Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 6a[1]

QR factorization: Householder reflections

Let

$$A = \begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}.$$

Then

$$\boldsymbol{u}_1 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \qquad \boldsymbol{u}_1\boldsymbol{u}_1^{\mathrm{T}} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ -2 & 2 & 4 \end{bmatrix}, \qquad \Omega_1 = I - 2\frac{\boldsymbol{u}_1\boldsymbol{u}_1^{\mathrm{T}}}{\|\boldsymbol{u}_1\|^2} = \tfrac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \\ 2 & -2 & -1 \end{bmatrix},$$

$$R_1 = \Omega_1 A = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 0 & 3 \\ 0 & 3 & 3 \end{bmatrix},$$

$$\boldsymbol{u}_2 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \qquad \boldsymbol{u}_2\boldsymbol{u}_2^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \qquad \Omega_2 = I - 2\frac{\boldsymbol{u}_2\boldsymbol{u}_2^{\mathrm{T}}}{\|\boldsymbol{u}_2\|^2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$R = \Omega_2 R_1 = \begin{bmatrix} 3 & 3 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

Finally,

$$Q = (\Omega_2\Omega_1)^{\mathrm{T}} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & -2 \\ 2 & -2 & -1 \end{bmatrix} \right)^{\mathrm{T}} = \frac{1}{3}\begin{bmatrix} 2 & 1 & 2 \\ 1 & -2 & -1 \\ 2 & 2 & -2 \end{bmatrix}^{\mathrm{T}} = \frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ 1 & -2 & 2 \\ 2 & -1 & -2 \end{bmatrix}$$

[1] Corrections and suggestions to these notes should be emailed to a.shadrin@damtp.cam.ac.uk. All handouts are available on the WWW at the URL http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html.