FREE BOUNDARY PROBLEMS AND APPLICATIONS. (L16)

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The mathematical treatment of processes occurring in nature and industry involves the creation of models that are often translated into equations involving the partial derivatives of the function(s) that describe the process. However, the traditional framework of solving PDEs on fixed domains is sometimes insufficient - due to the nature of many processes, the domains on which the equations should be solved move or change. These fall into the category of Free Boundary Problems (FBPs).

During this course we will familiarize with the modeling of such processes, including most of the following:

- Melting and solidification
- Deformation of a membrane
- Image development and processing
- Tumor growth
- Superconductivity
- Jets and cavities
- Put call options and derivatives in finance.

Another important goal of the course is to familiarize with a selection of important PDE and analysis methods and learn how to apply them on the example of FBPs. These might include: Green's formulae and boundary value problems, second order elliptic PDEs, maximum and comparison principles, Harnack inequality, facts from functional analysis, function spaces, some techniques from variational inequalities used to prove the existence, uniqueness and regularity of the solutions.

The lectures will be supplemented by example sheets and example classes.

Desirable Previous Knowledge

The course will be designed to be mostly self contained, still some basic knowledge of analysis and a ODE/PDE courses will be required.

Introductory Reading

A. Friedman. Free boundary problems in science and technology. Notices Amer. Math. Soc. (2000) vol. 47 (8) pp. 854-861

Reading to complement course material

Due to the nature of the course a variety of of sources will be used, and the lecture notes will be complemented by hand outs and photocopies of review articles.

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