Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 5a[1]

QR factorization: Givens rotations

Let

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$

Then $R \leftarrow A$,

$$R \leftarrow A = \begin{bmatrix} \boxed{2} & 4 & 5 \\ \boxed{1} & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

$$R \leftarrow \Omega^{[1,2]} R = \underbrace{\frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}}_{\Omega^{[1,2]}} \begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{5}} \begin{bmatrix} \boxed{5} & 7 & 11 \\ 0 & -6 & -3 \\ \boxed{2\sqrt{5}} & \sqrt{5} & -\sqrt{5} \end{bmatrix}$$

$$R \leftarrow \Omega^{[1,3]} R = \underbrace{\frac{1}{3\sqrt{5}} \begin{bmatrix} 5 & 0 & 2\sqrt{5} \\ 0 & 3\sqrt{5} & 0 \\ -2\sqrt{5} & 0 & 5 \end{bmatrix}}_{\Omega^{[1,3]}} \frac{1}{\sqrt{5}} \begin{bmatrix} 5 & 7 & 11 \\ 0 & -6 & -3 \\ 2\sqrt{5} & \sqrt{5} & -\sqrt{5} \end{bmatrix} = \frac{3}{\sqrt{5}} \begin{bmatrix} \sqrt{5} & \sqrt{5} & \sqrt{5} \\ 0 & \boxed{-2} & -1 \\ 0 & \boxed{-1} & -3 \end{bmatrix},$$

$$R \leftarrow \Omega^{[2,3]} R = \underbrace{\frac{1}{\sqrt{5}} \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}}_{\Omega^{[2,3]}} \frac{3}{\sqrt{5}} \begin{bmatrix} \sqrt{5} & \sqrt{5} & \sqrt{5} \\ 0 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 0 & -3 & -3 \\ 0 & 0 & -3 \end{bmatrix}.$$

Finally,

$$Q^{\mathrm{T}} = \Omega^{[2,3]} \Omega^{[1,3]} \Omega^{[1,2]} = \frac{1}{\sqrt{5}} \begin{bmatrix} \sqrt{5} & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix} \frac{1}{3} \begin{bmatrix} \sqrt{5} & 0 & 2 \\ 0 & 3 & 0 \\ -2 & 0 & \sqrt{5} \end{bmatrix} \times \Omega^{[1,2]}$$

$$= \frac{1}{3\sqrt{5}} \begin{bmatrix} 5 & 0 & 2\sqrt{5} \\ -2 & 6 & \sqrt{5} \\ -4 & -3 & 2\sqrt{5} \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 2 \\ -2 & 2 & 1 \\ -1 & -2 & 2 \end{bmatrix}.$$

and

$$Q = \frac{1}{3} \begin{bmatrix} 2 & -2 & -1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$

(Compare with Example 4a and notice non-uniqueness of the QR factorization.)

[1] Corrections and suggestions to these notes should be emailed to `a.shadrin@damtp.cam.ac.uk`. All handouts are available on the WWW at the URL `http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html`.