Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 6[1]

QR factorization: Householder reflections

Let

$$A = \begin{bmatrix} 1 & 1 & 3 \\ -2 & 0 & 1 \\ 4 & 2 & -3 \\ -2 & 1 & 1 \end{bmatrix}.$$

Then $R \to A$,

$$\boldsymbol{u} = \begin{bmatrix} 6 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \qquad \Omega_1 = I - 2\frac{\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}}{\|\boldsymbol{u}\|^2} = \begin{bmatrix} -0.2000 & 0.4000 & -0.8000 & 0.4000 \\ 0.4000 & 0.8667 & 0.2667 & -0.1333 \\ -0.8000 & 0.2667 & 0.4667 & 0.2667 \\ 0.4000 & -0.1333 & 0.2667 & 0.8667 \end{bmatrix},$$

$$R \to \Omega_1 R = \begin{bmatrix} -5.0000 & -1.4000 & 2.6000 \\ 0.0000 & 0.8000 & 1.1333 \\ 0.0000 & 0.4000 & -3.2667 \\ 0.0000 & 1.8000 & 1.1333 \end{bmatrix},$$

$$\boldsymbol{u} = \begin{bmatrix} 0.0000 \\ 2.8100 \\ 0.4000 \\ 1.8000 \end{bmatrix}, \qquad \Omega_2 = I - 2\frac{\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}}{\|\boldsymbol{u}\|^2} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.3980 & -0.1990 & -0.8955 \\ 0.0000 & -0.1990 & 0.9717 & -0.1275 \\ 0.0000 & -0.8955 & -0.1275 & 0.4263 \end{bmatrix},$$

$$R \to \Omega_2 R = \begin{bmatrix} -5.0000 & -1.4000 & 2.6000 \\ 0.0000 & -2.0100 & -0.8159 \\ 0.0000 & 0.0000 & -3.5441 \\ 0.0000 & 0.0000 & -0.1153 \end{bmatrix},$$

$$\boldsymbol{u} = \begin{bmatrix} 0 \\ 0 \\ -7.0902 \\ -0.1153 \end{bmatrix}, \qquad \Omega_3 = I - 2\frac{\boldsymbol{u}\boldsymbol{u}^{\mathrm{T}}}{\|\boldsymbol{u}\|^2} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & -0.9995 & -0.0325 \\ 0.0000 & 0.0000 & -0.0325 & 0.9995 \end{bmatrix},$$

$$R \to \Omega_3 R = \begin{bmatrix} -5.0000 & -1.4000 & 2.6000 \\ 0.0000 & -2.0100 & -0.8159 \\ 0.0000 & 0.0000 & 3.5460 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

Finally,

$$Q = (\Omega_3\Omega_2\Omega_1)^{\mathrm{T}} = \begin{bmatrix} -0.2000 & -0.3582 & 0.9102 & -0.0561 \\ 0.4000 & -0.2786 & -0.0754 & -0.8699 \\ -0.8000 & -0.4378 & -0.3602 & -0.1964 \\ 0.4000 & -0.7761 & -0.1899 & 0.4490 \end{bmatrix}.$$

[1] Corrections and suggestions to these notes should be emailed to a.shadrin@damtp.cam.ac.uk. All handouts are available on the WWW at the URL http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html.