Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 1[1]

LU factorization

Let

$$A = \begin{bmatrix} -3 & 2 & 3 & -1 \\ 6 & -2 & -6 & 0 \\ -9 & 4 & 10 & 3 \\ 12 & -4 & -13 & -5 \end{bmatrix}.$$

Hence

$$\boldsymbol{\ell}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}, \qquad \boldsymbol{u}_1^{\mathrm{T}} = [-3, 2, 3, -1], \qquad A - \boldsymbol{\ell}_1 \boldsymbol{u}_1^{\mathrm{T}} = \left[\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ \hline 0 & 2 & 0 & -2 \\ 0 & -2 & 1 & 6 \\ 0 & 4 & -1 & -9 \end{array}\right],$$

$$\boldsymbol{\ell}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \qquad \boldsymbol{u}_2^{\mathrm{T}} = [0, 2, 0, -2], \qquad A - \boldsymbol{\ell}_1 \boldsymbol{u}_1^{\mathrm{T}} - \boldsymbol{\ell}_2 \boldsymbol{u}_2^{\mathrm{T}} = \left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -5 \end{array}\right],$$

$$\boldsymbol{\ell}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \qquad \boldsymbol{u}_3^{\mathrm{T}} = [0, 0, 1, 4],, \qquad A - \boldsymbol{\ell}_1 \boldsymbol{u}_1^{\mathrm{T}} - \boldsymbol{\ell}_2 \boldsymbol{u}_2^{\mathrm{T}} - \boldsymbol{\ell}_3 \boldsymbol{u}_3^{\mathrm{T}} = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -1 \end{array}\right],$$

$$\boldsymbol{\ell}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \boldsymbol{u}_4^{\mathrm{T}} = [0, 0, 0, -1],$$

and we deduce that

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 \\ -4 & 2 & -1 & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} -3 & 2 & 3 & -1 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

[1] Corrections and suggestions to these notes should be emailed to a.shadrin@damtp.cam.ac.uk. All handouts are available on the WWW at the URL http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html.