Mathematical Tripos Part IB: Easter 20001

Numerical Analysis – Detailed Example 3[1]

Cholesky factorization

Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 2 & 5 & 2 & 0 & 0 \\ 0 & 3 & 10 & 4 & 0 \\ 0 & 0 & 4 & 17 & 5 \\ 0 & 0 & 0 & 5 & 26 \end{bmatrix},$$

hence A is both symmetric (actually, as we'll see, positive definite) and banded. Using the matrix itself to store columns of L as we form them, we have

$$\boldsymbol{\ell}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A = \left[\begin{array}{c|cccc} 1 & 2 & 0 & 0 & 0 \\ \hline 2 & 1 & 3 & 0 & 0 \\ 0 & 3 & 10 & 4 & 0 \\ 0 & 0 & 4 & 17 & 5 \\ 0 & 0 & 0 & 5 & 26 \end{array}\right],$$

$$\boldsymbol{\ell}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \qquad A = \left[\begin{array}{cc|ccc} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 \\ \hline 0 & 3 & 1 & 4 & 0 \\ 0 & 0 & 4 & 17 & 5 \\ 0 & 0 & 0 & 5 & 26 \end{array}\right],$$

$$\boldsymbol{\ell}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \qquad A = \left[\begin{array}{ccc|cc} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 \\ 0 & 3 & 1 & 4 & 0 \\ \hline 0 & 0 & 4 & 1 & 5 \\ 0 & 0 & 0 & 5 & 26 \end{array}\right],$$

$$\boldsymbol{\ell}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 5 \end{bmatrix}, \qquad A = \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 \\ 0 & 3 & 1 & 4 & 0 \\ 0 & 0 & 4 & 1 & 5 \\ 0 & 0 & 0 & 5 & 1 \end{array}\right],$$

$$\boldsymbol{\ell}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \Rightarrow \qquad A = LL^{\mathrm{T}}, \qquad L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 \\ 0 & 0 & 0 & 5 & 1 \end{bmatrix}.$$

[1] Corrections and suggestions to these notes should be emailed to `a.shadrin@damtp.cam.ac.uk`. All handouts are available on the WWW at the URL `http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html`.