Part III - Lent Term 2005

Approximation Theory

1. Course description

Here is the content of the course as it was given the year before, but I am planning some changes in the second half.

1. **Basic concepts**
   The best approximation. Linear approximation and projection. Degree of approximation.
2. **The Weierstrass theorems**
   Linear positive operators. Korovkin theorem.
3. Bernstein polynomials. Fejer sums. The Weierstrass theorems.
4. **Existence and unicity of best approximation**
   Finite-dimensional subspaces. Strictly convex spaces. Examples of nonexistence.
5. **Best approximation in $C(K)$**
   Kolmogorov criterion. Haar spaces.
6. Chebyshev alternation theorem. Haar unicity theorem. Loss of Haar.
7. **Chebyshev polynomials**
   Chebyshev polynomials. Estimates outside the interval. Application to the iterative methods.
8. **Trigonometric approximation**
   Moduli of continuity. Convolution. Jackson's theorems.
9. Bernstein inequality. Inverse theorems.
10. Lipschitz and Zygmund classes. Approximation by algebraic polynomials.
11. **Lagrange interpolation**
    Lagrange interpolation formula. Polynomials with interlacing zeros. Inequalities for derivatives.
12. Markov inequality. Duffin-Schaeffer refinement.
13. Error bounds for Lagrange interpolation. Peano kernel. Numerical differentiation.
14. **B-splines**
    Splines. Basis of truncated powers. Divided differences.
15. B-splines. Normalization. The recurrence relation.
16. Lee's formula. Marsden identity. B-splines as basis functions.
17. **Dual functionals**
    de Boor-Fix functional. B-spline expansions.
18. Quasi–interpolants. Degree of spline approximation.
19. **Spline interpolation**
    The Schoenberg-Whitney theorem.
20. Knot insertion. B-splines with multiple knots. Sign changes.
21. Spline interpolation projector. Total positivity of the collocation matrix.
    Inverse of a totally positive matrix.
22. Minimization of the norm of inverse. Optimal interpolation points.
    Chebyshev splines. Demko's theorem.
23. **Orthogonal spline projector**
    Least squares approximation. Exponential decay of the Gram inverse.
24. Max-norm of the L_2 spline projector. Douglas-Dupont-Wahlbin theorem.

2. Lecture notes in the class and on the web

⋆ I will try to have every handout available on the web-site of the Numerical Analysis Group at <www.damtp.cam.ac.uk/user/na/na.html> a few days before the corresponding lecture. You will get it in the class anyway.

3. Example classes

⋆ Example classes will be given *each* week, i.e., after each 3 lectures, the time to be agreed. There will be 2-5 exercises enclosed to each handout.

4. Appropriate books

1. M. J. D. Powell, Approximation theory and methods, Cambridge Univeristy Press, 1981.

2. E. W. Cheney, Approximation theory, MgGrow-Hill, New-York, 1966.

3. R. A. DeVore, G. G. Lorents, Constructive Approximation, Springer-Verlag, Berlin, 1993.

4. C. de Boor, Lecture Notes on Approximation Theory, <www.cs.wisc.edu/~deboor>

⋆ To a large extent, the course follows the Lecture Notes [4] where you can find much more details on each subject. In the first half, it is also based on [2] with some extracts from [1] and [3].

5. Communication

- **Surgery hours:** Mondays, 2:00-3:00 (to be agreed) at CMS, Room F2.03,
  or, at other time, by appointment.
- **E-mail address:** <a.shadrin@damtp.cam.ac.uk>

6. Essay

⋆ I offer the essay title: "Orthonormal bases of compactly supported wavelets"
(which is *not* in the existsing list). You can contact me for details.