Pure Maths fun

crazy mathematical ideas?
what good are they to us?

Newnham Maths Summer School

Orsola Rath Spivack
University of Cambridge

April 17, 2021

Outline

1. What is mathematics about?
2. Crazy mathematical ideas
   - Crazy ideas in history
3. Maths that is good for us

Mathematics and mathematicians

Have you heard some people say

- Mathematics is boring
- Mathematics is not creative ("I don't do maths, because I'm a creative person!")
- Mathematics is not useful, like engineering, medicine, architecture, ...

... but we know better!

Mathematics and mathematicians

Mathematicians say:

- "Mathematics is not a contemplative but a creative subject." (G.H. Hardy)
- "It is by the aid of statistics that the law in the social sphere can be ascertained and codified." (Florence Nightingale)
- "Mathematics is no more computation than typing is literature." (John Allen Paulus)
- "It is impossible to be a mathematician without being a poet in soul." (Sofia Kovalevskaya)

Mathematics and mathematicians

Mathematicians say:

- "Mathematics is not a contemplative but a creative subject." (G.H. Hardy)
- "It is by the aid of statistics that the law in the social sphere can be ascertained and codified." (Florence Nightingale)
- "Mathematics is no more computation than typing is literature." (John Allen Paulus)
- "It is impossible to be a mathematician without being a poet in soul." (Sofia Kovalevskaya)

- "Therefore, o student, study mathematics and do not build without foundation" (Leonardo da Vinci)

What is mathematics about?

- Mathematics is about spotting patterns (*e.g. about numbers: try to predict the last digit of* 2^n),
  - then making a guess,
  - then proving universal truths.
  - then generalise to other, perhaps more abstract objects (i.e. let your imagination run wild!)
  - also use the new truths, to improve engineering, medicine, architecture, ...

- "In Mathematics the art of asking questions is more valuable than solving problems" (Georg Cantor)

- "Math. It's just there ... You're either right or you're wrong. That's what I like about it." (Katherine Johnson)

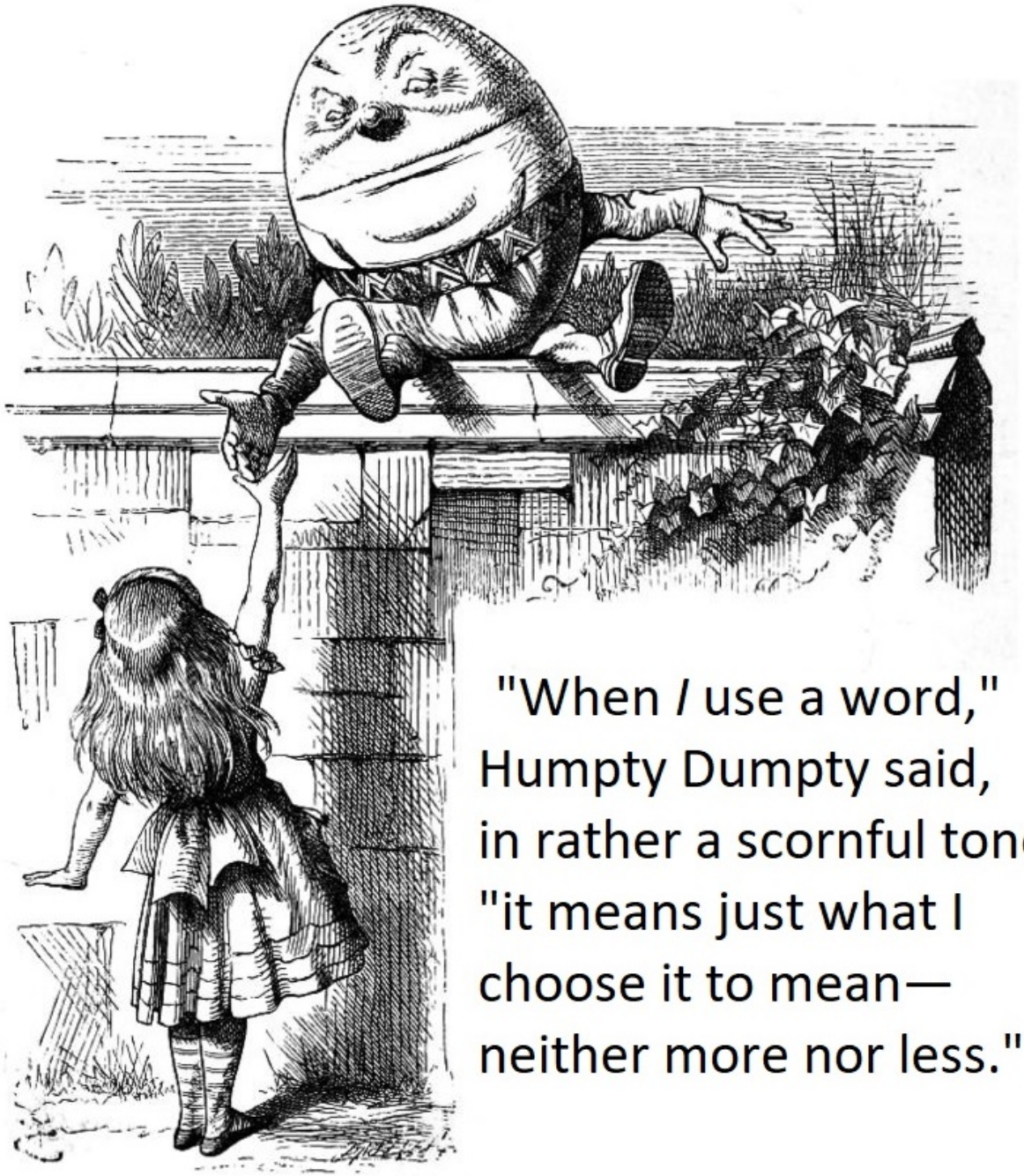
What is mathematics about?

Maths is fun!

but it's been sometimes a bumby ride:

- some concepts that are for us just natural, and are learnt in primary schools, were considered completely crazy - or worse - when first introued!
- some mathematical ideas that are now used in very practical everyday applications (e.g. video games), seemed very abstract ideas with no practical use when first introduced!

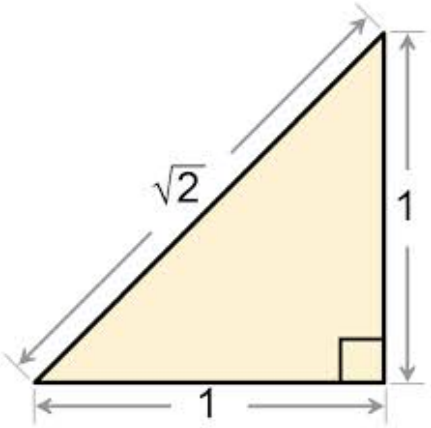
Whole new worlds



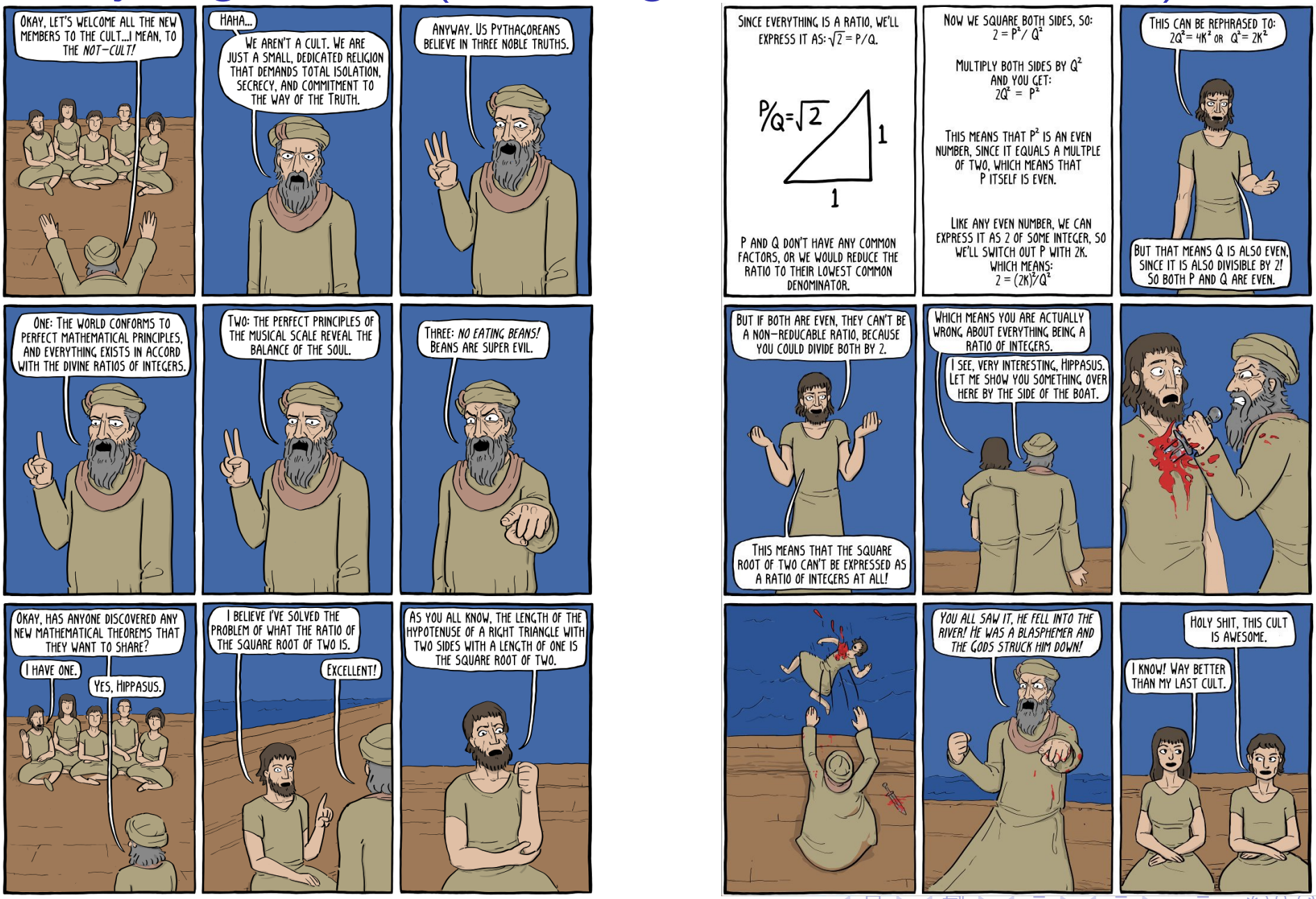
Irrational numbers

The dangerous ratio!!

- Pythagoras (c.570-c.495 B.C.E.)
  "All is number" the world explained by the 'divine ratio of integers'
- Hippasus (c. 500 B.C.E.)
- $\sqrt{2}$ is not a ratio of integers - blasphemy! (prove it!)
- irrational numbers and their uses
  - π
  - Euler's number e
  - the Golden Ratio $\phi = \frac{1+\sqrt{5}}{2}$



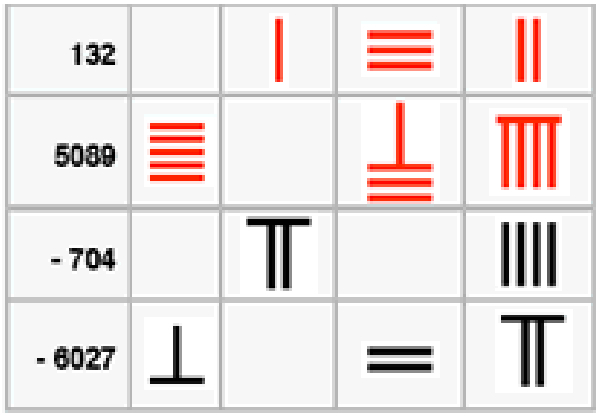
The Pythagoreans - (according to Existential Comics)



Negative numbers

" Negative numbers darken the very whole doctrines of the equations and make dark of the things which are in their nature excessively obvious and simple." (Francis Maseres, as late as 1758!)

- The Chinese (c. 200 BCE), *buying & selling, taxes ...*
- Diophantus (c. 200-c.284) solution to $4 = 4x + 20$ is 'absurd' !
- Brahmagupta (c.598 - c. 668) first set of rules for operations with negative numbers, and definition of the number 0

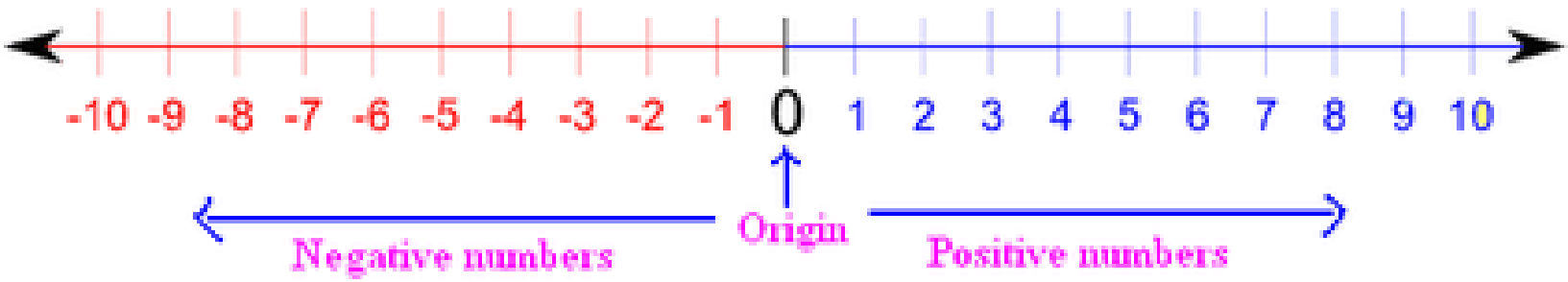


- Al-Khwarizmi (800–850) used negative numbers as 'debts' and fortunes in inheritance calculations, but considered equations which have a negative solution as 'meaningless'.
- Girolamo Cardano (1501-1576) used negative solutions, but still called positive numbers 'real numbers', and negative numbers 'fictitious numbers' !

Negative numbers

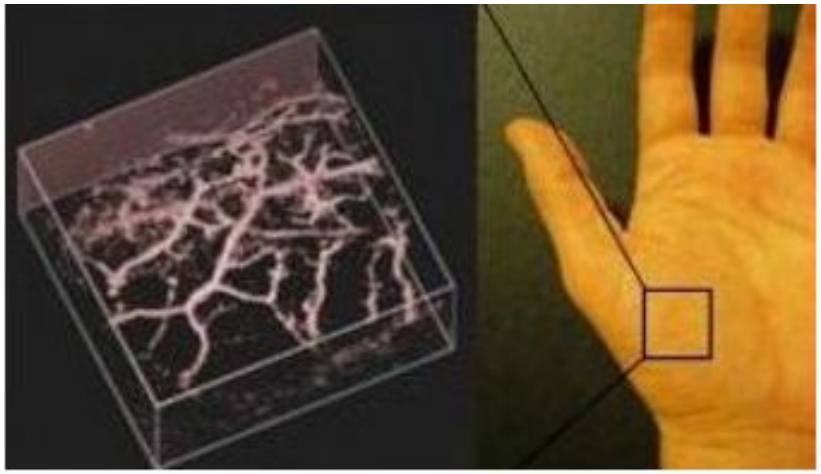
Towards accepting negative numbers.

- Bhaskara (c.600 - c.680) first *positional representation* of numbers
- John Wallis (1616 - 1703) - giving meaning to negative numbers by inventing the number line



Maths that is good for us

Maths is everywhere!


Maths that explains all the physics

- Emmy Noether (1882 - 1935)
- "Yesterday I received a very interesting paper on invariants from Miss Noether. I'm impressed that these things can be seen in such a general way. It would do the old guard at Göttingen no harm to be sent back to school under Miss Noether. She knows her stuff." (*Albert Einstein, 1918*)
- Noether's Theorem:
  symmetry $\Leftrightarrow$ conservation law

Not enough time today to see this!

Thanks!