further Examples

This document contains discussion for the examples from Curve Interrogation onward.

Curve Interrogations

Newton iteration

Follow the Newton iteration seeking for a root of $0 = x - x^3$ starting from various initial values of x

a) between 0.1 and 0.5
b) between 1.1 and 1.5
c) at 0.9
d) between 0.5 and 0.71

What can you say about the different patterns of convergence ?

Now look for a root of $0 = 1 - x + x^2$ with initial values

e) 0
f) 0.1
g) 0.01

Parametric Surface Definitions

Implicit Surfaces

These are not parametric but it is worth having one question on them.

What are the surfaces defined by

$$f1(P) = x^2 + y^2 - 1$$
$$f2(P) = z$$
$$f3(P) = 4 - x^2 + y^2 + z^2$$

and the solid defined by

$$f4(P) = min(f1(P), f2(P), f3(P)) >= 0$$

Tensor Product: Inheritance of various properties

Prove that, if $f_i(u)$ is non-negative for all $i \in 1..m$ and all $u \in [0..1]$ and $g_j(v)$ is non-negative for all $j \in 1..n$ and all $v \in [0..1]$, then $f_i(u)g_j(v)$ is non-negative for all $i \in 1..m$, all $j \in 1..n$ and all $(u,v) \in [0..1]^2$.

Prove that, if $\sum_{i \in 1..m} f_i(u) = 1$ for all $u \in [0..1]$ and $\sum_{j \in 1..n} g_j(v) = 1$ for all $v \in [0..1]$, then

$$\sum_{i \in 1..m} \sum_{j \in 1..m} f_i(u)g_j(v) = 1 \forall (u,v) \in [0..1]^2$$

Prove that, if $f_i(u)$ is Cn over $[0..1]$ and $g_j(v)$ is Cn over $[0..1]$, then $f_i(u)g_j(v)$ is Cn over $(u,v) \in [0..1]^2$

Bezier triangle theorems

Prove that the functions

$$\frac{n!}{i!j!k!} u^i v^j w^k \ , \ i+j+k=n \ , \ i,j,k \geq 0$$

are non-negative over the triangle $u+v+w=1, \ u,v,w \geq 0$ and that they sum to unity.

Positions of maxima

Identify the values of u and v (in $[0..1]^2$) where

$$\frac{m!n!}{i!(m-i)!j!(n-j)!}(1-u)^{m-i}u^i(1-v)^{n-j}v^j$$

has its maximum.

Check this for $i=0$ and $i=m$

Identify the values of (u,v,w) in $u+v+w=1, \ u,v,w > 0$ where

$$\frac{n!}{i!j!k!} u^i v^j w^k$$

has its maximum.

For what values of $i,j,k, \ i+j+k=n$ do these maxima have their largest and smallest values for $n=6$?

How large is the smallest maximum ?

Parametric Surface Interrogations

Design a parametric surface patch such that you can find points Q so that
a) the nearest point is at a vertex
b) the nearest point is on an edge
c) the nearest point is in the interior
d) there is a point in the surface for which the Newton matrix is not positive definite

and report the position of an example Q for each case.

Hints: You can keep things simple by letting $x=u$ and $y=v$ and just designing the variation of z. Assume the default boundaries $u=0,1$ $v=0,1$.