Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 7[1]

Least squares

We wish to find $\boldsymbol{x} \in \mathbb{R}^3$ that minimizes $\|A\boldsymbol{x} - \boldsymbol{b}\|$, where

$$A = \begin{bmatrix} 5 & 3 & 9 \\ -7 & -2 & -1 \\ 5 & 6 & -3 \\ 1 & -2 & -6 \\ 0 & 3 & 4 \\ 7 & 5 & 4 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ -7 \\ 1 \\ 1 \\ -5 \end{bmatrix}.$$

We QR-factorize,

$$Q = \begin{bmatrix} -0.4096 & 0.0159 & 0.6347 & 0.3490 & -0.2690 & -0.4847 \\ 0.5735 & -0.4226 & 0.0308 & 0.4934 & -0.4720 & 0.1592 \\ -0.4096 & -0.5301 & -0.6075 & -0.0280 & -0.2158 & -0.3671 \\ -0.0819 & 0.4763 & -0.4294 & 0.7352 & 0.1955 & -0.0566 \\ 0.0000 & -0.5460 & 0.1933 & 0.2609 & 0.7723 & -0.0055 \\ -0.5735 & -0.1234 & 0.0728 & 0.1591 & -0.1536 & 0.7757 \end{bmatrix}$$

$$R = \begin{bmatrix} -12.2066 & -7.5369 & -4.8335 \\ 0.0000 & -5.4950 & -3.3795 \\ 0.0000 & 0.0000 & 11.1452 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix}.$$

We thus need to solve $\tilde{R}\boldsymbol{x} = \tilde{\boldsymbol{b}}$, where

$$\tilde{R} = \begin{bmatrix} -12.2066 & -7.5369 & -4.8335 \\ 0.0000 & -5.4950 & -3.3795 \\ 0.0000 & 0.0000 & 11.1452 \end{bmatrix}$$

is the upper 3×3 block of R and

$$\tilde{\boldsymbol{b}} = \begin{bmatrix} 5.8165 \\ 3.8510 \\ 4.3183 \end{bmatrix}$$

is made out of the three top elements of $Q^{\mathrm{T}}\boldsymbol{b}$. The outcome is

$$\boldsymbol{x} = \begin{bmatrix} -0.0501 \\ -0.9391 \\ 0.3875 \end{bmatrix}.$$

[1] Corrections and suggestions to these notes should be emailed to `a.shadrin@damtp.cam.ac.uk`. All handouts are available on the WWW at the URL `http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html`.