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Exciton–polariton condensation
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We review, aiming at an audience of final year undergraduates, the phenomena observed in, and properties of, microcavity exciton–polariton condensates. These are condensates of mixed light and matter, consisting of superpositions of photons in semiconductor microcavities and excitons in quantum wells. Because of the imperfect confinement of the photon component, exciton–polaritons have a finite lifetime, and have to be continuously re-populated. Therefore, exciton–polariton condensates lie somewhere between equilibrium Bose–Einstein condensates and lasers. We review in particular the evidence for condensation, the coherence properties studied experimentally, and the wide variety of spatial structures either observed or predicted to exist in exciton–polariton condensates, including quantised vortices and other coherent structures. We also discuss the question of superfluidity in a non-equilibrium system, reviewing both the experimental attempts to investigate superfluidity to date, and the theoretical suggestions of how it may be further elucidated.

Keywords: exciton–polariton; BEC; polariton laser; pattern formation; superfluidity

1. Introduction

Microcavity exciton–polaritons are quasi-particles that result from the hybridisation of excitons (bound electron hole pairs) and light confined inside semiconductor microcavities. At low enough densities, they behave as bosons according to Bose–Einstein statistics, and so one may investigate Bose–Einstein condensation (BEC) of these particles, and the phenomena associated with it, such as increased coherence, superfluidity, quantised vortices, pattern formation. One particularly notable feature of BEC is that the many particle quantum system can be represented by a classical complex-valued field $\Psi$, so the dynamics of the system can be described by essentially classical equations of nonlinear physics.

Exciton–polaritons are one of several examples of quasi-particles inside solids which present opportunities to explore condensation and related effects. One advantage of studying condensation of quasi-particles in solids is that they have effective masses that can be controlled by the design of the material. These effective masses are typically much smaller than atomic masses. Since the critical temperature of the BEC is inversely proportional to the effective mass, solid state condensates can often be realised at relatively high temperatures, and in some cases, even at room temperature. This is in contrast with the micro-Kelvin temperatures needed to condense dilute atomic gases.

Because polaritons are quasi-particles that involve light, they also have a finite lifetime, as light can escape from the cavity. This means that any polariton condensate requires continuous injection of new polaritons to balance the loss. As such there is naturally a close relation to lasing.

This article reviews some of the phenomena of exciton–polariton condensation, aiming in particular to illustrate the behaviour that has been seen experimentally. As the field of microcavity exciton–polariton condensation has been a very active field for several years, there already exist a large number of review articles [1–7], as well as several books [8–10] and collections or edited volumes [11–14]. The particular focus of this article is that it is aimed at an audience of final year undergraduates, and concentrates on discussing the phenomena that have been observed, or are predicted to be observed, in exciton–polariton condensates. As such, we do not discuss theoretical techniques, but we will try to make connections between the behaviour of the polariton system and other well-studied fields, such as quantum hydrodynamics [15–17], lasing [18,19], and nonlinear optics and pattern formation [20,21]. Furthermore, by arranging our discussion according to phenomena, we will not discuss the historical development of the subject.

This article is arranged as follows. The remainder of this section provides an introduction to Bose–Einstein

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condensation, and to microcavity exciton polaritons as a system of quasi-particles in solids where condensation may be studied. Following this, the article is then arranged around classes of phenomena seen in exciton–polariton systems. Section 2 discusses those features indicating condensation, including energy and momentum distribution of polaritons and coherence properties. An aspect of condensation which has been studied particularly extensively concerns the spatial structure of condensates, and the effects of finite polariton lifetime on this structure; these are discussed in Section 3. Section 4 considers features associated with exploring which aspects of superfluidity can be seen in polariton systems, an area which is still not fully resolved. Finally, Section 5 concludes, and provides a brief summary of some of the topics likely to feature significantly in future work which we have not had space to discuss in this article.

1.1. Bose–Einstein condensation

This subsection provides a brief overview of some of the essential features of Bose condensation, which will play an important rôle in the following discussion of exciton–polariton condensation. More comprehensive discussions of Bose condensation can be found in many textbooks [15,16].

Bose–Einstein condensation of a non-interacting Bose gas occurs because for a Bose–Einstein distribution in three dimensions, there is a finite density at which the chemical potential reaches zero. Thus, as one increases density at a fixed temperature, or decreases temperature at a fixed density, the required chemical potential eventually reaches the bottom of the density of states. Since the Bose–Einstein distribution function $n_B(E) = \frac{\exp[\beta(E - \mu)] - 1}{\beta}$ diverges at $E = \mu$, the point at which $\mu$ reaches zero then implies that there is a macroscopic population of the $E = 0$ modes. The temperature and density at which $\mu$ first reaches zero thus describes the point where the phase transition to the condensed state occurs. By finding the density and temperature at which $\mu = 0$, one produces the non-interacting transition temperature $T_{\text{BEC}} = \left(\frac{2\hbar^2}{mk_B}\right)^{2/3}$, where $m$ is the atomic mass and $\rho$ is the number density.

In more complicated situations, one can rigorously define condensation as the appearance of a single particle state with a macroscopic occupation [22]. Macroscopic occupation of a single particle state can be determined by considering the one particle density matrix (i.e. the density matrix of the system after tracing out all but one particle), and determining its eigenvalues. For a regular thermal gas, the eigenvalues will all be much less than one indicating that most single particle states are occupied by a small number of particles. As the density increases, or temperature decreases, some of the eigenvalues become of order one, and one has a degenerate quantum gas, where the differences between bosonic and fermionic particles become apparent. When condensed, one eigenvalue becomes macroscopic, indicating that many particles are in the same state. In a homogeneous system, this state can be expected to be the zero momentum state, and this immediately leads to long range coherence.

This macroscopic occupation means that there is a single complex classical function $\Psi$ that describes the interacting many particle system. The explicit many particle wavefunction has the form $\Psi(r_1, \ldots, r_N) = \prod_{i=1}^N \Psi(r_i)$, i.e. there are many particles in exactly the same quantum state. The function $\Psi(r)$ can be seen in several other ways: it is the eigenvector of the density matrix with macroscopic occupation. If $\Psi(r)$ is normalised so that $\int d^3 r |\Psi(r)|^2 = N$, it can be seen as the order parameter for the Bose condensed phase. In the second quantised approach (which we will otherwise avoid in this review), $\Psi$ can be seen as replacing the quantum field creation and annihilation operators with a classical field describing the condensate mode. This replacement is appropriate when the occupation of this mode is much larger than one and so the non-commutativity of the quantum field operators can be neglected.

Such a ‘classical field’ description also occurs when one goes from quantum electrodynamics to the classical description of electric and magnetic fields, which obey Maxwell’s equations. Moreover, it has been demonstrated [23,24] using a general analysis of the kinetics of a weakly interacting bosonic field, that all low energy states with macroscopic occupation can be described as an ensemble of classical fields with corresponding classical-field equations. The requirement of weak interaction is essential here, since in a strongly interacting system it is impossible to divide single-particle modes into highly occupied and practically empty ones: these modes are always coupled to the rest of the system.

The existence of long-range phase coherence is also associated with the idea of breaking of phase symmetry. This is because in the condensed state a large number of particles occupy the same quantum state, and so it is possible to see macroscopic interference effects, meaning that a well-defined quantum mechanical phase can exist. A pivotal relation in the theory of condensates is that the velocity of the condensate flow $v(r)$ at the position $r$ is proportional to the gradient of the phase of the wavefunction $\Psi = |\Psi| \exp i\phi(r)$

$$v(r) = \frac{\hbar}{m} \nabla \phi(r).$$

(1)
Several fundamental properties follow from this result. Firstly, as one proceeds around any closed path in a simply-connected condensate, the phase $\phi$ can only change by a multiple of $2\pi$ in order that $\Psi$ remains single-valued along this path. If the phase $\phi$ does indeed change by, say, $2\pi$, then there has to be at least one point (in 2D) or a line (in 3D) inside this path where $\phi$ takes any value between 0 and $2\pi$. To prevent such phase singularity, the amplitude of $\Psi$ has to vanish at this point (line). These points or lines constitute quantised vortices with the unit of circulation $\oint \mathbf{v} \cdot d\mathbf{l} = 2\pi \hbar / m = h / m$. Secondly, it follows from Equation (1) that the condensate cannot support rotation, since the vorticity $\omega = \nabla \times \mathbf{v} = 0$ away from phase singularities. The flow is thus a real example of potential flow and, therefore, can be described by the equations of classical irrotational hydrodynamics. Moreover, on distances larger than the size of the vortex core (the length-scale over which the number density of the condensate, $|\Psi|^2$ heals back to the unperturbed value) the compressibility of the flow can be neglected implying $\nabla^2 \phi = 0$. This is again directly related to the motion of point vortices and vortex lines in classical Eulerian fluid. In this representation vortices move as material lines (points) according to the classical Biot–Savart Law. The same law is used to compute the magnetic field generated by steady current $\mathbf{j}$.

In two dimensions (which many of the solid state condensate systems are), the meaning of condensation is rather different. For a non-interacting gas in two dimensions, no phase transition occurs. For an interacting gas, a transition does occur, but there is no macroscopically occupied state. Instead, there is a transition between a low temperature superfluid phase with power law decay of correlations, and a high temperature normal phase. This power law decay of correlations occurs due to the effects of long wavelength fluctuations of the condensate phase $\phi(r)$ – at any non-zero temperature, these fluctuations are sufficiently large that one cannot consider a well-defined phase, and the correlation between the phases at two distant points has the form $\langle \exp[i\phi(r)]\exp[-i\phi(r')] \rangle \sim |r - r'|^{\eta}$, with $\eta = k_B T \hbar / 2\pi \rho \hbar^2$. Although no one single state acquires a macroscopic occupation in an infinite two-dimensional system, there is a notably increased population of a range of low momentum states. If the system is also confined by a trap within the two-dimensional plane, then many features of condensation can be recovered, in particular a single particle state will become macroscopically occupied. One may note that this transition involves many body effects, in that the transition temperature is much larger than the energy level spacing for a single particle in the same trap.

Many other physical systems are closely linked with Bose condensation. A superconductor is a charged superfluid, where the Bose condensate is formed by Cooper pairs – quasi-particles with integer spin (normally zero) consisting of electron pairs. The wave function of such a charged Bose condensate is once again a complex classical field. In nonlinear optics the order parameter is the electric field of the (polarised) wave. The close connection between broad-aperture lasers, superconductors, BEC, and superfluids has been recognised since the late 1960s [25] by establishing that all these systems can be described by the same order parameter equation – the complex Ginsburg–Landau equation. This fundamental equation, which can be viewed as the driven dissipative nonlinear Schrödinger equation with relaxation, will be discussed in Section 3.

1.2. Introduction to exciton–polaritons

Exciton–polaritons are the quasi-particles that result when one allows strong coupling between excitons and photons [26,27]. Excitons are the bound states resulting from the Coulomb interaction between electrons and holes in a semiconductor. Excitons can be created by shining light on a semiconductor in its ground state, exciting an electron from the valence to the conduction band, leaving a hole in the state the electron was excited from. Excitons can therefore also decay, by the electron and hole recombining to emit a photon. ‘Strong coupling’ occurs when the rate at which photons are converted to excitons and vice versa exceeds the rate at which photons escape from the system. In this case, the normal modes of the system are not excitons and photons, but rather superpositions of these, i.e. polaritons.

The idea of polaritons was originally considered [26,27] in bulk materials. However, for polariton condensation, it is necessary to consider microcavity polaritons. These are formed when the photons are confined inside a microcavity, i.e. a planar Fabry–Pérot resonator formed by two Bragg mirrors. As illustrated in the cartoon in Figure 1(a), the Bragg mirrors which form the cavity consist of quarter-wavelength layers of alternating refractive index, and so they can provide a high quality mirror over a limited range of wavelengths of light. By confining photons, one allows polaritons to potentially survive long enough to cool and condense. Furthermore, one can engineer stronger coupling between excitons and photons by locating the excitons (confined in quantum wells) at anti-nodes of the confined light mode in the cavity.

For photons that are confined inside the microcavity, their state can be described by a wave-vector in...
the two-dimensional plane of the cavity, the ‘in-plane wavevector’ \( k \), and an index \( N \) labelling the excitation number in the direction perpendicular to the plane. In terms of these variables, the angular frequency of such a mode can be written as 
\[
\omega_k = (c/n)[k^2 + (2\pi N/L_w)^2]^{1/2},
\]
where \( n \) is the refractive index, \( c \) the speed of light in vacuum, and \( N \) labels the transverse mode in a cavity of transverse size \( L_w \). For small in-plane wavevector \( k \), this frequency can be expanded to give a quadratic dispersion \( \hbar \omega_k = \hbar \omega_0 + \hbar^2 k^2/2m \). The photon mass appearing in this expression \( m = \hbar n(2\pi N/L_w) \) is thus set by the size of the cavity, however this size is in turn fixed by the requirement that the bottom of the photon band, \( \omega_0 = (c/n)(2\pi N/L_w) \), should be close to resonance with the energy required to create a bound exciton. For typical materials, this sets the photon mass to be of the order of \( m = 10^{-3} m_e \) (where \( m_e \) is the free electron mass.) Compared to this steep photon dispersion, the exciton dispersion can be almost neglected; excitons have a mass set by the electron and hole mass in a semiconductor which is thus typically somewhere between \( 0.1 m_e \) and \( m_e \).

For a sufficiently high quality cavity, the polariton states can be found by solving the Schrödinger equations for coupled exciton and photon wavefunctions:

\[
\hbar \partial_t \psi = \left( \frac{\hbar \omega_0}{2} + \frac{1}{2} \gamma \right) \psi + \left( \frac{1}{2} \gamma + \frac{\hbar \omega}{2} \right) \psi_{\text{ex}},
\]

(2)

where \( \gamma \) describes the rate of interconversion between excitons and photons; \( \gamma \) thus depends on the dipole matrix element for a single photon to excite a single exciton. The eigenstates of this equation are the new normal modes, the lower and upper polaritons,

\[
E_{k_{\text{LP,UP}}} = \frac{1}{2} \left( \hbar \omega_0 + \frac{\hbar^2 k^2}{2m} \right) \pm \left[ \left( \hbar \omega_0 - \frac{\hbar^2 k^2}{2m} \right)^2 + \gamma^2 \right]^{1/2}.
\]

(3)

The dispersion of these two modes are shown in Figure 1(b). For small in-plane wavevector, the lower polariton has an almost quadratic dispersion, with an effective mass of the order of the photon effective mass (at resonance, \( \hbar \omega_0 = \hbar \epsilon \), one has \( m_{\text{pol}} = 2m \)). At large momenta, the lower polariton dispersion has a point of inflection, before eventually approaching the limit of the exciton dispersion. Realisation of the strong-coupling regime in semiconductor microcavities [28,29] can be clearly seen experimentally, since the transmission and reflection of the microcavity as a function of energy and incident angle can be measured, as discussed below in Section 1.3, allowing direct observation of the modified spectrum.

Polaritons are not, however, ideal, non-interacting bosons. Because of their excitonic component, there are polariton–polariton interactions. These arise both due to the interactions between the charged particles making up the exciton, and because of saturation of the exciton–photon interaction. In a clean sample, the dominant interaction between polaritons at low-momenta is the relatively short ranged electron–electron exchange interaction [30] (i.e. it results from the process in which two excitons exchange their electrons). For low densities, as a first approximation.
to the full form of this interaction, one may consider a pseudo-potential, \( U(r) \rightarrow U \delta(r) \), describing a contact interaction between two polaritons. A typical scale for \( U \) is of the order \( 10^{-3} \) meV \( \mu \text{m}^2 \). In addition, because the strength of coupling between excitons and photons is comparable to the temperatures at which condensation is studied, the internal structure of the polariton cannot be entirely neglected. Polaritons also have a relatively short lifetime (of the order of 5 – 10 ps) because the photonic component is not perfectly confined by the Bragg mirrors, i.e. their reflectivity is not perfect. The fact that polaritons escape from the microcavity is, however, what allows one to experimentally image the properties of the condensate, as discussed below.

Another complicating feature of polaritons is that they have two possible polarisation states, corresponding to the left- and right-circularly polarised photon states. A comprehensive review of polarisation phenomena in polaritons can be found in [7], and a review of polarisation dynamics in [31]. In many cases, however, coupling between mechanical strain in the sample and the energy of electron and hole states breaks this polarisation symmetry and favours a particular linear polarisation – i.e. a particular superposition of left- and right-circular polarised polaritons. In these cases, the polarisation degree of freedom can then be ignored. In this article, we will only mention polarisation in a few cases where it leads to interesting new physics.

### 1.3. Experiments on exciton–polaritons

The basic outline of almost all experiments on exciton–polariton condensates are similar: light is used to create excitations inside the semiconductor microcavity, these excitations relax and scatter to form a polariton condensate, and because of the finite polariton lifetime, light then escapes the cavity and is detected. Since the escaping light conserves the energy and in-plane momentum of the polaritons, it allows one to image both the real-space and momentum-space shape of the condensate, the polariton dispersion, the line-shape, and the coherence properties of the polaritons. The conserved in-plane momentum of the photon can be written in terms of its angle of emission as \( \mathbf{k} = E_k^{LP} \sin(\theta) \), hence one may refer to the polariton momentum, wavevector or emission angle somewhat interchangeably. Figure 2 illustrates schematically the various setups for imaging and coherence measurements on the condensate.

Experiments on polaritons are performed in a number of materials, and with a variety of different ways of injecting polaritons. The choice of semiconductor material that the cavity and mirrors are made from affects the various parameters describing polaritons, such as the exciton binding energy, the exciton–photon coupling, the effective exciton–exciton interaction, and the photon lifetime. For most of the experiments discussed in this review, the system is either made from CdTe, doped with Mg or Mn to make the Bragg mirrors and the cavity, or GaAs doped with Al or In.

The way in which new polaritons are injected has a more pronounced effect on the behaviour that can be observed. In the same way that the light escaping from the cavity conserves the energy and in-plane momentum of polaritons, one can create polaritons with a pump laser incident on the cavity, and thereby control what states are populated by varying the energy and in-plane momentum (i.e. incident angle) of the pump. Schemes to inject polaritons include: directly creating zero momentum polaritons with a coherent pump laser; coherently creating polaritons at a ‘magic angle’ from where they can parametrically scatter directly into the ground state; coherently creating polaritons at large angles so that many scattering events are required to reach the ground state; using a pump laser detuned from the polariton resonance to create incoherent populations of electrons and holes which subsequently relax; and in some recent experiments, injecting electrons and holes by electrical currents in a light-emitting diode configuration. This list is ordered starting from cases where the coherence, polarisation, and details of the pump have the most profound effect on the properties of the resulting polariton state, and...
ending with cases where such details of the pump have almost no effect. However, with the exception of coherently populating the zero-momentum state, all the other methods can be said to show some type of condensation, and so we will discuss experiments with all these cases of polariton injection below.

1.4. Other solid state quantum condensates

Exciton–polaritons are not the only quasi-particles in solids in which condensation has been sought or realised, and so this subsection provides a very brief summary of some of the other quasi-particles in solids which have been studied.

Magnons, which are elementary excitations – quantised spin waves – of a magnetic system have also been observed to condense. Magnon condensation has been seen in two quite different systems; ferromagnetic insulators [32–35] and within superfluid phases of $^3\text{He}$ [36,37]. Similar to the state of non-condensed Bose gas, the ‘normal’ states of these magnetic materials, without magnon condensation, are disordered paramagnetic states. In the magnon condensate, spins develop a common global frequency and phase of precession.

Magnon condensation in room temperature ferromagnetic insulators, in particular yttrium-iron garnet (YIG) films, requires a combination of an in-plane magnetic field, and driving with microwave radiation. A microwave photon excites two primary magnons that then relax, forming a magnon gas. As for the incoherently pumped polariton system, the chemical potential increases with pump power. When the microwave power exceeds a threshold value, the magnon population condenses in the minimum energy state. However, for magnons, this minimum energy occurs at non-zero momentum. This is due to the combined effects of the magnetic exchange interaction and the magnetic dipole interactions. The formation and structure of condensate can be analysed by scattering of light. A quasi-equilibrium state with a non-zero chemical potential can be realised because the magnon lifetimes, set by the spin–lattice relaxation times (\(\sim 1\ \mu s\)), are long compared to the magnon–magnon thermalisation time (\(\sim 100\ \text{ns}\)). Magnon condensation in helium is studied using nuclear magnetic resonance to create and then probe the magnon condensate.

Excitons on their own have also long been considered as candidates for condensation, (see, e.g. the articles in [38]). In such experiments, there is no Fabry–Pérot cavity, and it is preferable to reduce the exciton–photon coupling strength, so that the exciton has a long lifetime before it decays by recombination. Various different ways to achieve long lifetimes have been considered. One way involves choosing excitonic states for which recombination is dipole forbidden, meaning that selection rules forbid the decay of excitons; this is the case in cuprous oxide [39]. An alternate way to enhance exciton lifetimes is to spatially separate electrons and holes, by having two parallel quantum wells, and apply an electric field to trap electrons and holes in different wells [40–42]. These exciton systems have shown a significant change in their behaviour at a temperature where condensation would be expected, including evidence of enhanced spatial coherence [43], however, a full understanding of the observed behaviour remains to be found. A review of much of the physics of such excitonic systems is given in [44]. There are also close connections between excitons in coupled quantum wells, formed of bound electron–hole pairs, and the properties of quantum-Hall bilayers [45,46] – these systems consist of electron–electron pairs in strong perpendicular magnetic fields.

2. Condensation, Bose stimulation and coherence

Condensation – in the broad sense described above – of exciton polaritons manifests itself by macroscopic accumulation in low momentum states, increased temporal and spatial coherence, and evidence of Bose stimulation in producing the macroscopic occupation of low momentum states. This section will review these underlying features of condensation; features which serve as prerequisites for the phenomena discussed in subsequent sections.

2.1. Stimulated scattering

When microcavity systems are pumped sufficiently strongly, a threshold is seen above which the emission at low momenta increases sharply. The mechanism of such accumulation differs according to how the system is pumped. In particular, it matters whether pumping creates an incoherent population of polaritons which then scatter to the ground state, or whether polaritons are created coherently at wave-vectors which allow for parametric scattering. We will thus next discuss these two cases separately.

2.1.1. Incoherent pumping

In the first case, with an initially incoherent polariton population, the threshold behaviour is relatively simple. The relaxation of polaritons towards the low momentum state becomes stimulated due to the increasing population of the low momentum states [47]; i.e. the bosonic nature of polaritons leads to final state stimulation of the relaxation. One may also directly probe the stimulated scattering toward the ground state by seeing how the system responds...
to an additional coherent pump at zero momentum [48,49]. By measuring the response of either the population at higher angles from which polaritons are scattered, or the state to which they are scattered, one can see that scattering is stimulated by the final state population.

2.1.2. Parametric scattering

If polaritons are injected coherently at angles near the point of inflection in the lower polariton spectrum, a rather different behaviour is seen. As shown in Figure 3(a), there is a momentum state from which a pair of polaritons can scatter into one zero momentum and one high momentum state in a process which conserves energy and momentum. This means that for a pump at this angle (sometimes called the ‘magic angle’), and a weak probe at zero momentum, there will be strong amplification of the probe [51,52] due to stimulated scattering. This process is referred to as optical parametric amplification (OPA), as it is analogous to parametric amplification in nonlinear optics [18,19], where a pump beam can scatter in an energy and momentum conserving way into a signal and idler mode, and a probe in the signal mode would be amplified.

In addition to parametric amplification, it is also possible to realise optical parametric oscillation (OPO), where there is no probe beam present. In this case, above a threshold pump power, the state with polaritons only at the pump wave-vector becomes unstable, and the population of the signal and idler modes grows [51,53]. To understand the conditions under which OPO can be realised, one must take account of the energy shifts that occur with increasing density. These energy shifts mean that both the detuning between the pump laser frequency and the polariton state at the pump wave-vector is affected by the polariton density. The energy shifts also affect the detuning between the pump state and the signal and idler states. If one detunes the pump laser below the polariton mode, then as pump intensity increases, resonance becomes increasingly worse, and this limits the ability of parametric scattering to occur. If the pump is detuned above the polariton mode, the energy shift drives the system towards resonance. This leads to another kind of instability of the pump-wavevector-only state, Kerr bistability. This bistability occurs between two possible states, one with the pump mode having a low occupation, and thus a small blueshift, and so remaining far from resonance; the other with a higher occupation, and thus a larger blueshift, pushing this state into resonance with the pump (see Figure 3(b)). With an appropriate choice of detuning, these two states may also compete with the parametric instability, hence there is a range of pumping strengths in Figure 3(c) where there are both multiple states in which only the pump mode is occupied (grey, solid line) as well as states with all three modes (signal, idler and pump) occupied [50,54] (red dashed line).

While the parametric oscillator state looks very different from an equilibrium Bose condensate, it does however share with it the important idea of breaking of phase symmetry. Although the external pump sets the phase of the polariton field at the pump wavevector, the parametric scattering to signal and idler fields allows for an arbitrary phase $\phi_{\text{signal}} - \phi_{\text{idler}}$, i.e. the only requirement is that $\phi_{\text{signal}} + \phi_{\text{idler}} = 2\phi_{\text{pump}}$. This free phase allows in principle for some of the phenomena normally associated with condensation.

![Figure 3](https://example.com/figure3.png)

Figure 3. (a) Lower polariton spectrum, illustrating the idea of parametric scattering; pairs of polaritons scatter toward a signal state (near zero momentum) and a higher momentum idler state. Grey dashed lines represent the phonon relaxation mechanism for incoherently injected polaritons (see Section 2.2). (b) Illustration of the conditions for Kerr bistability, where the interaction induced blue-shift of the polariton can drive it closer to resonance with a coherent pump. (c) Input–output relation for the pump field, showing the pump only state (grey, solid line) and the parametric oscillation (OPO) state (red, dashed line). The pump-only state (grey) displays Kerr bistability for a narrow range of pump strengths, while the OPO state exists for a wider range of pumping strength. Also shown is the signal field intensity (blue dashed line) for the OPO state. One should note that the lines shown represent steady states of the equations of motion, however, these steady states may either be stable or unstable. After [50].
to be seen in this very non-equilibrium system. In particular, the spectrum for fluctuations on top of the OPO state shares features with the spectrum of a condensate [4,54,55] – these will be discussed further when discussing superfluidity in Section 4.

2.2. Momentum distribution and thermalisation

For incoherent pumping, observation of stimulated relaxation and accumulation of polaritons in low momentum modes leaves open several questions regarding the nature of this state. Because polaritons have a finite lifetime, the resulting steady state is a balance between particle loss, replenishment by the pump, and processes whereby polaritons thermalise and cool by collisions among themselves and interactions with other particles such as phonons. The steepness of the polariton dispersion at low energies, which is responsible for the elevated transition temperatures, also means that the process of cooling via phonon emission is slow. Phonons have a shallow dispersion, so momentum and energy conservation require emission of phonons with small energies (see dashed lines in Figure 3(a)). This means many phonons are required for such relaxation, leading to a ‘bottleneck effect’ [56], in which polaritons accumulate at the point where the dispersion switches from exciton-like to photon-like.

This bottleneck effect prevented the investigation of coherence properties for a long time, by hampering large accumulation of polaritons at low energies, and producing very non-thermal distributions of polaritons. A combination of experimental improvement and theoretical simulations using the Boltzmann equation [57–59] led to the observation that the bottleneck effect could be considerably reduced by a combination of detuning the photon slightly above the exciton mode and increasing the polariton density, at the expense of increasing the polariton temperature. The detuning leads to more exciton-like lower polaritons, increasing the strength of the interaction between polaritons. Combined with higher densities, this allows polariton–polariton scattering to thermalise the polariton distribution. One should, however, note that while such collisions establish a thermal distribution, they do not remove energy from the system – thermalisation and cooling are separate processes.

By reducing the bottleneck effect, it became possible to see a reasonably thermalised distribution of polaritons, along with an accumulation of polaritons in low energy modes [60] (see Figure 4). The processes of thermalisation and cooling have also been studied experimentally. By observing the dynamics of the polariton distribution [61] following a pulsed excitation, one may determine the time required to reach a quasi-thermal momentum distribution, and see how this depends on the exciton–photon detuning. One may also consider the system with a continuous pump [62], and study how the threshold pump power depends on detuning and on temperature of the semiconductor. Comparison to Boltzmann equation modelling then allows one to interpret this dependence in terms of the two effects of detuning the photon above the exciton: increased rate of thermalisation and reduced critical temperature for condensation (due to the increased polariton mass).

Figure 4. (a) Upper row: Energy and momentum distribution, Lower row: momentum distribution of polaritons, for three different pump powers, from below to above threshold. (b) Energy distribution with increasing pump power. At threshold, an exponential (i.e. Maxwell–Boltzmann) distribution describes the population, and above threshold, polaritons accumulated in low energy states, without any appreciable change to the temperature describing the decay of the tail. Adapted from [60], results are for a CdTe microcavity, held at a cryostat temperature of 5 K.
2.2.1. Non-equilibrium condensation versus lasing

Since polariton condensates are non-equilibrium steady states emitting coherent light, the question of whether they are more properly described as condensates or as lasers is one that is frequently asked. There are several criteria by which one might distinguish equilibrium condensation from simple lasers, and for most of these criteria, polariton condensates are somewhere between the two extremes.

Considering the criterion of whether there is a thermal distribution, the polariton distribution is set by a balance of pumping, decay and relaxation. The smaller both the decay and pumping rates become, the closer the system approaches the equilibrium state. In this sense, there is a smooth crossover between equilibrium Bose–Einstein condensation and the behaviour of a polariton condensate. With a non-vanishing pump rate, the kinetics of thermalisation involves stimulated scattering into the ground state. One may thus try to describe the system by writing the ‘quantum’ Boltzmann equation, i.e. a kinetic equation, describing the time evolution of the populations of the states, taking account of stimulated scattering due to the population of the states particles are scattered into. For a vanishing pump rate, the steady state of this equation is just the equilibrium Bose–Einstein distribution. Lasers also involve stimulated scattering, but typically have a much larger pump rate, so are further away from equilibrium. This illustrates how a smooth crossover between lasing a condensation might occur, however, there are also some characteristic differences between polariton condensates and typical photon lasers. Firstly, for polaritons, the stimulated scattering is within the set of polariton modes, rather than the stimulated emission of photons which occurs for regular lasers. Secondly, polariton condensation can occur without any inversion of the gain medium, which is required for normal laser operation, i.e. polariton condensation occurs with a quasi-thermal distribution of polaritons in the system, while regular lasing would require an inverted (negative temperature) distribution of some part of the gain medium in order for gain to exceed absorption. This idea provided one of the original motivations for polariton condensation [63].

One may also note that the same microcavity systems that show exciton–polariton condensation can also show regular lasing, when they cross over from strong to weak coupling at higher temperatures and pumping strengths. In order to check whether such a transition to weak coupling has occurred, one should investigate whether the emission follows the lower polariton dispersion, or the bare photon dispersion. Because lower polaritons have repulsive interactions, the energy of the low momentum states does increase with increasing density, but near threshold, this shift is typically small compared to the polariton splitting.

2.3. Coherence and correlation measurements

As a polariton condensate produces coherent light, it is of interest to characterise this coherence, i.e. to measure how the coherence decays in time and in space. Such measurements do several things: they provide further evidence that there is a sharp threshold for the appearance of coherent light; they provide measurable quantities which allow one to position the polariton system between simple lasers and equilibrium condensates; and they can provide information about the kinetics of the polariton system. Such measurements aim to determine the first- and second-order correlation functions of the electromagnetic field:

\[
g_1(r, r', t, t') = \frac{\langle E'(r, t')E(r, t) \rangle}{\langle E(r', t')E(r, t) \rangle} \frac{\langle E(r, t) \rangle}{\langle E(r', t') \rangle}, \tag{4}\]

\[
g_2(r, r', t, t') = \frac{\langle E'(r, t')E'(r, t)E(r, t)E(r', t') \rangle}{\langle E(r', t') \rangle^2 \langle E(r, t) \rangle^2}. \tag{5}\]

In an infinite steady state system, such correlations clearly depend only on \(|r - r'|, t - t'|\), but since most experiments to date involve relatively small clouds, coherence depends also on the position within the cloud. In the following, except where explicitly noted, we consider coherence measurements of the incoherently pumped system, as this case has been studied more extensively.

2.3.1. Temporal coherence

Temporal coherence – i.e. \(g_1(t) \equiv g_1(r, r, t + \tau, t)\) can either be measured directly, using e.g. an interferometer (see Figure 2(d)), or can be inferred from the line-width of the polariton emission, by using the Wiener–Khinchin theorem to relate the power spectrum to the temporal correlation function. As such, increased temporal coherence should be seen by a line narrowing above threshold, which is indeed seen in experiments [60,64]. However, at yet higher powers, the line-width was then seen to increase. Such an effect, due to the phase noise induced by interactions between particles, had been anticipated [65,66], but the broadening observed in experiments [60,64] was later found to be due to an extraneous source of noise. In [60,64], a multi-mode pump laser was used, meaning that the intensity of injected polaritons fluctuated in time. Because of interactions between the condensate polaritons, and the higher energy excitonic states created by
the laser, noise in the pumping intensity translates to fluctuations of the energy of the condensate, broadening the line-width.

Experiments using a single mode pumping laser revealed a coherence time that increased above threshold and then remained constant [67] (see Figure 5(b)). These experiments showed a Gaussian time dependence of the correlation function which can be explained [68] by considering the dynamics of condensate density fluctuations, which in turn produce energy shifts, reducing the phase coherence. If one allows for a non-zero relaxation time for the density fluctuations $\tau_r$, then one has a contribution to the decay $g_1(\tau) \propto \exp(-U \rho_0 \tau_r^2 \exp(-\tau/\tau_r) + \tau/\tau_r - 1)$, where $U$ is the polariton–polariton interaction and $\rho_0$ is the condensate number density. The dependence on $\tau/\tau_r$ accounts for the spectrum of intensity fluctuations due to the non-zero relaxation time. This approach, along with [65,66] focused on the dynamics of the single condensate mode.

For an equilibrium two-dimensional condensate, the first-order coherence is expected to show power law decay of correlations as a function of either distance or time, with a form $g_2(t) \sim t^{-\eta} \eta = k_B T_m/2\pi \rho_0 h^2$, the same power law as discussed for spatial correlations in Section 1.1. Attempts to calculate the coherence decay in a non-equilibrium two-dimensional system have been made in [55,69]. In a finite system, these approaches eventually reproduce the Gaussian form of the single mode problem discussed above. Other approaches to calculating the coherence function have made use of simulations of the quantum kinetic equation [70].

Further information on the processes that lead to dephasing, and which thus control the coherence time, can be found by measuring the intensity–intensity correlation function, i.e. $g_2(\tau) \equiv g_2(r, r, t + \tau, t)$. For a thermal state, $g_2(\tau) = 2$ (by Wick’s theorem), while for a coherent state, $g_2(\tau = 0) = 1$, and $g_2(\tau)$ then tends toward 2 with increasing delay time. Measuring $g_2(0)$ therefore can be used as a confirmation of appearance or disappearance of a coherent state. Early measurements [71] were limited by their finite time resolution, meaning that the experiment actually records $(1/T_m) \int_0^{T_m} dt' g_2(\tau')$, where $T_m$ is the measurement time. This averaging makes interpretation of such results challenging. A combination of improved time resolution [72], and use of single mode rather than multi-mode pumping [67] as above gives a much clearer signature of $g_2(\tau)$, and allows extraction of the time-scale for relaxation of intensity fluctuation. Again, such results have been reproduced by single mode models [68] and by quantum kinetic approaches [70]. There have also been investigations of the growth and decay of the line-width, and degree of coherence, i.e. studying the time dependence of $g_2(r, r, t, t)$ following pulsed excitation [73].

2.3.2. Spatial coherence, $g_1(|r|) \equiv g_1(r_0 + r, r_0, t, t)$

Spatial coherence is investigated by interfering light from different points in real space. One way this can be studied is by interfering an image of the condensate with a rotated copy of the same image. As such, each point on the detector corresponds to light emerging from two different points on the sample, one from the rotated image, and one from the non-rotated image. By inserting a delay for one copy of the image, one then has either constructive or destructive interference, depending on the delay (as in Figure 2). The visibility of these interference fringes depends on the coherence between the light coming from the two different points on the sample. The fringe visibility thus provides a map of the coherence of the condensate. At the point on the detector corresponding to the centre of rotation, the light from both images comes from the same point on the sample and so the coherence is maximum. As one moves away from the centre, the distance between the two points on the sample increases, and the coherence decreases. Such a map of coherence, as measured in [60] is shown in the lower panels of Figure 5. Top: temporal coherence properties of CdTe when pumped with a single mode laser, from [67]. Reprinted figure 3 with permission from Love et al., Phys. Rev. Lett. 101 (2008), 067404. Copyright (2008) by the American Physical Society. Bottom: spatial coherence of CdTe, from [60], showing maps of fringe visibility as a function of displacement between source points on the sample. Left panel is below threshold, right figure is above threshold.
Figure 5. One should note that this technique gives a map of the unnormalised coherence, $G_1(2|\rho|) = \langle E(r, t)E(-r, t) \rangle$. An alternative way to study spatial coherence is with a Young’s double slit experiment [74,75], which examines how the fringe visibility is affected by the spacing between the two slits.

In most of the experiments to date, the small size of the polariton condensate has made it hard to extract useful information about the decay of spatial coherence (e.g. the coherence map shown in Figure 5 mainly reflects the density profile of the condensate (see Section 3 for a discussion). However, recent experiments with higher quality samples, having both less disorder, and longer polariton lifetimes, have allowed for notably larger condensates [76] in which coherence can be measured on sufficient length-scales to investigate the functional form of decay of spatial coherence.

3. Spatial pattern formation

Spontaneous emergence of spatio-temporal order – pattern formation – in non-equilibrium systems represents one of the key mechanisms of self-organisation in nature [77]. Many different physical systems (chemical, biological, hydrodynamical, optical, etc.) are described by similar order parameter equations and, therefore, may share similar pattern forming properties. Often, two types of patterns are distinguished: localised (coherent) structures and extended (possibly periodic) solutions.

There are several aspects of polariton condensates that suggest that they are capable of pattern forming. Polariton condensates are non-equilibrium steady states, meaning that spatial inhomogeneity generally implies the existence of steady state currents, which in turn modify the spatial pattern. Polaritons also have a non-trivial dispersion, with a point of inflection (see Section 4.1), meaning that the coherently pumped system can display interesting solitonic structures, that result from a complex interplay between dispersion, dissipation, forcing and nonlinear interactions. Finally, polaritons can live in a non-trivial potential landscape, either due to intrinsic disorder in the material, or due to deliberately designed potentials. These potentials may induce stabilising (or destabilising) currents further facilitating symmetry breaking in the system. Both coherently and incoherently pumped systems can show (different kinds of) interesting pattern formation, and so both will be discussed below.

Much of this spatial pattern formation can be understood in terms of the complex Ginsburg–Landau equation (cGLE) [15,78] – the universal equation that describes the behaviour of systems in the vicinity of an instability and symmetry-breaking. The cGLE for the order parameter $\psi(r, t)$ takes a generic form:

$$i\partial_t \psi = c_1 \nabla^2 \psi + c_2 |\psi|^2 \psi + c_3 \psi,$$

(6)

where $c_1$, $c_2$ and $c_3$ are complex parameters.

For an equilibrium condensate system $c_1$, $c_2$ and $c_3$ are real, leading to an ubiquitous equation of nonlinear physics called the nonlinear Schrödinger equation (NLSE) (also known as the Gross–Pitaevskii equation). For a weakly interacting Bose gas, this equation can be derived microscopically from the Heisenberg representation of the many-body Hamiltonian. To emphasise the generic nature of this fundamental equation we introduce it here by using a fully classical argument. From general considerations of a symmetry breaking transition in an equilibrium system, one can consider an energy functional that reaches a minimum in a stationary state. The energy of this state can be written as $E = \int L \text{d}r$, where the Lagrangian $L$ can be expressed through the order parameter field $\psi$ and its spatial derivatives. The Lagrangian containing only algebraic functions of $\psi$ and of its first derivative is called the Ginsburg–Landau (GL) Lagrangian. The lowest order (i.e. simplest) GL Lagrangian that corresponds to a state of broken symmetry, and preserves the symmetry of the system under global phase rotations of the order parameter $\psi$, is

$$L = \frac{1}{2} |\nabla \psi|^2 + \frac{1}{4} (1 - |\psi|^2)^2.$$

The quartic potential represents the simplest form for which the disordered state $|\psi| = 0$ can become unstable leading to spontaneous symmetry breaking and a new ordered state (corresponding to $|\psi| = 1$) emerges. The dynamic equation of motion is then obtained directly as the Euler–Lagrange equation coming from this Lagrangian yielding the NLSE

$$-i\partial_t \psi = \nabla^2 \psi + (1 - |\psi|^2)\psi.$$

(7)

It is also instructive to see that the NLSE can be obtained as a non-relativistic limit of the Klein–Gordon equation – the simplest equation consistent with special relativity [17].

In non-equilibrium systems, such as exciton–polariton condensates, $c_1$, $c_2$ and $c_3$ can become complex with the imaginary parts representing the processes of pumping and dissipation. The equation for the macroscopically occupied polariton state $\Psi(r, t)$ becomes:

$$i\hbar \partial_t \Psi = \left[ E(i\nabla) + U|\Psi|^2 + V(r) \right] \Psi$$

$$+ i \left[ \frac{P_{coh}(r,t)}{} + \left( P_{inc}(r) - \kappa - \sigma |\Psi|^2 \right) \Psi \right] \Psi,$$

(8)
3.1. Condensation at finite momentum

A particularly simple way to create a non-trivial spatial pattern in a condensate, and one which in fact was unintentionally realised in several early experiments is to pump with a sufficiently small pumping spot, and with the exciton–photon detuning close to, or just greater than zero [80]. This system has been studied both with quantum kinetic equations [81] and with the above Ginsburg–Landau approach [82]. In the latter approach, one can understand transparently the reason for the finite momentum condensation: due to repulsion the high density at the centre of the pump spot shifts the polariton energy. If one wants to find a steady state profile \( \Psi(r, t) = \Psi_0(r) \exp(-i\mu t) \), then one is forced to have \( \mu = U|\Psi_0(r = 0)|^2 \) at the centre of the spot. Away from the centre, the density decreases (as loss exceeds pumping), and so to maintain the same energy, one requires \( \Psi_0(r) \simeq \exp(ik \cdot r)|\Psi_0(r)| \) with \( h^2k^2/2m_{\text{pol}} = \mu - U|\Psi(r)|^2 \). Alternatively, this can be re-interpreted as strong repulsion repelling polaritons, giving them a finite outward velocity, thus producing a condensate at a ring of finite momentum values. The two descriptions are equivalent. Note that the finite momentum ring still shows features such as coherence [80], in that light emitted at different azimuthal angles (from polaritons with different directions of in-plane momenta) shows interference fringes. Similar condensation at finite momenta has also been seen recently in a clean, long polariton lifetime, 1D microcavity structure [76]. In this case, for some conditions of pumping, more than one condensate was seen at once; we will discuss this below in Section 3.2.

3.2. Multimode coexistence

In the thermal equilibrium condensed state of weakly repulsive bosons, there is only one macroscopically occupied mode, i.e. there is no ‘fragmentation’ [15]. Many experiments on incoherently pumped polariton systems have however shown simultaneous coexistence of several macroscopically occupied modes [67,76,83,84]; i.e. strong emission is visible at a number of distinct frequencies, indicating a state \( \Psi(r, t) = \sum_n \Psi_n(r) \exp(-i\mu_{nt}) \). Spectrally resolving the emission and then determining the real and momentum space profiles of the resultant signal allows one to determine the functions \( \Psi_n(r) \). These mode profiles show that the occupied modes are related to the series of single particle modes in a disorder-induced effective trap (for experiments on CdTe, as in [67,83,84]), or from different plane-wave-like states in a 1D waveguide geometry (for the cleaner GaAs system of [76]). There is, however, evidence in momentum space images of these condensates that the occupied modes are strongly
influenced by pumping and decay. Without pumping and decay, the system would be time-reversal symmetric, and so the real-space wavefunctions can be made real, implying that $\Psi(k) = \Psi^*(−k)$, and so the intensity pattern in momentum space, $|\Psi(k)|^2$ should show mirror symmetry. Measurements of the momentum space profile of the coexisting macroscopically occupied states shows an asymmetry in momentum space [84,85]. Thus, the modes occupied in such a multimode condensate seem to be related to the single particle modes, but modified by the particle flux due to pumping and decay.

To understand what conditions are required for coexistence, one may first consider competition between the different single particle states which are fed by the same reservoir of incoherently created excitons [86]. In this approach, multimode coexistence is possible if the density profiles of the modes have sufficiently small overlap that they can be simultaneously fed by different spatial regions of the reservoir. There are, however, other features that can affect whether multimode coexistence will occur: synchronisation of modes, and relaxation effects.

Synchronisation refers to the fact that if particles can coherently move between the different spatial mode profiles, then this leads to a term that favours two different spatial modes having their phases lock together, and oscillating at a common frequency [87]. To describe this, suppose one starts with a system $\Psi_1(r)\exp(−i\theta_1) + \Psi_2(r)\exp(−i\theta_2)$, where a simple coexistence of independent modes would have $\theta_1 = \mu_1t$, $\theta_2 = \mu_2t$, then the question of whether two potentially coexisting modes synchronise or not depends on whether $\theta_{12} = \theta_1 - \theta_2$ takes a fixed value, or a time dependent value. The equation satisfied by $\theta_{12}$ can be reduced to the form [87,88]:

$$\frac{d^2\theta_{12}}{dt^2} + \gamma \left( \frac{d\theta_{12}}{dt} - \Delta_{12} \right) = -ju(\bar{n}_1\bar{n}_2)^{1/2}\sin(\theta_{12}),$$

where $\gamma$ is a damping term arising from the pumping and decay rates, $\Delta_{12}$ is the energy difference between the single particle states, $J$ is the rate of tunnelling between the two states, $\bar{n}_{1,2}$ are the average populations of the two modes and $u$ is an effective average interaction strength between particles in one state and particles in the other. If $\Delta_{12} > ju(\bar{n}_1\bar{n}_2)^{1/2}$ then $\theta_{12} \simeq \Delta_{12}t$, and one has multimode coexistence, but for small enough $\Delta_{12}$, the modes lock together. In this case, there can, however, be interesting dynamics of collective oscillations of polaritons between the two states after a pulsed excitation; this has been investigated using disorder localised states in CdTe [89].

Recent work has also begun to investigate the effect of energy relaxation on the possibility of multimode condensation. In its simplest form, the cGLE does not account for relaxation, in that the extent to which modes are populated does not necessarily favour lower energy modes. Experiments on extended 1D waveguides appear to suggest a process of relaxation is important, in allowing multimode condensation of modes that have no overlap with the pump spot, but which might couple via intermediate macroscopically occupied states [76,90]. Such relaxation can be incorporated in the cGLE by adding a term proportional to $−i\partial_t\Psi$ inside the second bracket on the right-hand side of Equation (8) [91]. This approach is very similar to how relaxation has been incorporated to account for the interactions of a superfluid with a thermal cloud [92] in superfluid helium.

### 3.3. Vortices

In equilibrium condensates, the observation of quantised vortices has been viewed as evidence for a macroscopically occupied quantum state, as such a state can only be made to rotate by inserting phase singularities, i.e. vortices, in the wavefunction. A superfluid condensate that has been set into rotation also demonstrates the ability of a superfluid to show persistent metastable flow; i.e. to remain in a metastable state for astronomically long times if the transition to the ground state would involve introducing phase twists into the macroscopically occupied wavefunction. In polariton condensates, several other aspects of vortices and rotation are interesting: in particular the interplay between steady state currents and vortex formation, and the ability to induce vortices by appropriate pumping, for both coherently and incoherently pumped systems.

#### 3.3.1. Vortices with incoherent pumping

Numerical simulations of an incoherently pumped system, with toroidal pumping and no trapping potential have suggested it should be possible to induce a rotating polariton condensate if it is seeded with a sufficiently strong coherent pump to inject angular momentum. The rotation then persists for some time until vortex motion out of the polariton cloud stops the rotation [93]. Other numerical simulations have suggested that in the presence of a harmonic trapping potential, one can in fact engineer situations where the reverse behaviour occurs – the non-rotating solution is made unstable by the current flow in the inhomogeneous density profile, and the solution with vortices is the stable solution [94]. Figure 6(a) shows the simulation starting from an originally non-rotating cloud, with vortices moving in from the edge of the polariton cloud. Experimentally, vortices have been seen in incoherently pumped systems, almost certainly arising due to current
flows caused by the intrinsic disorder potential in CdTe [95]. Such quantised vortices can be clearly observed in interference images (see schematic experiment in Figure 2), where phase winding at a point leads to a fork in the interference pattern, see Figure 6(b). In contrast with equilibrium trapped condensates, in which vortices spiral out of the condensate due to the Magnus force (force in the direction orthogonal to the density gradients) and interactions with the thermal cloud [96], the exciton–polariton condensates’ vortices are stabilised (pinned) by inward fluxes at the local minima of the disordered potential.

3.3.2. Vortices and the polarisation degree of freedom

If one accounts for the polarisation degree of freedom of polaritons then one may consider more interesting kinds of vortex. The simplest description of a polarised polariton condensate neglects weaker effects such as the frequency difference between photon modes with transverse electric and transverse magnetic polarisations (TE–TM splitting), and the anisotropy whereby the strain field splits the energies of different linearly polarised exciton states. This leaves, however, a strong effect due to interactions between polaritons: the repulsion between opposite polarisation polaritons is weaker than that between equal polarisation polaritons. Therefore, the ground state has equal populations of left- and right-polarised polaritons; i.e. it is a linearly polarised state [97,98]. If one then considers vortices on top of this polarised state, one finds that the minimum energy configuration for a single vortex, balancing the interaction energy with kinetic energy, is a ‘half vortex’ [99], i.e. either a vortex of either left- or right-circular polarisation.

The left- and right-circular vortices are not however entirely independent; the TE–TM splitting in a cavity adds a term $H_{TE-TM} \propto \Psi^\dagger \left( \partial_x - i \partial_y \right)^2 \Psi + \Psi^\dagger \left( \partial_x + i \partial_y \right)^2 \Psi$ to the Hamiltonian of the equilibrium system. This produces a long-range interaction between left- and right-vortices with opposite circulation (i.e. between a left vortex and right anti-vortex, or vice versa) [100]. Experiments have seen such individual half vortices of left- and right-circular polarisation as well as aligned half vortices of opposite circulation at various fixed positions in a disorder potential [101]. These experiments studied the interference patterns of light after it had passed through a polarising filter, thereby observing that there are points where the left-circular light has a vortex but the right-circular light has no vortex, or vice versa. Any energetic splitting between linear polarisation states can destabilise half vortices; as mentioned earlier, such a splitting can arise due to strain fields, and may also occur intrinsically in quantum wells in non-centrosymmetric semiconductors. The existence of a disorder potential and associated supercurrents may help to stabilise half vortices. It also was shown numerically [88] that it is possible to stabilise the co-existing right- and left-circular vortices of opposite circulation, by applying a magnetic field perpendicular to the cavity. A full understanding of the conditions under which half vortices are stable, and the features present in the experiment which allows their observation, remains an open question [102,103].

3.3.3. Vortices with parametric pumping

With coherent pumping in the OPO configuration, since there is a free phase between the phase of the signal and that of the idler, it is possible to seed vortices in this relative phase. This means that one can have a situation where, e.g. there is a vortex in the signal beam, and an associated anti-vortex in the idler [104]. Experiments have shown that a vortex in the signal can be observed in the presence of a probe beam containing a vortex at either the signal or idler momentum [85]. Further experiments have also studied the behaviour following a pulsed probe.
beam that seeds a vortex [105]. If such a pulsed probe is injected in a coherently pumped system below threshold (i.e. with a pump power too low to show OPO), then a large transient amplified signal is seen, and the vortex survives as long as the transient signal. If the system was pumped above threshold, then the vortex may either survive for long times, or decay (by escaping out of the pump spot). To create a long lived vortex in this way requires a sufficiently strong probe.

3.4. Confining polaritons

One way to explicitly manipulate the pattern creation is to control the external potential \( V(r) \) by producing a spatial variation of either photon or exciton energy. To create a spatial variation of the photon energy one can modify the mirrors. Varying the width of the cavity, one can create shallow in-plane traps for photons [106]. One can also shift the photon energy by adding metallic layers on top of the Bragg mirrors [75]. Very strong confinement of photons can also be produced by etching to produce a narrow pillar [107], and relying on dielectric contrast for reflection from the edge of the pillar. A spatial variation of exciton energy can be produced by applying localised stress to the sample, which modifies the electronic band structure, and thus changes the energy of the excitonic states [108].

A different approach to engineering an effective trapping potential is to use the repulsion between polaritons to dynamically create an effective potential [76,109,110]. For incoherent injection, there is typically a large population of reservoir excitons at high energies, and the low energy polariton states will be repelled by these states, as well as by other low energy polaritons. Thus, a spatial profile of the pump field has two effects: it both creates a spatially dependent source for the polariton condensate and also creates a potential that affects the polariton condensate.

Even when no potential is deliberately created, there may be an effective potential due to disorder. This can either come from spatial variation of the exciton energy, i.e. roughness of the quantum well interfaces [111], or from spatial variation of the photon [112] energy, roughness of the Bragg mirrors. While both excitonic and photonic disorders are present, the photonic disorder seems to play the dominant role in trapping polaritons in most current experiments; the relative role of both types of disorder is reviewed in [113].

4. Superfluidity

One of the more remarkable features of quantum condensates is superfluidity; the ability of an interacting condensate to flow without mechanical resistance, as long as the flow is below a critical velocity. This occurs because the condensate is unable to respond to the transverse drag force exerted by the walls of a container. This in turn arises from a combination of two effects. Firstly, as mentioned in the introduction, the condensate can only be made to rotate by creating vortices, and so it is not able to respond to the transverse force arising from the wall. Secondly, as will be discussed further below, the spectrum of single particle excitations in the presence of a condensate is linear, defining a critical velocity. For speeds lower than this critical velocity, it is not energetically favourable to create excitations. At non-zero temperatures, a population of quasi-particle excitations will however already be present, and these quasi-particles may respond to drag forces. This leads to a two-fluid description, where the behaviour of the system can be described by a combination of a normal and a superfluid part. The fact that the condensate can only be made to rotate by exciting vortices also means that if one has a non-simply-connected geometry (e.g. a ring of superfluid) that is made to rotate, and then cooled below the transition, the condensate persists in a rotating state.

In an equilibrium interacting condensate, all the phenomena named in the previous paragraph are seen together as aspects of superfluidity (for a more complete review of aspects of equilibrium superfluidity see e.g. [16,114]). However, in non-equilibrium polariton condensates it is not necessarily the case that these aspects need all appear together. For example, the excitation spectrum of single particle excitations can be significantly modified by finite particle lifetime, requiring a re-defining of a critical velocity, yet, as already discussed above, quantised vortices can still be clearly observed in polariton condensates. Table 1 shows a possible checklist of different aspects of superfluidity shown by equilibrium condensates, and what has been observed in both coherently and incoherently pumped polariton systems. Some of these aspects of superfluidity, such as the observation of quantised vortices, and the possibilities for solitary wave propagation have already been discussed in Sections 3.3 and 3.1, respectively. The remainder of this section will discuss various other aspects of superfluidity that have been studied in polariton condensates.

4.1. Spectrum

In an equilibrium condensate, the Bogoliubov spectrum comes from considering fluctuations of the form \( \Psi(r,t) = \exp(-i\epsilon_{k}\tau/h)[\Psi_{0} + \sum_{k}\Psi_{k}\exp(-i\epsilon_{k,t} + ik \cdot r) + v_{k}\exp(i\epsilon_{k,t} - ik \cdot r)] \), and finding a self-consistent set of equations for \( \Psi_{k} \) and the frequency \( \epsilon_{k} \). Substituting this ansatz for fluctuations into Equation (8) without any pumping or decay, and with a quadratic
Table 1. Superfluidity ‘checklist’, adapted and updated from [115], showing the phenomena expected to be seen, or that have been seen in different classes of potentially superfluid systems (see also [116]).

<table>
<thead>
<tr>
<th></th>
<th>Quantised vortices</th>
<th>Landau critical velocity</th>
<th>Metastable persistent flow</th>
<th>Two-fluid hydrodynamics</th>
<th>Local thermal eqbm.</th>
<th>Solitary waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superfluid $^4$Hecold atom BEC</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Non-interacting BEC</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
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<td>Classical irrotational fluid</td>
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<td>Incoherently pumped polariton condensates</td>
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<td>Parametrically pumped polariton condensates</td>
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dispersion, the result for small $k$ is the real mode-energy spectrum $\hbar^2 \omega_k \simeq \hbar k (\mu/m_{pol})^{1/2}$ with $\mu = U \langle |\Psi_0|^2 \rangle$. This linear dispersion defines the critical velocity such that the energy in a moving frame $\xi_k = \xi_0 + \mathbf{v} \cdot \mathbf{k}$ remains positive.

The spectrum of single particle excitations in a nonequilibrium polariton condensate has been theoretically studied extensively, both for incoherent [69,117] and parametric pumping [55,118]. The basic behaviour is quite similar for both cases because both involve the same ingredients, of having a free phase (such as that between the signal and idler modes) and a finite particle lifetime. For incoherent pumping, with a net pumping rate $\eta$, the above linear dispersion becomes instead $\hbar^2 \omega_k \simeq -i \hbar \eta + \sqrt{\hbar^2 (k^2/m_{pol}) - \hbar^2 \eta^2)}^{1/2}$. This form means that the real part of the spectrum is zero for $k < \eta (m_{pol}/\mu)^{1/2}$. The diffusive behaviour at small $k$ means that a simplistic identification of superfluidity as a property of the real part of the spectrum $\xi_k$ would imply that the non-equilibrium system is not superfluid. However, since the separate aspects of superfluidity are not necessarily related in the same way as in equilibrium, such a statement is premature.

There is an important difference in the role of the spectrum between equilibrium and non-equilibrium condensates. In equilibrium, one is interested in the energetic criterion of whether creating quasi-particles costs or gains energy; in a general non-equilibrium case, the question is rather whether superfluid flow is dynamically stable [119]. If one allows a degree of thermalisation by adding a relaxation mechanism, the energetic and dynamic criteria for stability would become linked [91], in that the combination of relaxational dynamics and excitations which would reduce energy can combine to give a dynamical instability above a critical velocity.

Experiments have only begun to explore the spectrum, and as yet do not have sufficient resolution to clarify differences between equilibrium and non-equilibrium spectra. The experiments so far are consistent with there being a change between a quadratic spectrum in the non-condensed state and a Bogoliubov-like form when condensed [120].

4.2. Scattering from disorder

A more direct probe of superfluidity is to consider the behaviour of polaritons flowing in a disordered potential, and ask whether the polaritons suffer frictional drag. For flow past a weak potential, there will be a simple connection between the observed behaviour, and the spectrum discussed above, in that the linear response of the polariton system to the weak potential will be involved. However, it is experimentally simpler to investigate scattering off a strong disorder potential, which can complicate the interpretation of experimental results. A further source of complication is that even if the system were partially superfluid (i.e. superfluid, but not at zero temperature, so a two-fluid description was required), then the observed behaviour would be a mixture of superfluid and normal behaviour. There would thus still be drag on a defect, and similarly there would still be some scattering off disorder.

Experiments on such scattering have only been performed with resonant pumping, and in two quite distinct experimental configurations, which we will describe.

4.2.1. Polariton ‘bullets’ in parametrically pumped systems

The experiments in [121] considered a parametrically pumped system (i.e. an injected signal), below the threshold for OPO. On top of a steady pump beam, a weak idler beam was injected, which caused stimulated parametric scattering to the signal and idler states. The packet of polaritons in the signal state can be created with a non-zero group velocity, and thus be made to propagate through a larger region where the pump exists. One can then study how such packets of polaritons interact with disorder. No scattering is seen, and the packet maintains a well-defined single group wavevector.

4.2.2. Rayleigh scattering of pump beam

An alternate approach to studying how the excitation spectrum affects the propagation of polariton
wavepackets can be found by considering a different experiment. The experiment in [122], following a proposal of Carusotto and Ciuti [123], uses a single pump beam, in a configuration where there is neither a Kerr nor parametric instability, and so the polariton population rises smoothly with pump power. When such a polariton beam interacts with disorder, it can only scatter to other momentum states if such states exist. Because of the nonlinear interactions among polaritons, at large enough densities, the spectrum may become linear around the pump wavector and thereby remove other states that the system might scatter into. In experiment it was observed that for the range of pump wavevectors and pump intensities where this linearisation should occur, scattering off disorder disappeared.

5. Conclusions and outlook

In the last few years, experiments have gone beyond simply attempting to prove that exciton–polariton condensates exist, and instead have begun to explore the properties they display. Future experiments seem likely to further explore these properties. There are also likely to be significant developments in several areas we have not been able to discuss here. One direction concerns polariton condensation in organic materials supporting polaritons, which have recently shown polariton lasing [124,125]. These materials are interesting both because the exciton states involved are rather different from those in inorganic semiconductors, being far more localised, and because they offer the promise of condensation at far higher temperatures. There are also inorganic materials, wide bandgap semiconductors such as ZnO and GaN, which have much stronger exciton–photon coupling, and so offer the possibility of stable polaritons at room temperatures [126–128]. These materials would therefore offer the possibility of making collective quantum coherence far easier to realise. It is therefore imperative to understand what kind of coherence these systems do produce, and how this may be used. (Note added in proof: very recently experiments on organic dyes in optical cavities with weak light-matter coupling have reported a thermalised “photon condensate” at room temperature [J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Bose–Einstein condensation of photons in an optical microcavity, Nature 468 (2010) p. 545].)

Another subject which we have only briefly mentioned is that of studying effects associated with the polarisation degree of freedom of the polaritons. In Section 3.3.2, we mentioned one effect of the polarisation degree of freedom, that of allowing the possibility of having separate vortices of left and right polarisation. In addition, there can be interesting effects where an applied magnetic field favours circular polarisation, and thus competes both with the interactions that favour linear polarisation, and with any anisotropy from strain fields [88,131–133]. There are also a rich variety of phenomena associated with spin dynamics in the process of parametric scattering. These arise from competition between a number of effects: polarisation rotation due to the anisotropy and TE–TM splitting, polarisation rotation due to the effective magnetic field produced by polariton–polariton interactions, and spin-dependent rates of parametric scattering which arise inevitably from the spin-dependent polariton–polariton interaction. The variety of behaviour that can be seen due to these effects are discussed extensively in [7].

In this review we have discussed many experimental observations and theoretical predictions of various phenomena that take place in exciton–polariton condensates. Due to the coupled light–matter nature of exciton–polaritons, they are capable of showing behaviour both related to that of equilibrium condensates and lasers. They can also show kinds of behaviour that differ from both of these systems, as manifested, for instance, by the dynamics of vortices, the temporal coherence properties, or the behaviour seen in the parametrically pumped system. The finite polariton lifetimes puts these condensates in a regime distinct from cold atomic gases or superfluid helium. At the same time, the behaviour is not that of a simple laser, as can be seen from the lack of population inversion, and the clear effects of polariton–polariton interactions. By differing from both the simple laser and the equilibrium condensate, exciton polariton condensates offer the opportunity to study novel aspects of condensation, pattern formation and superfluidity. They, therefore, prompt profound questions about what superfluidity can mean in a non-equilibrium system, and offer a venue to explore the range of conditions under which collective quantum coherence can be achieved.

Notes

1. Earlier experiments had also seen aspects of this behaviour, e.g. [134].
2. Given that the reservoir states of higher energy excitons have a much higher mass than the polaritons, and given the existence of disorder in typical quantum wells, it is reasonable to assume that the reservoir excitons do not diffuse, thus the dynamics of the reservoir, balancing creation by the pumping laser with scattering into lower energy polariton states, can be assumed to be spatially local.
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References


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