi$, where Ξ is a $p \times N$ random matrix with independent, identically distributed, and zero-mean entries, and consider the limit $p, N \to \infty$ while their ratio p/N is kept finite. For the limit to be meaningful, the variance of the entries of Ξ should be $O(N^{-1})$. We first describe the replica analysis in the case where the entries of Ξ follows a fixed distribution as $N \to \infty$, in which case we observe that the replica analysis successfully reproduces the Marcenko-Pastur law. We next discuss the case where the entries of Ξ is nonzero with probability proportional to N^{-1}, thereby yielding sparse matrices as $N \to \infty$. The replica analysis on the sparse case turns out to be difficult, so that we follow an approximation scheme, which is called the effective medium approximation, and was introduced by Biroli and Monasson in their analysis on a different random matrix ensemble, to obtain an approximate result on the limiting eigenvalue distribution.

Feature Selection via Random Matrix Theory

Lior Wolf

The Center for Biological and Computational Learning,
McGovern Institute for Brain Research,
Department of Brain & Cognitive Sciences,
Massachusetts Institute of Technology.
E-mail : liorwolf@mit.edu

The problem of selecting a subset of relevant features in a potentially overwhelming quantity of data is classic and found in many branches of science. Examples in computer vision, text processing and more recently bio-informatics are abundant. In text classification tasks, for example, it is not uncommon to have 10^4 to 10^7 features of the size of the vocabulary containing word frequency counts, with the expectation that only a small fraction of them are relevant. Typical examples include the automatic sorting of URLs into a web directory and the detection of spam email.

We model irrelevant features as random variables and are able to derive the following results:

1. Feature selection in this setting cannot be invariant to orthogonal transformations.

2. We link the spectral properties of the features' measurement matrix to the relevancy of the variables. The feature selection process is then based on a continuous ranking of the features defined by a least-squares optimization process.

3. The optimization process results in a positive set of weights (relevancy scores) for each variable. This result is a randomized extension of the well known Perron-Frobenious theorem.

4. The procedure results in a sparse set of relevancy scores, i.e., most of the scores are near zero as is expected from a feature selection procedure.

The presentation is based on joint work with Amnon Shashua. Ofer Zeitouni, Michael Ben-Or and Alex Samorodnitsky assisted with the proof of the 3rd result.

POSTER SESSION

ORGANISERS:

Bernd Bank & Marc Giusti

Cryptography in BSS Computational Model

Ilir Çapuni[2]

Department of Mathematics and Computer Science
University of Montenegro, Cetinjski put bb
Podgorica, Serbia and Montenegro.
ilir@cg.ac.yu

Most of cryptography is based on the classical Turing model of computation. We consider the possibility of cryptographic primitives over BSS model of computation where the traditional finite field F_n is replaced with the field $\mathbf{Q}$. The very basic cryptographic primitives are the one-way functions (i.e. functions that are "easy" to compute but "hard on average" to invert). Unlike in the classical model where it is believed that one-way functions exist, in this model they do exist. It is proved that in classical BSS some basic cryptographic paradigms like encryption do not exist. We show that the situation on the "weak BSS model" differs.

In this work, we initiate theoretical investigation of cryptography on the weak BSS model over an arbitrary field (where $\mathcal{P} \neq \mathcal{NP}$), hoping that by studying cryptology in this model, a new insight for the classical model of cryptography could be obtained.

Poster presentation period: 4–9 July 2005

On the complexity of the D5 principle

X. Dahan, É. Schost[a], M. Moreno Maza, W. Wu, Y. Xie[b]

[a]LIX, École polytechnique, 91128 Palaiseau, France {dahan, schost}@lix.polytechnique.fr

[b]ORCCA, University of Western Ontario, London, Ontario, Canada
{moreno, wwu25, yxie}@orcca.on.ca

The standard approach for computing with an algebraic number using its irreducible minimal polynomial over some base field k. However, many algebraic numbers may appear when solving a polynomial system; applying them this approach requires possibly costly factorization algorithms. Della Dora, Dicrescenzo and Duval introduced "dynamic evaluation" techniques (also termed "D5 principle") [2] as a means to compute with algebraic numbers, avoiding factorization. This approach leads one to compute over *direct products of field extensions of* k, instead of only field extensions.

We address complexity issues for such computations. Let $\mathbf{T} = T_1(X_1), T_2(X_1, X_2), \ldots T_n(X_1, \ldots, X_n)$ be polynomials such that $k \to K = k[X_1, \ldots, X_n]/\mathbf{T}$ is a direct product of fields. We write δ for the dimension of K over k. Using fast polynomial arithmetic, it is a folklore result that for any $\varepsilon > 0$, the operations $(+, \times)$ in K can be performed in $c_\varepsilon^n \delta^{1+\varepsilon}$ operations in k, for some constant c_ε. Using fast Euclidean

[2]Joint work with Prof. Žarko Mijajlović, Faculty of Mathematics, University of Belgrade, Studentski trg 16, 11000 Belgrade, Serbia and Montenegro; e-mail: zarkom@eunet.yu; and Prof. Slobodan Vujošević, Department of Mathematics, University of Montenegro, Cetinjski put bb, 81000 Podgorica, Serbia and Montenegro; e-mail: vslobo@cg.yu

algorithm, a similar result carries over to inversion, *in the special case when K is a field.*

Our main results are similar estimates for the general case. Following the D5 philosophy, meeting zero-divisors in the computation will lead to *splitting* $\mathbf{T}$ into a family thereof, defining the same extension. Inversion is then replaced by *quasi-inversion*: a quasi-inverse [4] of $\alpha \in K$ is a splitting of $\mathbf{T}$, such that α is either zero or invertible in each component, together with the corresponding inverses. We obtain similar result for gcd computation with coefficients in K. Again, the notion of a gcd has to be adapted: a gcd of two polynomials F and G in $K[y]$ consists of a splitting of $\mathbf{T}$, together with *monic* polynomials that form gcd's of F and G over each factor.

Theorem. *Let $\varepsilon > 0$. There exists $C_\varepsilon > 0$ such that addition, multiplication and quasi-inversion in K can be done in $C_\varepsilon^n\, \delta^{1+\varepsilon}$ operations in k. There exists $C' > 0$ such that one can compute a gcd of degree d polynomials in $K[y]$ using $C'\, C_\varepsilon^n\, d^{1+\varepsilon}\, \delta^{1+\varepsilon}$ operations in k.*

In both cases, the main difficulty comes from handling splittings: if $\mathbf{T}$ has been split into a family $\mathbf{T}_1, \ldots, \mathbf{T}_s$, this corresponds to making effective the map $k[X_1, \ldots, X_n]/\mathbf{T} \to \prod_{i=1}^{s} k[X_1, \ldots, X_n]/\mathbf{T}_i$. This operation has a quasi-linear complexity when $n = 1$; $n > 1$, a similar result lacks. However, it is possible to extend the result from the univariate case when $\mathbf{T}_1, \ldots, \mathbf{T}_s$ satisfy a regularity condition, the absence of *critical pairs*. To reduce to this case, we have to remove critical pairs. This is done by introducing a new algorithm for *coprime factorization* of univariate polynomials [1] (this tool that was already used in [3] for parallel complexity estimates in a similar context).

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Strang Splitting for the Time-Dependent Schrödinger Equation on Sparse Grids

Vasile Gradinaru

Mathematisches Institut, Universität Tübingen, Auf der Morgenstelle 10, D-72076 Tübingen

The time-dependent Schrödinger equation in moderately many dimensions is discretized in space by a sparse grid pseudo-spectral method. The Strang splitting for the resulting evolutionary problem features first or second order convergence, depending on the smoothness of the potential and of the initial data. In contrast to the full grid case, where the frequency domain is the working place, the proof of the sufficient conditions for the convergence goes in the space realm and uses approximations results in Korobov spaces.

POSTER PRESENTATION PERIOD: 30 JUNE – 8 JULY 2005.

Numerical solution for anti-planar shear model for granular flow using Runge-Kutta Discontinuous-Galerkin method with adaptive mesh refinement

Aleksander Grm[3]

Fraunhofer Institute for Industrial Mathematics (ITWM)
D-67663 Kaiserslautern, Germany

Email: grm@itwm.fhg.de

The flow and handling of granular materials is of major importance to many industries. We consider model of granular flow as elasto-plastic continua. In this picture, the granular material flows as a plastic with a frictional yield condition, and deforms as an elastic solid otherwise. It is clear, that there are serious mathematical difficulties with the equations for time-dependant elasto-plastic granular flow. We describe the approach to numerical solution of simplified time-dependant granular flow in two dimensions where shear bands are presented. We consider elasto-plastic deformation in anti-planar shear motion. That is, while the deformation is homogeneous in the z-direction (the state variables only depend on x and y direction) the motion is only in the z direction. This model is the simplest model which still describes the 3D behavior of granular flow with shear bands. Deformation is described by a hyperbolic system of equations. The points, where hyperbolicity is lost, are points where shear banding occurs. Tracking this points we need high resolution of the motion. Runge-Kutta Discontinuous-Galerkin (RKDG) method seems to be very suitable for this kind of problem. It is high order and mesh adaptivity can be easily adopted. This properties of the RKDG method gives us high resolution of the motion in whole domain, which is crucial for the simulation of the problem we have.

We will show numerical solutions in the square domain for different boundary conditions, different polynomial orders and different error estimators.

POSTER PRESENTATION PERIOD: THURSDAY, 30 JUNE – SATURDAY, 2 JULY 2005.

[3]Supervisor: Prof. Dr. Axel Klar, Department of Mathematics, TU Kaiserslautern, D-67653 Kaiserslautern, Germany, klar@itwm.fhg.de

From Graphs to Manifolds - Strong Pointwise Consistency of Graph Laplacians

Matthias Hein[a], Jean-Yves Audibert[b], and Ulrike von Luxburg[c]

[a] Max Planck Institute for Biological Cybernetics, Tübingen, Germany

[b]CERTIS, ENPC, Paris, France

[c]Fraunhofer IPSI, Darmstadt, Germany

In recent years methods for clustering and semi-supervised learning based on graph Laplacians have become increasingly popular in statistical learning theory. These methods are often motivated by properties derived from the Laplace-Beltrami operator. However for the setting where the graph is generated by random samples drawn from a probability measure P on a submanifold M in $\mathbb{R}^d$ no rigorous connection has been established yet. We show under mild assumptions that for every function on M the so called 'normalized' graph Laplacian converges pointwise almost surely in the interior of M towards the weighted Laplace-Beltrami operator of M. We only use the knowledge of the Euclidean distance in $\mathbb{R}^d$ for the edge weights of the graph; no knowledge of the intrinsic distance of the submanifold M is required. Based on our convergence results we then argue against using the so called 'unnormalized' graph Laplacian since it converges only up to a function of the density of P towards the weighted Laplace-Beltrami operator of M.

Poster presentation period: Periods 2 and 3 FOCM'05.

Complexity of Integer Quasiconvex Polynomial Optimization

Sebastian Heinz

Humboldt-Universität zu Berlin
Mathematisch-Naturwissenschaftliche Fakultät II
Institut für Mathematik

We study a particular case of integer polynomial optimization:
Minimize a polynomial $\hat{F}$ on the set of integer points described by an inequality system $F_1 \leq 0, \ldots, F_s \leq 0$, where $\hat{F}, F_1, \ldots, F_s$ are quasiconvex polynomials in n variables with integer coefficients.
We design an algorithm solving this problem that belongs to the time-complexity class $O(s) \cdot l^{O(1)} \cdot d^{O(n)} \cdot 2^{O(n^3)}$, where $d \geq 2$ is an upper bound for the total degree of the polynomials involved and l denotes the maximum binary length of all coefficients. The algorithm is polynomial for a fixed number n of variables and represents a direct generalization of Lenstra's algorithm in integer linear optimization. In the considered case, our complexity-result improves the algorithm given by Khachiyan and Porkolab for integer optimization on convex semialgebraic sets.

Invariant Computation for Linear Hamiltonian Vector Fields

Guillem Huguet

Departamento de Matemática e Informática
Universidad Pública de Navarra
E 31006 Pamplona, Spain
guillem.huguet@unavarra.es [4]

We classify all possible Williamson normal forms related to a quadratic polynomial Hamiltonian of n degrees of freedom, with n arbitrary. Then, given a semisimple part of the quadratic Hamiltonian, we compute a fundamental set of invariants as well as a basis of its linearly independent invariants for a given degree.

We consider Hamiltonian functions of the form:

$$\mathcal{H}(\mathbf{x}) = \mathcal{H}_0(\mathbf{x}) + \mathcal{H}_1(\mathbf{x}) + \ldots, \tag{10}$$

where $\mathbf{x}$ is a $2\,n$-dimensional vector in the coordinates $x_1, \ldots, x_n$ and respective momenta $X_1, \ldots, X_n$. Each $\mathcal{H}_i$ is a homogeneous polynomial in $\mathbf{x}$ of degree $i+2$. We present a combinatorial method to generate all possible normal forms corresponding to any $\mathcal{H}_0$ with n arbitrary. This classification is based on the type and number of indecomposable eigenspaces of the matrix A associated with the linear differential system derived from $\mathcal{H}_0$, and the number of normal forms is closely related to the number of partitions of the dimension of the system.

Once determined all possible normal forms of the Hamiltonian, we compute all polynomials invariant under the action of the uniparametric group associated with $\mathcal{H}_0$. The number of linearly independent polynomial invariants of a certain degree is given by the coefficients of the Hilbert-Poincaré series associated with the action of the group aforementioned. Then, these invariants are found after solving a system of Diophantine equations. In this case we have restricted ourselves to semisimple normal forms.

Poster presentation period: Period 2 of FoCM'05

[4] Advisor: Jesús F. Palacián, Departamento de Matemática e Informática, Universidad Pública de Navarra, Spain.

Relative Periodic Solutions of the Complex Ginzburg-Landau Equation

Vanessa López[5]

Department of Computer Science, University of Illinois, Urbana, IL, USA.
Current affiliation: Lawrence Berkeley National Laboratory, Berkeley, CA, USA.
vlopez@hpcrd.lbl.gov

We consider the problem of numerically computing *relative periodic solutions* of chaotic partial differential equations (PDEs) with symmetries, that is, solutions that are periodic in time, up to a transformation by an element of the equations' symmetry group. In particular, we work with the complex Ginzburg-Landau equation (CGLE) with cubic nonlinearity in one spatial dimension, with periodic boundary conditions. A spectral-Galerkin method, where solutions are represented by truncated Fourier series modified to include the group element as an unknown, was used to discretize the CGLE in both space and time. Thus, the problem of computing solutions to the PDE was reduced to one of computing solutions of an underdetermined system of nonlinear algebraic equations. A large number of distinct relative periodic solutions were found in a chaotic region of the CGLE. These are new solutions and all have broad temporal and spatial spectra. The value of the time period for the relative periodic solutions found ranges between 0.02 and 0.46 and the value of the largest Lyapunov exponent of each solution varies from 1.88 to 17.20. The solutions exhibit a great deal of variety in their spatio-temporal profiles[6].

[5] Joint work with Philip Boyland (Mathematics Department, University of Florida, Gainesville, FL, USA), Michael T. Heath (Department of Computer Science, University of Illinois, Urbana, IL, USA), and Robert D. Moser (Department of Theoretical and Applied Mechanics, University of Illinois, Urbana, IL, USA. Current affiliation: Mechanical Engineering Department, The University of Texas at Austin, Austin, TX, USA).

[6] V. López, P. Boyland, M. Heath, and R. Moser. Relative periodic solutions of the complex Ginzburg-Landau equation. To appear in *SIAM Journal on Applied Dynamical Systems*. (Preprint: http://arxiv.org/abs/nlin.CD/0408018).

The Combinatorial Geometry of General Flag Manifolds

Sofía Pinzón[7]
Escuela de Matemáticas
Universidad Industrial de Santander
Apartado Aéreo 678
Bucaramanga Colombia
sofia@matematicas.uis.edu.co

We give and interesting review about some geometrical properties on general flag manifold $\mathbb{F}$ in connection with some differential structures like almost complex structures and f-structures. A tensor field $\mathcal{F}$ of type (1,1) on a Riemannian manifold is called an *f-structure* if $\mathcal{F}^3 = -\mathcal{F}$, and *almost complex* if $\mathcal{F}^2 = -I$. We call $\mathcal{F}$ (1,1)-symplectic if the $(+,-)$ part of the derivative $d^\nabla \mathcal{F}$ vanishes. Although this property is different than the $(1,2)$-symplectic property, the two are identical if $\mathcal{F}$ is almost complex. An f-structure $\mathcal{F}$ on a Riemannian manifold will be called (1,1)-admissible if it is (1,1)-symplectic with respect to some metric on the manifold. Admissibility on a general flag manifold $\mathbb{F}$ is known to play an important role in the analysis of complex harmonic functions with values in $\mathbb{F}$, but its characterization is known when $\mathcal{F}$ is almost complex or when $\mathbb{F}$ is the classical full flag manifold $\mathbb{F}(n)$. We provide a simple characterization for admissibility, valid for any invariant f-structure on any general flag manifolds. Namely, we reduce admissibility to the local transitivity of an associated intersection graph.

[7]In collaboration with:
Nir Cohen,
Department of Mathematics
IMECC – UNICAMP
Cx. Postal 6065, 13083-970
Campinas – SP Brazil
E-mail address: nir@ime.unicamp.br

Counting Irreducible Components of Algebraic Varieties

Peter Scheiblechner[8]

Dept. of Mathematics
University of Paderborn
D-33095 Paderborn
Germany
pscheib@upb.de

The problem of *counting the connected components* of complex algebraic varieties defined by integer equations is complete in FPSPACE, the class of functional problems that can be solved by a Turing machine in polynomial space.

It is natural to ask the corresponding question about the problem #IC of *counting the irreducible components* of complex algebraic varieties. Our main result states that #IC lies in FPSPACE. Moreover, we also show that #IC can be solved in parallel polynomial time assuming unit cost for the arithmetic operations.

Results of Giusti and Heintz show that one can compute the Chow forms of the equidimensional components of a variety in polynomial space, which gives a polynomial space reduction to the problem #IF of *counting the absolutely irreducible factors* of a multivariate polynomial.

We generalize a new method of Gao for absolute factorization of bivariate polynomials to several variables and use it to show that #IF can be solved in parallel polylogarithmic time, when the polynomial is given in dense representation. Moreover, when the polynomial is given as a straight-line program, we show that one can still solve #IF in polynomial time in the model of unit cost for the arithmetic operations.

POSTER PRESENTATION PERIOD: ALL THREE PERIODS OF THE CONFERENCE.

[8] Guiding senior researcher Peter Bürgisser