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Preface

Nonlinear phenomena in degenerate quantum gases

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A B S T R A C T

In this introductory survey, we give an overview of the main physical problems and corresponding themes of research addressed in this Special Issue. We also briefly discuss some avenues of potential interest for future research in degenerate quantum gases.

1. The background

Fourteen years ago – a very long time, in terms of modern science– three experimental groups were able to cool down bosonic gases (^{87}Rb, ^{7}Li, and ^{23}Na, respectively) below the Bose–Einstein transition temperature [1–3]. These experiments provided the first direct demonstration of Bose–Einstein condensation, a phenomenon predicted almost a century ago [4,5]. These achievements are now rightfully among the landmarks of modern physics.

The creation of Bose–Einstein condensates (BECs) in ultracold alkali gases was not only the advent of a long-sought state of matter and the verification of a fundamental theoretical prediction. Actually, this event had ushered a new era in condensed-matter physics, since these media have proven to be a universal laboratory for the study of phenomena previously predicted but never clearly observed in many areas of Physics, ranging from quantum fluids to the theory of quantum phase transitions [6–8].

While the original prediction of the BEC was made for noninteracting and spatially uniform systems, ultracold atomic condensates feature, and indeed require in order to condense, collisional interactions between atoms. Moreover, they are spatially inhomogeneous because of the effect of trapping fields. Typically, the atoms interact weakly enough for the condensate to remain as a dilute gas, but strongly enough to make nonlinear effects not only noticeable but even critical in the creation of the condensates and their dynamics.

On the theoretical level, a fundamental mean-field theory for boson condensates was developed independently forty years ago by Pitaevskii [9] and Gross [10] (originally, this was done with the intention to apply the resultant equations, currently known as *Gross-Pitaevskii equations* (GPEs), to quasi-particles in liquid helium). One of the basic conclusions made in the early days of development of this field was the fact that the experimental observations concerning the shape, dynamical expansion, and collective excitations of trapped condensates [11–13], can be indeed explained by means of the GPEs, in virtually all physical situations, with a surprisingly accurate precision and without the use of any adjustable parameters [14–21]. These results confirmed the significance of nonlinearities in the dynamics

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of BECs and stimulated an enormous number of studies of a plethora of nonlinear phenomena that arise in such degenerate quantum gases. Because of the broad scope of topics arising around BECs, the field has drawn a great deal of interest from people working in quantum fluids, quantum field theory, nonlinear and quantum optics, condensed-matter and statistical physics, classical hydrodynamics, applied mathematics, classical and quantum chaos, complexity theory, and others. This has led to a cross-fertilization through the exchange of ideas and techniques between different areas of Physics and Nonlinear Science relevant to the studies of quantum gases.

Many remarkable experiments have been conducted to observe manifestations of nonlinear phenomena in ultracold degenerate quantum gases. Among them, was the experimental generation of dark [22–25], bright [26–29] and gap solitons [30], as well as vectorial (two-component) solitons [31] (thus far, all the solitons have been created in effectively one-dimensional (1D) settings), vortices and related structures [32–36], vortex rings [37–39], dark ring-shaped waves [40], shocks [41,42], Faraday waves [43], blow-up phenomena (alias the wave collapse) [44–47], four-wave mixing [48], chaotic dynamics [49], suppression of dephasing [50,51], nonlinear tunneling phenomena [52–54], nonlinear self-trapping and related spontaneous symmetry breaking [55], the interplay between nonlinearity and disorder [56–58] and a related effect in the form of the Anderson localization of matter waves in disordered and quasi-periodic potentials [59,60], the interaction blockade [61], nonlinear properties in dipolar condensates [62–64], and other phenomena [65–68]. As for theoretical works, the corresponding list is virtually endless, and it is beyond the scope of this introduction to mention all essential theoretical publications in the field of BEC. To give an idea of that, it is relevant to mention that a recent review [69], devoted only to theoretical studies of nonlinear waves in BECs contains more than 500 references, comprising only a part of the field of nonlinear phenomena in degenerate quantum gases. Several focused meetings, as well as many topical sessions at more general conferences, have been devoted to the subject of nonlinear phenomena in BEC and, more generally, in degenerate quantum gases (that may include fermion gases and Bose-Fermi mixtures).

2. This Special Issue

This Special Issue of Physica D presents a collection of articles addressing important topics of current interest in various areas of the nonlinear science and their applications to degenerate quantum gases. Probably, the particular selection of topics covered in this Special Issue is somewhat biased towards the theory and applications of nonlinear waves to BECs dynamics. The articles included herewith are representative of a subfield of the study of degenerate quantum gases that is concerned with general work in the area of nonlinear phenomena, and are therefore expected to be of direct interest to the readership of Physica D. In fact, for a typical "nonlinear scientist" it is not enough that the model involved in the analysis be nonlinear (few models remain purely linear in the contemporary science), but that nonlinearities play a significant role, the character of which may be sometimes striking or counter-intuitive, in the phenomena under study.

In what follows we will describe the contents of the Special Issue.

2.1. Solitons

The generation of solitons in BEC [22,27,28] has been one of the most significant manifestations of nonlinear phenomena in these systems. Despite the numerous results already obtained in the field, there are still many open problems and there has been a burst of new experimental results in the last few years [24,25,29,31]. This Special Issue contains two articles dealing directly with the dynamics of one-dimensional solitons in different scenarios of interest in BEC.

First, Ref. [70] puts forward a theoretical proposal to use bright matter-wave solitons to probe and study quantum reflection from a solid surface at normal incidence. The idea of using BECs to probe interactions at the microscopic scale has been around for some time but this paper develops in detail a very precise idea with a clear focus on enabling its experimental implementation. Also in this Special Issue, Ref. [71] deals with the theoretical analysis of in multicomponent spinor models that are found to be integrable for particular values of the coupling constants, by dint of the inverse scattering method. They are related to symmetric spaces of BD.I-type $SO(2r+1)/SO(2)\times SO(2r-1)$ for $r = 2$ and $r = 3$. Finally, using a conveniently modified Zakharov–Shabat dressing procedure, generic one-soliton solutions are constructed. The paper also puts forward some ideas on how to extend the method to obtain explicit N-soliton solutions.

2.2. The BEC merging problem and shock waves

The formation of interference patterns during the interaction of initially separated BECs, the so-called BEC merging problem, is a fundamental setting which makes it possible to clarify the fundamental wave nature of matter. This problem was studied experimentally shortly after the production of BECs [72,73]. Although it has many aspects beyond "purely nonlinear" properties, many experiments have already demonstrated coherent nonlinear wave structures arising in BEC-merging experiments [37,38,41,42,74–76].

This Special Issue includes an article [77] dealing with the BEC-merging problem by considering, both theoretically and experimentally, the case of two elongated, anisotropically confined BECs from a hydrodynamic perspective. The results span the gap between the linear theory of matter-wave interference for small densities [78,79] and a nonlinear theory that describes the interference pattern as a modulated train of solitons.

2.3. Noise and nonlinearity

The interplay between disorder and nonlinearity and various, sometimes counter-intuitive, outcomes of their coexistence have been a subject of interest to researchers working in nonlinear dynamics for many years. In the field of nonlinear waves, topics such as exactly integrable nonlinear stochastic equations, dynamics of nonlinear waves in random media, evolution of random waves in nonlinear media, and many others have been discussed extensively both from a general point of view and in terms of applications, where such problems naturally arise in solid-state physics, hydrodynamics. nonlinear optics, plasmas, mathematical models of molecules and biological structures, etc. Many books and reviews on this general topic are available (see e.g. [80,81]).

Only recently BECs have been recognized as yet another setting able to display the coexistence of disorder and nonlinearity, thus allowing for the study of previously predicted phenomena in the framework of the wave theory, condensed-matter physics, etc., while at the same time presenting its own phenomenology. Since 2005, there has been a flurry of experimental activities aimed at studying BECs in disordered potentials. These experimental studies include disorder-induced inhibition of transport [82–84, 57,58], effects of disorder in terms of collective modes [82,58,85], Bose glass [86], granular BEC and phase coherence [58], density modulations [58,87], the disordered Bose-Hubbard model [88], and Anderson localization [59,60], between other topics. This Special Issue contains paper [89] which reviews recent studies of the effects of the disorder induced by a laser speckle field on a ^{7}Li BEC. In these systems, both the interaction, which gives rise to the nonlinearity in a BEC, and the disorder can be tuned experimentally. This opens many opportunities to study the interplay of the nonlinearity and disorder in both condensed-matter physics and nonlinear science, as described in the paper.

2.4. Driven Bose–Einstein condensates

As stated before, the response of a BEC to various types of an external drive has been a topic of active research since 1995, the literature on the topic being enormous. This special issue contains three articles dealing with driven condensates. The first one [90] theoretically investigates the diffraction management in BEC in 1D, 2D, and 3D geometries. The management technique considered in that work is based on the superposition of sinusoidal lattice potentials moving at a common speed but in different directions, leading to a sinusoidal spatio-temporal modulation of the potential. Another interesting application of spatio-temporally driven lattices in connection to BECs is the possibility of constructing a ratchet potential [91–93]. In Ref. [94] in this special issue, the dynamics of bright matter-wave solitons under the action of a time-periodic ratchet potential induced by a 1D bichromatic optical lattice is studied. Due to the broken space- and time-reversal symmetries of the potential, the soliton is transported with a nonzero average velocity. The dependence of the transport velocity on the initial state of the soliton and soliton collisions are studied. A third paper in this issue dealing with driven BECs considers the modulational instability of matter-waves in BEC under the action of a strong temporally periodic nonlinearity management [95]. This paper adds interesting results to the much studied subject of condensate driving through Feschbach management of the strength of inter-atomic interactions (see e.g. Refs. [96–106] and references therein).

2.5. Bose–Einstein condensates with spatially-modulated interactions

The use of the Feshbach-resonance management also opens the possibility of managing the interactions spatially. Although this has remained, thus far, a theoretical concept, nothing should prevent its implementation in experiments in which interactions are controlled by spatially nonuniform magnetic or optical fields. The consideration of such spatially managed systems has led to the prediction of many nonlinear phenomena in the course of the last few years [107–126].

In this Special Issue there are two articles complementing the existing literature dealing with systems based on spatially managed interactions. In Ref. [127] the existence of super-strong localization phenomena for the ground-state atom density around the zeroes of the scattering length is proven combining intuitive arguments with the analysis of exact solutions, numerical simulations and rigorous mathematical results. Another paper in this Issue dealing with spatially-modulated interactions [128] studies collisionally inhomogeneous BEC in double-well potentials for different combinations of in-well nonlinearities, and reports both analytically and numerically different types of solutions and bifurcations.

2.6. Nonlinear phenomena in optical lattices: One-dimensional configurations

Given their considerable flexibility, optical lattice potentials have been one of the tools of choice for matter-wave management. They have been used to realize experimentally in BEC settings a number of fundamental phenomena, previously known in condensed-matter physics. These include the Josephson effect [129], squeezed states [130], Landau-Zener tunneling and Bloch oscillations [131], and the transition between superfluidity and Mott insulation at both the classical [132] (predicted theoretically in [133]) and quantum levels [134] (see also the review [135]). This Special Issue includes several articles where the interplay of OL potentials and nonlinearities lead to interesting nonlinear phenomena. The first article [136] is a brief review of early results on the delocalizing transition, and addresses how to predict this transition in simple and realistic 1D scenarios, in continuous as well as discrete settings. The second article [137] investigates the effects of the application of an initial kick to 1D matter-wave solitons in a self-attractive condensate trapped in an OL, in the framework of nonpolynomial Schrödinger equations. Crossover from pinning to quasi-free motion of gap solitons is shown to depend on the size of the kick, the strength of the self-attraction, and the OL parameters. The third article is concerned with a variety of nonlinear effects, such as modulational instability and associated phenomena, in the photo-induced dynamics of site occupation in quasi-1D superlattices [138]. Finally, the fourth article Ref. [139] gives an example of the derivation and analysis of a GPE-type model, in this case for a fermionic superfluid in an optical lattice. Various types of one-dimensional gap solitons (as well as their two-dimensional counterparts) are studied in that paper.

2.7. Nonlinear phenomena in optical lattices: Multidimensional configurations

Multidimensional OLs have many more degrees of freedom than their one-dimensional counterparts and thus open even more possibilities for control of quantum matter. In this Special Issue there are several articles dealing with various aspects of BECs in multidimensional lattices.

The first paper devoted to that topic in this special issue [140] studies BECs in a honeycomb optical lattice and shows that in the long-wavelength, mean-field limit, it can be described by a nonlinear Dirac equation with broken symmetries. It is interesting to see how a quantum-electrodynamics model can arise in the context of ultracold (and, accordingly, ultra-slow) gases. Several articles in this Issue deal with vortices in higher-dimensional lattices. It is relevant to stress that the study of vortices, including their existence, stability and dynamical properties, has been one of the central themes of studies in BEC [6,8,69]. The first paper on our special issue [141] on that topic studies the existence, stability and dynamical evolution of dark vortex states in the 2D defocusing discrete nonlinear Schrödinger model. Interesting analogies are drawn between the stability of such dark vortex states and their one-dimensional counterparts. A second article on that topic in this Issue is Ref. [142], that provides a complete mathematical description of symmetrical solutions to the nonlinear Schrödinger equation with the nonlinearity depending on the absolute value of the field. Finally, a third paper on vortices in higher-dimensional scenarios included in this special issue [143] explores numerically a rich variety of 2D solitary vortices with different topological charges, and compound solitons of other types (such as quadrupoles) in systems with anisotropic optical lattices.

2.8. Collapse

The phenomenon of collapse (also termed blow-up, in the mathematical literature) belongs to the most spectacular effects observed in the mean-field dynamics of a BEC with a negative scattering length. After experimental observation of the threshold number of atoms in condensates of ^{7}Li [2,44], there was a lot of theoretical interest in the analysis of the largest possible number of atoms and dynamical properties of the blow-up on the basis of the proper GPE and other related models (see e.g. [14,21,144–157] and references therein) This body of work adds to the previously existing more mathematically oriented literature on blow-up phenomena (see, e.g., [158] and references therein), and to experiments directly demonstrating blow-up in BEC [29,45–47].

This Special Issue includes three articles adding new findings to the state-of-the-art in the field of blow-up phenomena. The first one [159], considers the possibility of blow-up during collisions of Bose–Einstein solitons due to a local increase of the density during the collision. It is found, somewhat surprisingly, how a good estimate can obtained from a simple variational model. A closely related work reported in this Special Issue [160] uses variational methods and numerical simulations to study the dynamics of multiple bright solitary waves in trapped BECs. A third article [161] derives rigorous blow-up estimates valid for general heteronuclear multicomponent systems and prove that such systems can be driven to blow-up either by a single unstable component or because of negative cross-interaction coefficients, even when individually all of the species are stable.

2.9. Atom lasers

Atom lasers are sources of coherent matter waves that use an ultra-cold gas of trapped alkali atoms as a reservoir from which coherent pulses of atoms are extracted. Several methods have been proposed to deliver atoms coherently from their confinement. Historically, the first of these devices used short radio-frequency pulses as the outcoupling mechanism, flipping the spins of some of the atoms to release them from the trap [162]. Later, other atom lasers were built leading to pulsed, semi-continuous or single-atom coherent sources [163–168].

The efficiency of a matter-wave laser is determined by the amount of atoms that can be delivered from the trap, and by the purity of the emission process. Concerning this point, it has been recently shown [169] that spin-flipping techniques present serious limitations in terms of the number of atoms that can be emitted. This effect is related to fluctuations at high flux due to the fact that the output-coupling mechanism populates all accessible Zeeman states. This problem is an essential drawback for practical applications of atom lasers to high-precision measurements, such as matter-wave gyroscopes [170]. In principle, one can use nonlinear mechanisms to avoid the above-mentioned problems. This motivation has driven a lot of research, and several models of atom lasers, based on nonlinear mechanisms have been elaborated theoretically [99,107,113,171–173].

This Special Issue presents two novel proposals to construct coherent matter-wave pulses from a reservoir, using nonlinear phenomena. The first scheme, studied in Ref. [174], is based on the mode-locking effect, well-known in the context of photonic lasers. This phenomenon helps them to produce ultra-short pulses that are global attractors of the laser system. In Ref. [174] the ideas of mode-locking are extended to atom lasers. The second article devoted to atom lasers in this special issue [175] explores certain aspects of the universality of the emission of "blips" in the course of macroscopic tunneling, that are used to propose ways of controlling the speed, duration and number of atoms in the emitted pulses.

2.10. Phase transitions

An important and sometimes overlooked feature of the GPE is that it gives an accurate microscopic description of the formation of BEC from the strongly degenerate gas of weakly interacting bosons. [176–182]. The model is able to trace the development of the large scale coherent regime at a certain stage of evolution that sets in after the breakdown of the regime of weak turbulence in a low-energy region of wavenumber space. It corresponds to the formation of the superfluid short-range order which is the state of superfluid turbulence with quasi-condensate local correlation properties. Ref. [183] included in this special issue discusses the extension of the previous ideas to multicomponent systems and describes the thermodynamic properties of this system at equilibrium, a study that extends far beyond the strict limit of degenerate quantum gases, due to the universality of the problem studied.

Finally, in Ref. [184] included in this Special Issue it is shown that a gas-to-liquid phase transition at zero temperature may occur in a coherent gas of bosons in the presence of competing nonlinear effects even in the absence of any quantum effects. Several interesting dynamical phenomena are also studied.

3. The future

Where do we go from here? The field is now mature with many different research lines and research groups working in different directions, thus the continuation of activities on all fronts of the intersection between nonlinear dynamics and the physics of degenerate quantum gases may be expected.

There are several topics that are especially promising, and will plausibly drive broad-scale research in the next few years. Here we try to briefly identify some of these subjects, although the list is far from being exhaustive.

3.1. Exciton polariton condensates and non-equilibrium systems

Semiconductor microcavities offer unique systems in which to investigate the physics of interacting composite bosons. Their elementary excitations, polaritons – strongly interacting light-matter quasi-particles – can accumulate in macroscopically degenerate states to form various types of condensate in a wide range of experimental configurations. Being a quantum superposition of photon and exciton, these quasi-particles have very small effective mass (about 0.0001 that of the electron) and can condense in the relative warmth of tens of kelvin, or perhaps even higher. Moreover, the coherence, polarization and population distribution of polaritons can be easily probed by analyzing the far-field emission. Both incoherent [185–187] and coherent [188,189] excitations have been used to create these systems. There are many essential differences between polariton condensates and ultra-cold atomic systems. The lifetime of polaritons is of the order of picoseconds, so the system has to be continuously repopulated and is in an out-of-thermal-equilibrium state. The polaritonic atoms are large: their size is comparable to the wavelength of light, and, therefore, overlap at very low density, which is quite different from dilute atomic gases whose interactions are short-range. Finally, the current experimental systems are inherently two-dimensional, so that a Berezhinskii–Kosterlitz–Thouless phase transition is relevant. Despite these differences, features similar to those observed in other superfluid systems have been predicted and/or observed, such as the formation of quantized vortices, vortex lattices, flow without resistance, and others [190–194]. Thus, the study of BEC built of polaritons in microcavities is a very promising field of research [195]. These systems are a new kind of laser, but unlike a regular laser they are formed of coherent quantum objects which puts them in the class of quantum devices [196].

From the point of view of mathematical modeling and nonlinear phenomena, GPEs again emerge as relevant models in this field, but now with additional loss and gain terms responsible for the non-equilibrium dynamics of the system, i.e., as a matter of fact, equations of the complex Ginzburg–Landau type [194,197]. Many results are already available in models of that type [198], in the form of dissipative solitons [199], and also in laser physics, where similar models arise to describe the transverse dynamics of lasers with a single longitudinal mode. However, many new problems arise in the context of exciton-polariton condensates that may readily result in a cross-fertilization between the fields of nonlinear dynamics and quantum fluids, condensed matter, laser physics and mathematical physics.

Because of the lack of experimentally demonstrated mechanisms for continuously and coherently refilling the BECs these phenomena cannot be observed in atomic BECs. Losses are always present in BECs [200,201,13], and can even be induced through the use of focused electron beams [202]. Dissipation is generally found to damp the excitations and act against the existence coherent structures, but it has been recently that it may play a "non-destructive" effect [157,203] on certain nonlinear structures and even play a constructive role in the generation of coherent structures [204]. Thus, although not as rich as in exciton-polariton systems, there are also possibilities for the analysis of dynamical non-equilibrium nonlinear phenomena in atomic BECs.

3.2. Applications of matter-wave solitons and nonlinear phenomena to precision measurements.

Despite the many studies of solitons in the course of the last forty years, their practical applications are scarce. Even in nonlinear optics, despite the enormous potential, there are few real applications of solitons. In that sense, the field of the Bose–Einstein condensation is in a similar situation: nonlinear waves are fascinating phenomena for the studies in fundamental physics, whose genuine applications are yet to be demonstrated.

However, the fact that bright matter-wave solitons manifest themselves as self-trapped condensates, where the usual wavepacket dispersion is exactly balanced by the presence of attractive atomic interactions, offers many potential advantages for applications in atom optics and interferometry [205]. The idea that nonlinear effects in ultracold atoms could be used for precision measurements has been around for some time, but now it is probably the relevant time to develop this framework, in both theoretical and experimental directions, aiming to bring it closer to the practical implementation. This would result in an increase of the interest in solitons outside the nonlinear-wave community. As mentioned above, this Special Issue contains an example [70] of a proposal to use solitons for probing atom-surface interactions. Several other ideas concerning the use of matter-wave solitons in precision measurements have been discussed in the current literature [206]. Nonlinearities have also been shown to play a relevant role in some previous proposals [207,208].

Another application of solitons which awaits experimental verification is the creation of atom lasers operating in the pulsed (solitonic) regime. As discussed in Section 2.9, atom lasers based on nonlinear properties of matter waves may feature properties needed for several specific applications. To the previously existing set of theoretical ideas elaborated for the design of such devices [99,171,107,103,113,172,173], this Special Issue adds two more [174,175].

For instance, the achievement of quasi-continuous atom lasers, now available through different procedures but also discussed to be achievable using nonlinear phenomena would open new perspectives for applications. An example is their proposed use for novel gravito-inertial sensors. Presently, they are loaded with cold atomic clouds, a feature which has drastically increased the accuracy of the measurement of gravitational acceleration to a few parts per billion [209,210]. However, the possibility to load these devices with a fully coherent and collimated matter source instead of the incoherent cold atomic samples used so far would even improve the precision [211]. Many of the nice applications of cold atoms [209–212] for interferometric and other similar purposes could naturally benefit from the ideas of nonlinear science in the BEC context.

3.3. Quantum complexity with Bose–Einstein condensates

The emergence of chaotic dynamics of classical systems governed by finite dimensional hamiltonian systems is well understood in general terms and the advances in the last 40 years have been essential to consolidate the theory. The quantum -linear- counterparts of these classical phenomena are not so well understood although it is well established that such systems display a series of signatures of chaos, sometimes denoted under the collective umbrella of the term "quantum chaos" [213,214].

The fact that the Gross-Pitaevskii is a nonlinear model different from those of linear quantum mechanics opens many avenues for investigation that only recently have started to be considered.

A first example is provided by the so called δ-kicked quantum systems, that have been extensively studied with cold atoms in the linear regime (see for example [215–219] and references therein). It is only recently that the role of interactions, fundamental in experiments with BECs, has been considered leading to many unexpected phenomena [208,220–222].

As a second example, we would like to mention that one of the ways of treating and interpreting the quantum-classical correspondence is the use of the discrete WKB approximation (well elaborated in atomic physics [223]) for the description of the Fock space. Then the inverse number of particles plays the same role as the Planck constant in the standard WKB theory [224,225]. This leads to the association of the dynamics in the Fock space, in particular of the energy eigenvalues, with the quasi-classical (or mean-field) evolution. Another fundamental problem of Quantum Mechanics in the core of the modern theory of quantum chaos is the dependence of the level distribution on the properties of the corresponding classical system [226,227]. Then the natural extension of this problem to the dynamics of a many-body system is the study of deformations of the true quantum spectra induced by changes of the control parameters. This approach has been put forward in a recent work [228], where it was shown that instabilities in the mean field dynamics are intimately related to the quantum phase transitions significantly affecting the quantum spectra. Further extension of these studies appears to be interesting form the point of view of quantum switching, construction of cat states, macroscopic manifestation of the quantum phase transitions, etc.

These are only a couple of examples, but the interplay of nonlinearities with quantum chaos in BECs is a promising field of research with many fundamental open questions.

Last, but not least, the fact that the nonlinear Schrödinger equation is able to support complex turbulent solutions in different scenarios has been put forward by different authors in the context of liquid helium problems [229–231]. Quantum turbulence is comprised of quantized vortices which are stable topological defects, being expected to give a prototype of turbulence much simpler than the classical traditional turbulence. In the field of cold atoms, adding the specifics of the presence of the trap, there have been few studies on quantum turbulence. There is only a theoretical proposal on how to make steady quantum turbulence in trapped Bose–Einstein condensates [232] that also predicts that the energy spectrum obeys the Kolmogorov law. Also there is a theoretical prediction of the transition to quantum turbulence in a Bose–Einstein condensate through the bending-wave instability of a single-vortex ring [233].

BECs could provide simple test systems to check many of the predictions proposed in theoretical and experimental studies on liquid helium and also to study new complex phenomena related to the trapping potentials, and many other effects taking into account the level of control achieved (scattering lengths, potentials, different components, etc.).

3.4. BECs: (Still) a fundamental playground for nonlinear waves

The above directions naturally extend the rich phenomenology, and the theoretical, as well as the experimental understanding obtained through the study of nonlinear phenomena of degenerate quantum gases, to other recently emerging areas of physics. However, one can also argue that the BECs remain a fundamental setting that can still considerably enrich our understanding of solitons and nonlinear waves, and their interactions with other such structures, with potentials, as well as their dynamical formation and its implications for a wide range of scales ranging from the microscopic one to the macroscopic one (associated with the formation of defects in the early evolution of the universe).

On the basis of the current trends discussed in this volume (and other recent studies), it would be natural to extend the above considerations to the following topics

- Solitary waves in the presence of disorder and their interaction with different types thereof, both at the level of solitons [234] and at the level of vortices. Experiments and analysis similar to those leading to the formation of solitons [26], but in the presence of controllable disorder and nonlinearity as evidenced in [58] would be particularly interesting and useful.
- Considerations of phase transitions in single- and multi-component systems considered [183] in contexts associated with vortices [235], as well as with gray solitons [236] and their potential connections with cosmological studies would be particularly interesting to explore further.
- Soliton interactions with different types of potentials (a theme pervading most contributions to this special issue, from lattices and superlattices to double wells, magnetic traps and ratchets), and also between solitons [24,25,39] still clearly have a lot to teach us and are far from completely understood.

These are only some of the many emergent topics and connections arising in this field, whose maturation can serve as a guiding light towards other related fields, but which is itself far from being saturated. We hope that this overview and, especially the state-of-the-art represented in this special volume, will be a motivating source of numerous new discoveries.

We hope that this review will make many more readers interested in this field and will maybe help build connections between various areas of nonlinear dynamics

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