Mathematical Tripos Part IB: Easter 20001

Numerical Analysis – Detailed Example 5[1]

QR factorization: Givens rotations

Let

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 0 & 1 \\ 4 & 2 & -3 \\ -2 & 1 & 1 \end{bmatrix}.$$

Then $R \to A$,

$$\Omega^{[1,2]} = \begin{bmatrix} \mathbf{0.4472} & \mathbf{-0.8944} & 0.0000 & 0.0000 \\ \mathbf{0.8944} & \mathbf{0.4472} & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad R \leftarrow \Omega^{[1,2]} R = \begin{bmatrix} 2.2361 & 0.4472 & 0.4472 \\ 0.0000 & 0.8944 & 3.1305 \\ 4.0000 & 2.0000 & -3.0000 \\ -2.0000 & 1.0000 & 1.0000 \end{bmatrix},$$

$$\Omega^{[1,3]} = \begin{bmatrix} \mathbf{0.4880} & 0.0000 & \mathbf{0.8729} & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ \mathbf{-0.8729} & 0.0000 & \mathbf{0.4880} & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad R \leftarrow \Omega^{[1,3]} R = \begin{bmatrix} 4.5826 & 1.9640 & -2.4004 \\ 0.0000 & 0.8944 & 3.1305 \\ 0.0000 & 0.5855 & -1.8542 \\ -2.0000 & 1.0000 & 1.0000 \end{bmatrix},$$

$$\Omega^{[1,4]} = \begin{bmatrix} \mathbf{0.9165} & 0.0000 & 0.0000 & \mathbf{-0.4000} \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ \mathbf{0.4000} & 0.0000 & 0.0000 & \mathbf{0.9165} \end{bmatrix}, \quad R \leftarrow \Omega^{[1,4]} R = \begin{bmatrix} 5.0000 & 1.4000 & -2.6000 \\ 0.0000 & 0.8944 & 3.1305 \\ 0.0000 & 0.5855 & -1.8542 \\ 0.0000 & 1.7021 & -0.0436 \end{bmatrix},$$

$$\Omega^{[2,3]} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & \mathbf{0.8367} & \mathbf{0.5477} & 0.0000 \\ 0.0000 & \mathbf{-0.5477} & \mathbf{0.8367} & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{bmatrix}, \quad R \leftarrow \Omega^{[2,3]} R = \begin{bmatrix} 5.0000 & 1.4000 & -2.6000 \\ 0.0000 & 1.0690 & 1.6036 \\ 0.0000 & 0.0000 & -3.2660 \\ 0.0000 & 1.7021 & -0.0436 \end{bmatrix},$$

$$\Omega^{[2,4]} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & \mathbf{0.5319} & 0.0000 & \mathbf{0.8468} \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & \mathbf{-0.8468} & 0.0000 & \mathbf{0.5319} \end{bmatrix}, \quad R \leftarrow \Omega^{[2,4]} R = \begin{bmatrix} 5.0000 & 1.4000 & -2.6000 \\ 0.0000 & 2.0100 & 0.8159 \\ 0.0000 & 0.0000 & -3.2660 \\ 0.0000 & 0.0000 & -1.3812 \end{bmatrix},$$

$$\Omega^{[3,4]} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & \mathbf{-0.9210} & \mathbf{-0.3895} \\ 0.0000 & 0.0000 & \mathbf{0.3895} & \mathbf{-0.9210} \end{bmatrix}, \quad R \leftarrow \Omega^{[3,4]} R = \begin{bmatrix} 5.0000 & 1.4000 & -2.6000 \\ 0.0000 & 2.0100 & 0.8159 \\ 0.0000 & 0.0000 & 3.5460 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Finally,

$$Q = (\Omega^{[3,4]}\Omega^{[2,4]}\Omega^{[2,3]}\Omega^{[1,4]}\Omega^{[1,3]}\Omega^{[1,2]})^{\mathrm{T}} = \begin{bmatrix} 0.2000 & 0.3582 & 0.9102 & 0.0561 \\ -0.4000 & 0.2786 & -0.0754 & 0.8699 \\ 0.8000 & 0.4378 & -0.3602 & 0.1964 \\ -0.4000 & 0.7761 & -0.1899 & -0.4490 \end{bmatrix}.$$

[1]Corrections and suggestions to these notes should be emailed to `a.shadrin@damtp.cam.ac.uk`. All handouts are available on the WWW at the URL `http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html`.