Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 8[1]

Interpolation: the Lagrange formula

Let $x_0 = 3$, $x_1 = 8$, $x_2 = 5$, $x_3 = 1$, $x_4 = 7$; $f_0 = -459$, $f_1 = -18864$, $f_2 = -3105$, $f_3 = -13$, $f_4 = -11263$. We wish to find $p \in \mathbb{P}_4[x]$ s.t. $p(x_k) = f_k$, $k = 0, \ldots, 4$. We evaluate the cardinal polynomials

$$L_0(x) = \prod_{l=1}^{4} \frac{x - x_l}{x_0 - x_l} = -\tfrac{1}{80}x^4 + \tfrac{21}{80}x^3 - \tfrac{151}{80}x^2 + \tfrac{411}{80}x - \tfrac{7}{2},$$

$$L_1(x) = \prod_{\substack{l=0 \\ l\neq 1}}^{4} \frac{x - x_l}{x_1 - x_l} = \tfrac{1}{105}x^4 - \tfrac{16}{105}x^3 + \tfrac{86}{105}x^2 - \tfrac{176}{105}x + 1,$$

$$L_2(x) = \prod_{\substack{l=0 \\ l\neq 2}}^{4} \frac{x - x_l}{x_2 - x_l} = \tfrac{1}{48}x^4 - \tfrac{19}{48}x^3 + \tfrac{119}{48}x^2 - \tfrac{269}{48}x + \tfrac{7}{2},$$

$$L_3(x) = \prod_{\substack{l=0 \\ l\neq 3}}^{4} \frac{x - x_l}{x_3 - x_l} = \tfrac{1}{336}x^4 - \tfrac{23}{336}x^3 + \tfrac{191}{336}x^2 - \tfrac{673}{336}x + \tfrac{5}{2},$$

$$L_4(x) = \prod_{\substack{l=0 \\ l\neq 4}}^{4} \frac{x - x_l}{x_4 - x_l} = -\tfrac{1}{48}x^4 + \tfrac{17}{48}x^3 - \tfrac{95}{48}x^2 + \tfrac{199}{48}x - \tfrac{5}{2}.$$

Therefore

$$p(x) = \sum_{k=0}^{4} f_k L_k(x) = -4x^4 - 5x^3 + 2x^2 - 6x.$$

[1]Corrections and suggestions to these notes should be emailed to a.shadrin@damtp.cam.ac.uk. All handouts are available on the WWW at the URL http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html.