Mathematical Tripos Part IB: Easter 2001

Numerical Analysis – Detailed Example 9[1]

Interpolation: the Newton formula

Let $x_0 = 3$, $x_1 = 8$, $x_2 = 5$, $x_3 = 1$, $x_4 = 7$; $f_0 = -459$, $f_1 = -18864$, $f_2 = -3105$, $f_3 = -13$, $f_4 = -11263$. We wish to find $p \in \mathbb{P}_4[x]$ s.t. $p(x_k) = f_k$, $k = 0, \ldots, 4$ and commence by forming the table of divided differences:

$$
\begin{array}{ccccccccc}
\mathbf{-459} & & & & & & & & \\
 & \searrow & & & & & & & \\
 & & \mathbf{-3681} & & & & & & \\
 & \nearrow & & \searrow & & & & & \\
-18864 & & & & \mathbf{-786} & & & & \\
 & \searrow & & \nearrow & & \searrow & & & \\
 & & -5253 & & & & \mathbf{-73} & & \\
 & \nearrow & & \searrow & & \nearrow & & \searrow & \\
-3105 & & & & -640 & & & & \mathbf{-4} \\
 & \searrow & & \nearrow & & \searrow & & \nearrow & \\
 & & -773 & & & & -89 & & \\
 & \nearrow & & \searrow & & \nearrow & & & \\
-13 & & & & -551 & & & & \\
 & \searrow & & \nearrow & & & & & \\
 & & -1875 & & & & & & \\
 & \nearrow & & & & & & & \\
-11263 & & & & & & & &
\end{array}
$$

Therefore

$$
\begin{aligned}
p(x) &= f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) \\
&\quad + f[x_0, x_1, x_2, x_3](x - x_0)(x - x_1)(x - x_2) \\
&\quad + f[x_0, x_2, x_3, x_4](x - x_0)(x - x_1)(x - x_2)(x - x_3) \\
&= -459 - 3681(x - 3) - 786 * (x^2 - 11x + 24) - 73(x^3 - 16x^2 + 79 * x - 120) \\
&\quad - 4(x^4 - 17x^3 + 95 * x^2 - 199 * x + 120) \\
&= -4x^4 - 5x^3 + 2x^2 - 6x.
\end{aligned}
$$

[1] Corrections and suggestions to these notes should be emailed to `a.shadrin@damtp.cam.ac.uk`. All handouts are available on the WWW at the URL `http://www.damtp.cam.ac.uk/user/na/PartIB/Handouts.html`.