Direct and Inverse Scattering of Waves

Part III course

Dept. of Applied Mathematics & Theoretical Physics
University of Cambridge, UK

Why bother?

UNIVERSITY OF CAMBRIDGE

Applications (some, direct)

- **Ocean acoustics**
  - **effect of noise on sealife, structures**
  - **efficient transmission**
  - **modelling turbulent medium**

- **Acoustics of urban environment**
  - **distribution of noise amongst buildings**
  - **scatter from surfaces – reduce noise, glare**

- **Extended random media**
  - **equations for the moments of the field**
  - **scintillation index**

UNIVERSITY OF CAMBRIDGE

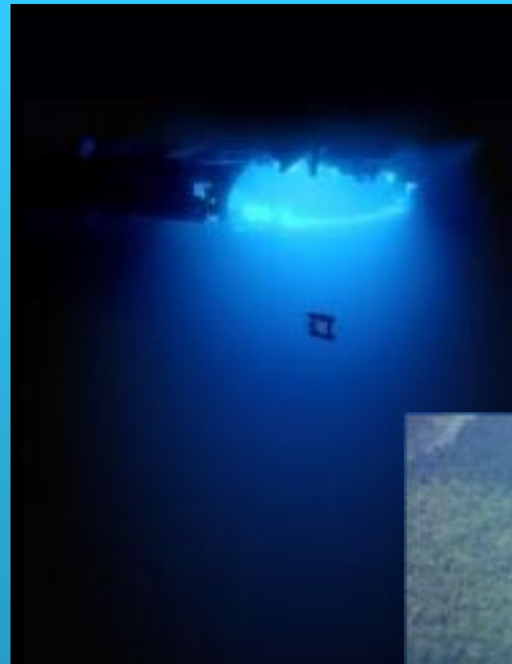
Applications (some, inverse)

- **Ocean acoustics**
  - reconstructing temperature changes (ATOC), monitoring icesheet, climate monitoring
  - submarine detection, monitoring seabed

- **Remote sensing**
  - road surface surveys, monitoring forest canopy, detecting and tracking oil slicks
  - scintillation of stars

- **Imaging**
  - medical ultrasound imaging, lithotripsy
  - nondestructive testing
  - landmine detection

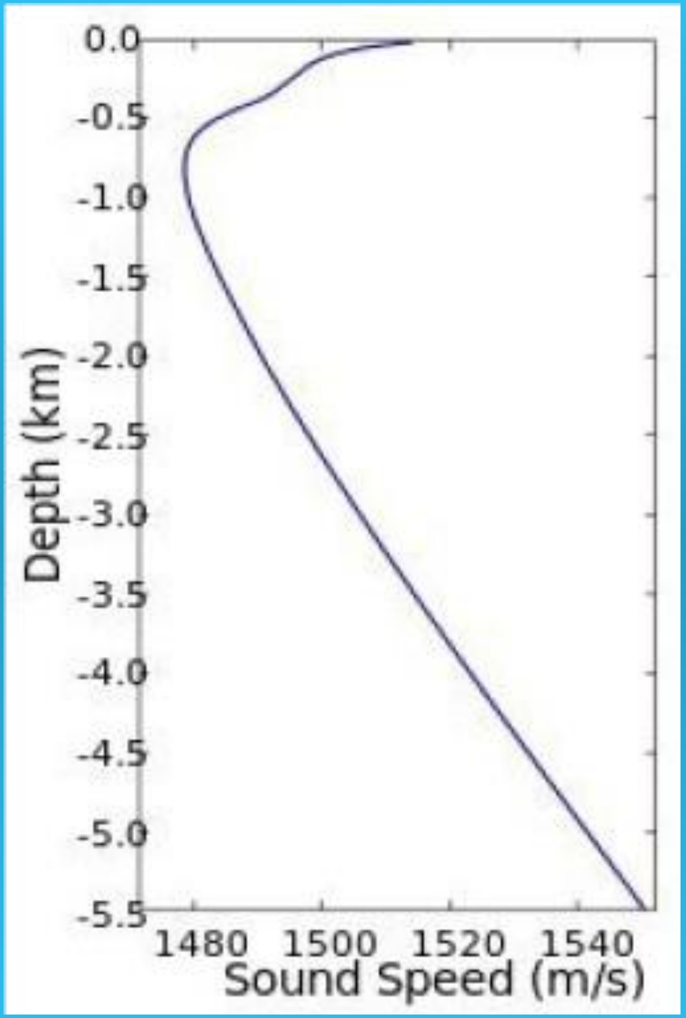
UNIVERSITY OF CAMBRIDGE



UNIVERSITY OF
CAMBRIDGE

The SOFAR channel

- Speed of sound affected by
  - temperature, pressure, ...
  - 20 Hz wave: 75m λ in water
    17m λ in air
- Axis of SOFAR channel:
  - varies from ~600m below
    to ~1200m below
    the sea surface
-



UNIVERSITY OF CAMBRIDGE

Topics in Part I of the course

- Green's function and Kirchhoff-Helmholtz eq.
- Paraxial approximation (P.E.)
- Perturbation approximations (Born, Rytov)

UNIVERSITY OF CAMBRIDGE

Topics in Part II of the course

- **Rough surfaces**
  - characterisation of randomly rough surfaces
  - scattered field, power spectrum
- **Extended random media**
  - equations for the moments of the field
  - scintillation index
  - time reversal

UNIVERSITY OF CAMBRIDGE

Topics in Part III of the course

- **The inverse scattering problem**
- **Inverse problems – general theory**
  - ill-posedness, Moore-Penrose generalized inverse
  - Tikhonov regularization, other regularisations
- **Solving inverse scattering problems**
  - the far field operator
  - linear sampling method
  - iterative methods
  - methods based on the Born or Rytov approximation
  - time reversal imaging

UNIVERSITY OF CAMBRIDGE

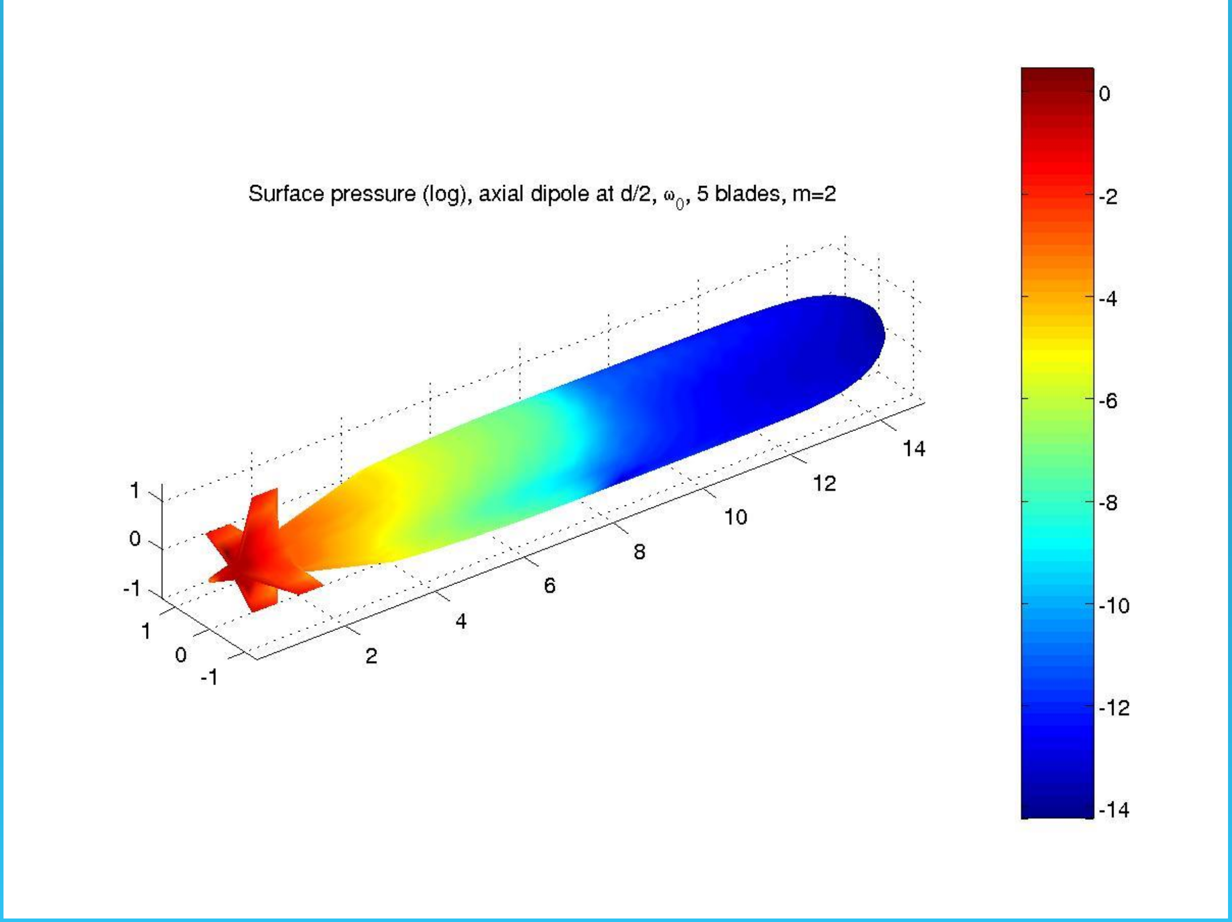
How useful?

- Green's function:
  - Solution (any PDE) with “point source”
  - Gives solution for inhomogeneous PDE, with any source

- Kirchhoff-Helmholtz equation
  - Greatly simplifies computation of problems with B.C.

Using the K-H equation

Acoustic pressure on hull



5-bladed propeller

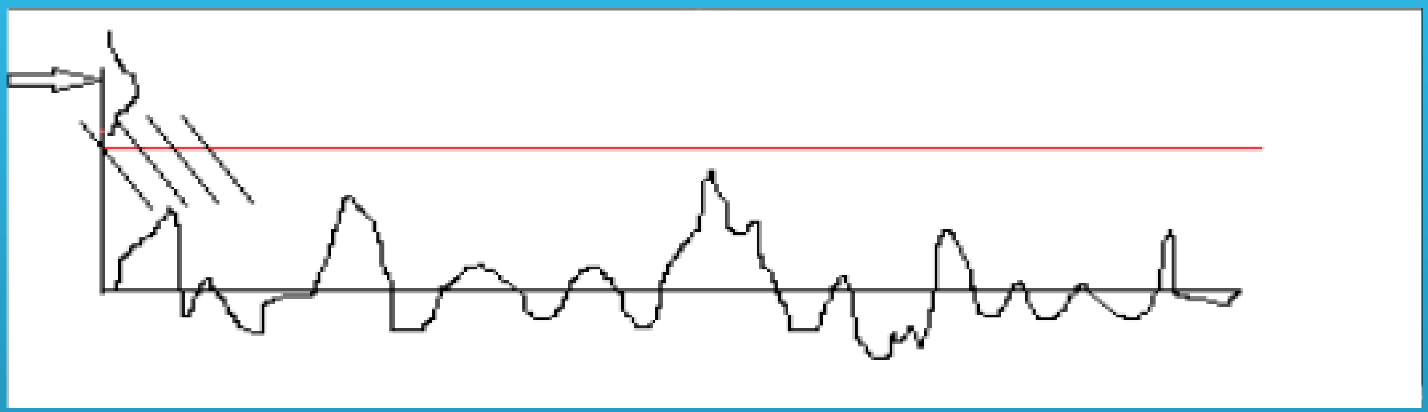
ω ~2Hz

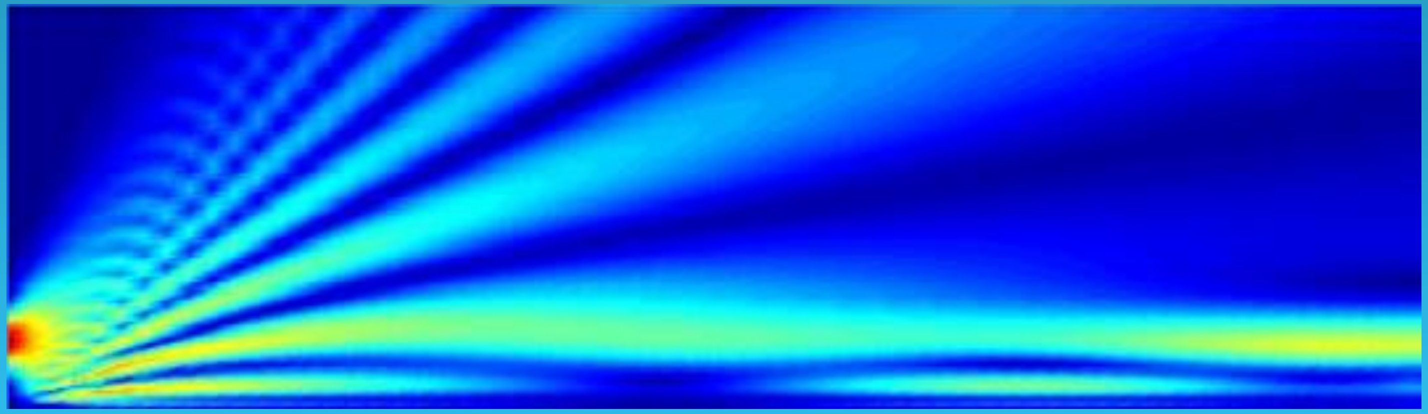
'loading' noise

UNIVERSITY OF CAMBRIDGE

Model with Helmholtz

justify P.E.





UNIVERSITY OF CAMBRIDGE

Extended random medium

measuring scintillation from stars

- atmosphere/plasma modelled as turbulent medium
- Kolmogorov model of turbulence: eddies as embedded inhomogeneities, multiple scales
- propagation of moments, measuring variation in intensity: scintillation

UNIVERSITY OF CAMBRIDGE

Extended random medium

measuring scintillation from stars

- **using scintillation to study atmosphere**

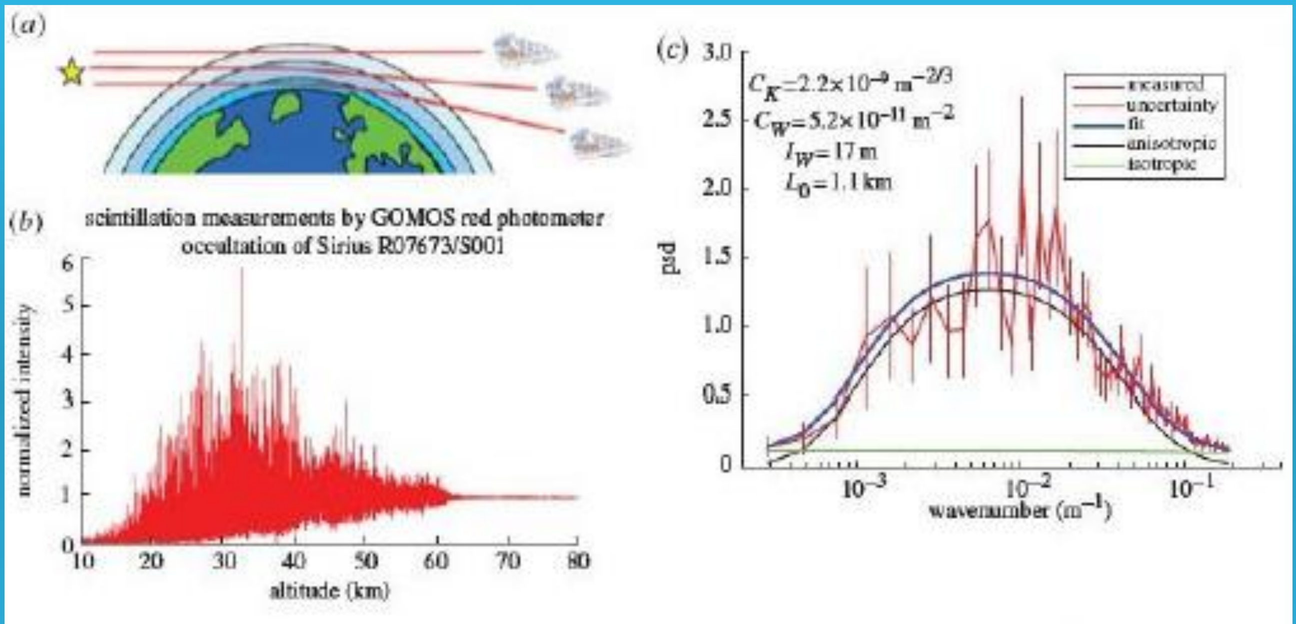


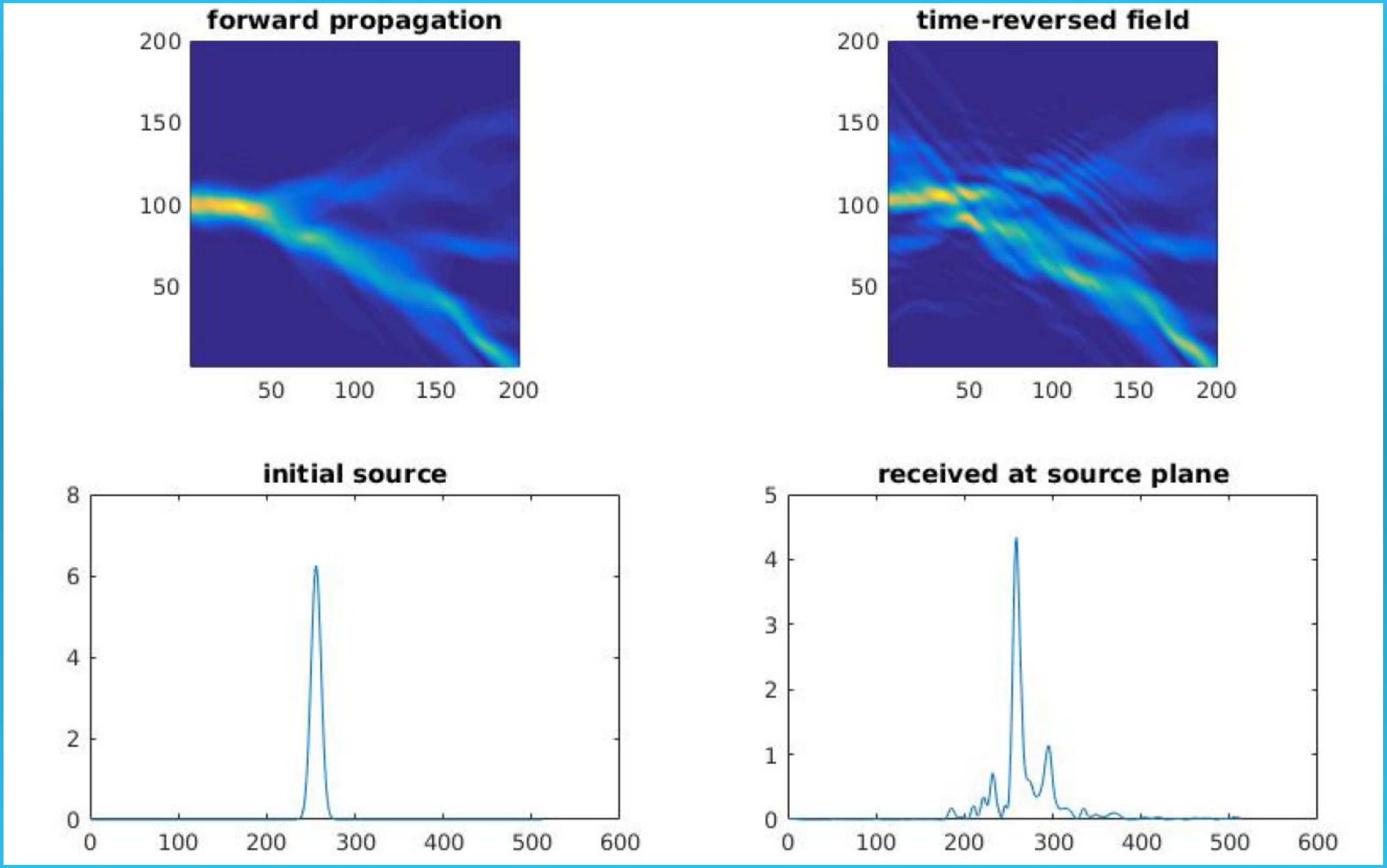
Figure 3. (*a*) A schematic of occultation measurements. (*b*) An example of scintillation records by the GOMOS photometers. (*c*) Scintillation spectrum (red) and retrieval results at 33 km altitudes for the occultation of R08738/S002, 12°N 155°E, carried out 1 November 2003, $\alpha = 50°$; blue—the fit with indicated anisotropic (black) and isotropic (green) components of the scintillation spectrum. The values of the retrieved GW and turbulence parameters are also indicated in the figure. (Online version

Sofieva *et al*
Phil Trans R Soc A, 2013

- **effect of earth's atmosphere on observation of moon**

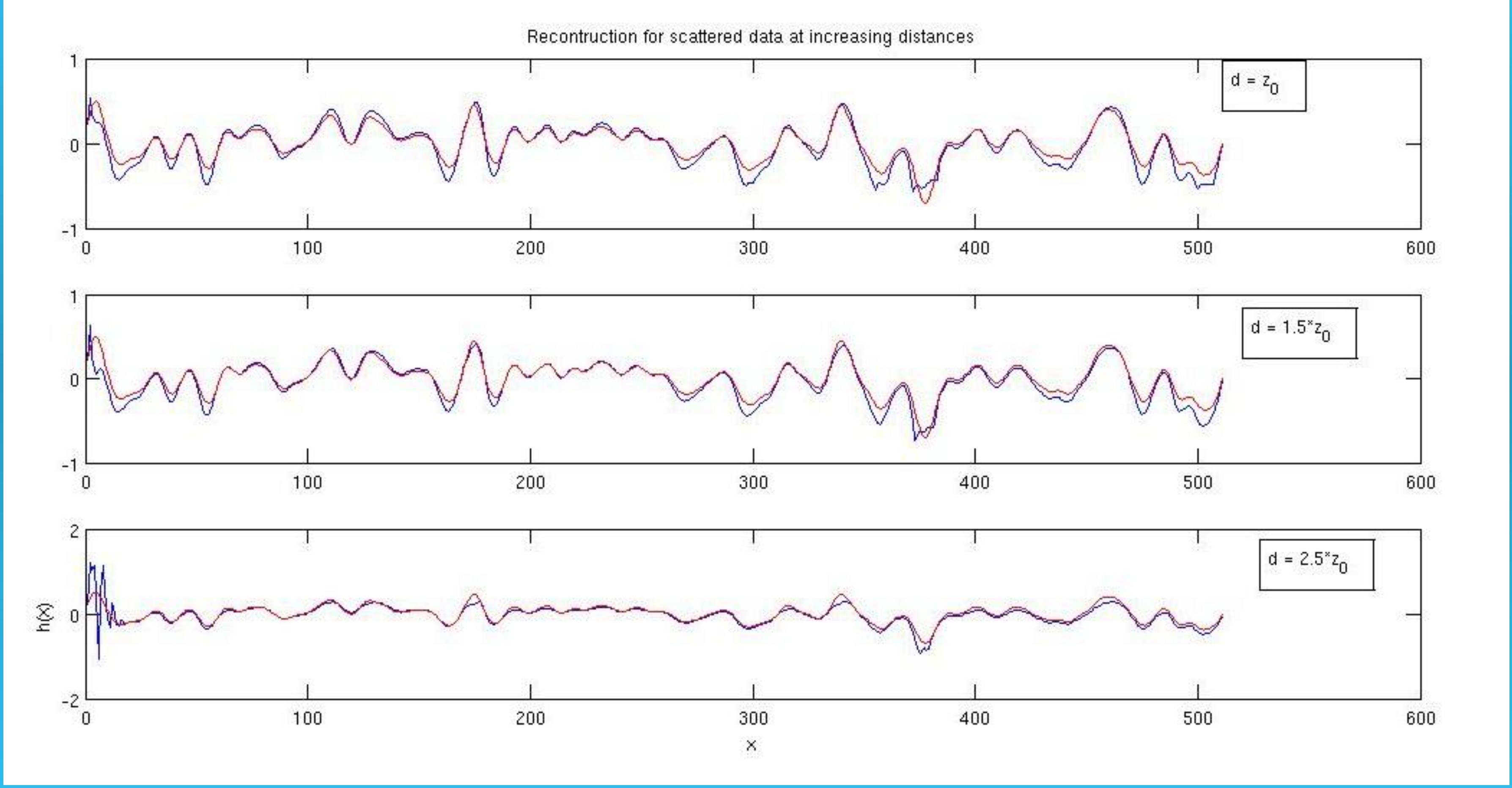
UNIVERSITY OF CAMBRIDGE

Inverse problems
time reversal in a random medium



Inverse problems

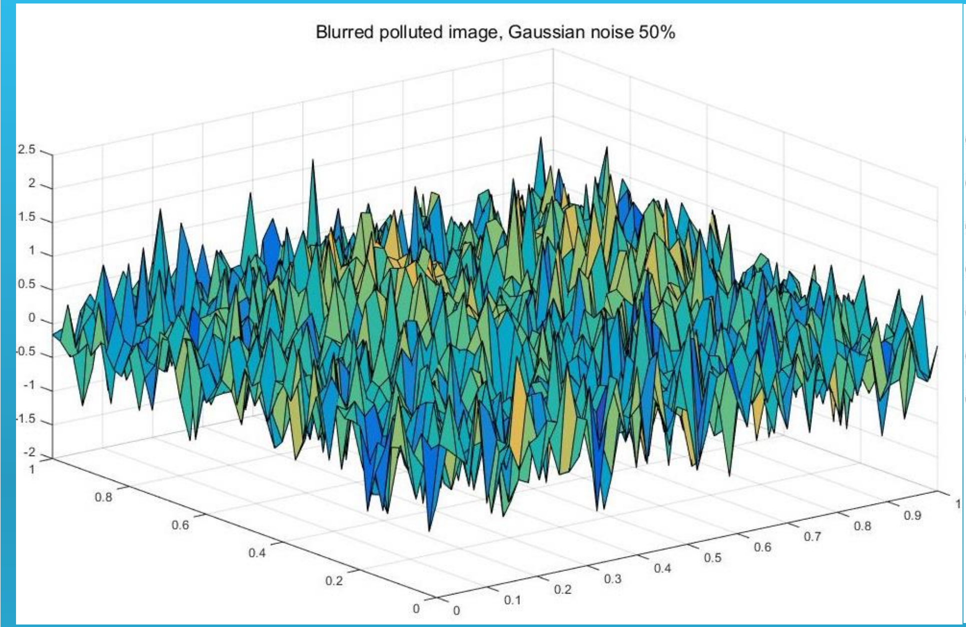
Reconstruct a random surface from scattered data

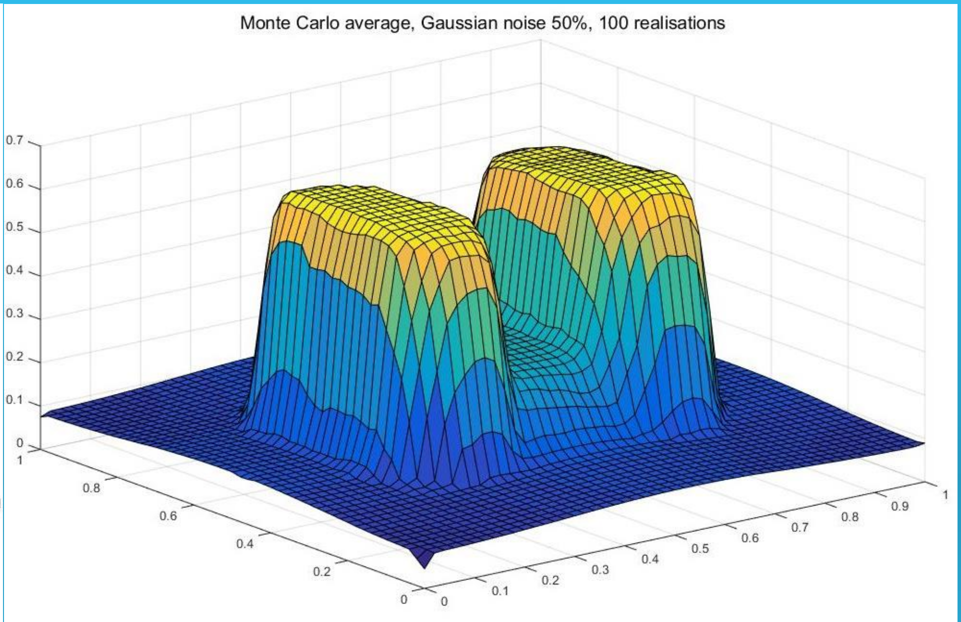


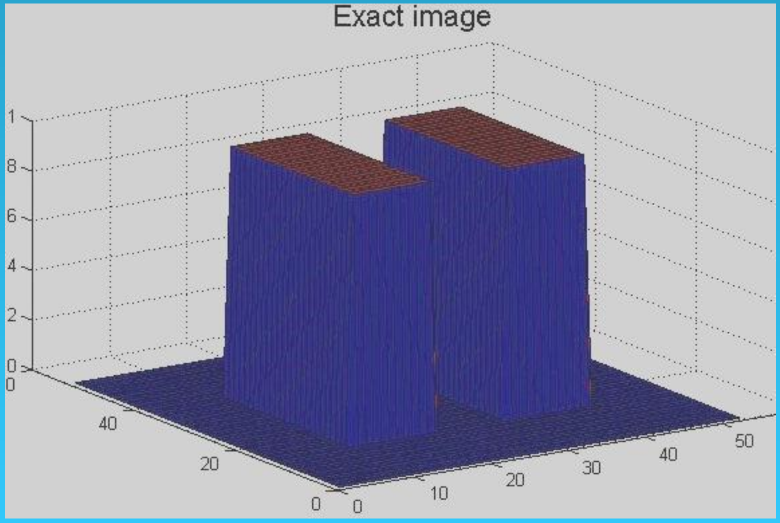
UNIVERSITY OF CAMBRIDGE

Inverse problems

Regularise the inversion of blurred and noisy data







UNIVERSITY OF CAMBRIDGE

Any Questions?

UNIVERSITY OF CAMBRIDGE