Operations on Sets

We have various kinds of entity:- numbers, points, displacements, coordinate systems.

We also have operations which create various combinations from these objects. For example, we can take the sum of two numbers.

We can also create sets, usually by collecting entities of the same kind together, and so we can talk of **intervals** (connected sets of values) of **point-sets** and of **displacement-sets**.

If we have a binary operation acting on two numbers, we can extend the definition to define a corresponding binary operation on two intervals.

The Minkowski sum of two intervals A and B is defined as the set of values

$$A + B \equiv \{c | c = a + b | a \in A, b \in B\} \tag{1}$$

Similarly we can make a Minkowski sum of two displacement sets. The definition look exactly the same as equation (1), we are merely interpreting the symbols a and b as displacements and A and B as displacement sets.

This principle can be extended to any binary operator. In particular, if we know that a given displacement vector a lies inside a certain set A (If A is convex we might call it a Hull) and b lies inside B, then we know that their cross product lies within a set which we call the cross product of A and B. Tautologically this is simply defined as

$$C = A \times B \equiv \{a \times b | a \in A,\, b \in B\}$$

It is straightforward to establish that if A and B are convex, so also is C.

Exercises:

1) prove this.

2) prove that if either A or B contains the origin, then so does $A \times B$.

The convexity result means that we can compute an explicit finite representation of C from the representations of A and B. We only need to take the convex hull of the combinations of extremes in A and B. So if A and B are boxes, we need only take the combinations of the corners of A with the corners of B and then take the convex hull (or the bounding box) of these 8×8 results to build a box known to contain all the possible cross products of elements taken one from each set.