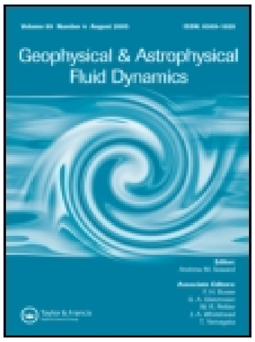

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SPIN-UP OF A TWO-LAYER FLUID IN A ROTATING CYLINDER

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'e report the results of experiments on the spin-up of two layers of immiscible fluid with a free upper ırface in a rotating cylinder over a wide range of internal Froude numbers. Observations of the evolution ' the velocity field by particle tracking indicates that spin-up of the azimuthal velocity in the upper layer ke much longer than in a homogeneous fluid. Initially, spin-up occurs at a rate comparable to that of homogeneous fluid but, at high internal Froude number, a second phase follows in which the remaining lative motion decays much more slowly. Quantitative comparison of these measurements to the theory ' Pedlosky (1967) shows good agreement.

Visualization of the interface displacement during spin-up detected the presence of transient azimuthal ıriations in the interface elevation over a wide range of Froude (F), Ekman (E), and Rossby (ε) number. nalysis of the occurrence of the asymmetric variations using the parameter space (Q, F), where $Q = E^{1/2}/\varepsilon$, ıggested by the baroclinic instability theory and experiments of Hart (1972), showed that the flow was able for $Q > 0.06$ with no discernable dependence on F. This result, together with the prediction of ədlosky's theory that radial gradient of potential vorticity in the two layers have opposite signs, suggests ıat the baroclinic instability mechanism was responsible for the asymmetries. The location and timing ' these instabilities may account for the discrepancies between the observations and the Pedlosky (1967) ıeory.

EY WORDS: Rotating cylinder, two-layer fluid, baroclinic instability, spin-up.

INTRODUCTION

ɪuch has been learned about the nature of the atmosphere and oceans by the study f simple mathematical models and controllable laboratory models of the dominant rocesses. The fundamental influence of boundary stresses on winds and currents, ›r example, can be explained by considering the response of fluid contained in a .osed cylinder mounted on a rotating table. If the fluid is homogeneous and the ıble is flat then, after a sufficiently long time, the fluid will rotate as a solid body 'ith angular velocity, Ω, equal to that of the cylinder, and with a surface deformation ıduced horizontal pressure gradient sufficient to balance the centripetal acceleration. . change in the rotation rate of the table and tank, to $\Omega + \Delta\Omega$, for example, will result in fluid motion in a rotating coordinate system that is subjected to boundary stresses. The subsequent evolution of the flow field depends on the aspect ratio of the cylinder, H/R (the tank depth divided by the radius) and the magnitude of the dimensionless parameters, $\varepsilon = \Delta\Omega/\Omega$ (the Rossby number) and $E = \nu/\Omega H^2$, (the Ekman number), where ν is the kinematic viscosity of the fluid. If, as in much of the ocean and atmosphere, the values of ε and E are small, then the laboratory flow is a useful analog of the effect of boundary stresses on the decay of geophysical scale flows.

This fundamental problem of geophysical fluid dynamics has become known as the "spin-up problem" and, with its many variants, has generated a considerable volume of literature. Greenspan (1968) has presented a thorough account of the development of theories for spin-up in a variety of geometries, and Benton and Clark (1974) provide a comprehensive review of the literature including the influence of density stratification, nonlinear, electromagnetic and thermal effects. More recently, Cederlof (1988) and O'Donnell and Linden (1991) have elucidated the role of a free surface in the decay of relative motion in a homogeneous fluid and van Heijst (1989) has explored spin-up in non-axisymmetric containers.

Since much of our understanding of the influence of stratification in geophysical flows is based on models that represent the vertical density structure of the fluid as a thin "interface" between two well mixed layers, a full appreciation of the role of friction in this type of flow is important. In this paper, we present the results of a comparison of velocity observations in a rotating, cylindrical laboratory tank containing two layers of immiscible fluids with a free upper surface to the predictions of a small Rossby number spin-up theory by Pedlosky (1967). Since the previous work of Linden and van Heijst (1984) and van Heijst (1989) suggested that baroclinic instabilities may influence the spin-up process, in this study we have examined spin-up Rossby numbers ranging from -0.5 to 0.3 and designed the experiments to enable visualization of the displacement of the interface so that azimuthal asymmetries, manifestations of flow instability, were obvious.

In the next section we qualitatively describe the predictions of Pedlosky (1967) and then outline the equipment and techniques employed in the experiments. In Section 4, we present a comparison of the measurements and the theory and in Section 5 we describe and discuss the unstable behaviour observed in some of the experiments.

2. THEORIES

Holton (1965) and Pedlosky (1967) have both tackled the theoretical analysis of the small Rossby number spin-up of two layers of fluid in a cylinder with a free upper surface and Holton also presented experimental verification of his predictions. In Holton's work the layers of fluid were miscible whereas Pedlosky's fluids were immiscible. The important difference between these theories, therefore, lies in the generation of the secondary radial circulation in the upper layer. Holton's theory does not include Ekman pumping at the bottom of the upper layer since, at the boundary between layers of miscible fluid, the stratification is continuous and the "interface" has a finite thickness determined by the diffusivity of the constituent controlling density. Even if this interface is thin compared to the layer depths, it may still be much thicker than the interfacial Ekman layer. The stratification would then suppress the vertical velocity produced by the interfacial Ekman layer pumping and eliminate the consequent radial circulation in the interior. In the absence of surface stresses then, vortex stretching due to the motion of the interface is the dominant process that modifies the vorticity in the upper layer and, as pointed out by Pedlosky (1967), conservation of fluid volume requires that the radial average of vorticity in the upper layer remains constant until viscous diffusion from the side walls becomes important.

In Pedlosky's (1967) theory for the spin-up of immiscible fluids, both the movement of the interface and pumping by interfacial Ekman layers couple the interiors of the homogeneous layers. The radial circulation in the interior of the upper layer then requires a boundary condition at the cylinder wall which balances the radial Ekman flux with an oppositely directed interior flux as in the lower layer. Solutions for small Ekman and Rossby numbers were obtained as the sum of an infinite Fourier-Bessel series.

In the following discussion of the characteristics of the solutions we have adopted a nomenclature slightly different from that of Pedlosky but which is now standard. The azimuthal velocity in layer i, scaled by the initial velocity at the edge of the cylinder, $\Delta\Omega R$, is denoted $u_{\theta i}$. The radial distance from the center of the tank, scaled by the tank radius, R, is denoted by r, and the time since the initiation of motion, scaled by the spin-up time for a homogeneous fluid, τ, is defined as t. Note that $\tau = E^{-1/2}\Omega^{-1}$ where E is the Ekman number $E = \nu/\Omega H^2$, ν is the kinematic viscosity of water, Ω is the final rotation rate of the tank, and $H = H_1 + H_2$ is the total depth of the fluid. The internal Froude numbers, $F_i\,(i = 1,2)$ are defined $F_i = 4\Omega^2 R^2/g\Delta\rho/\rho H_i$, where $\Delta\rho/\rho$ is the fractional density difference between the layers and g is the vertical acceleration imposed by gravity. For layers of equal thickness, $F = F_{1,2}$ is therefore the square of the ratio of the radius of the tank to the internal deformation radius.

Figure 1 shows an approximation to Pedlosky's solution for the magnitude of the upper and lower layer azimuthal velocities as functions of time for five values of radius. These were computed using the first forty eigenfunctions. The calculation presented assumes the layers have equal thickness and aspect ratio, $k_{1,2} = R/H_{1,2} = 2.5$. The velocities are scaled by the initial value, $[u_{\theta 1}(r = 0, t) = r]$, and are therefore equivalent to the angular velocity. Three values of the internal Froude number, (a) and (b) $F = 100.0$, (c) and (d) $F = 10.0$ and (e) and (f) $F = 1.0$ are presented, and the spin-up rate for a homogeneous fluid is shown for reference.

It is evident from the figure that the ultimate spin-up of both layers takes place more slowly than in a homogeneous fluid, irrespective of r and F, and that the evolution is strongly controlled by the Froude number. The spin-up process can be divided into three stages. At high Froude number, during the initial phase of spin-up, the lower layer accelerates rapidly compared to a homogeneous fluid, see Figure 1(b), until, at $t \approx 2$, the second phase begins and there is a reduction in the rate of spin-up accompanied by the development of more radial structure in the velocity field. Spin-up in the upper layer during these first two stages is slower than in a

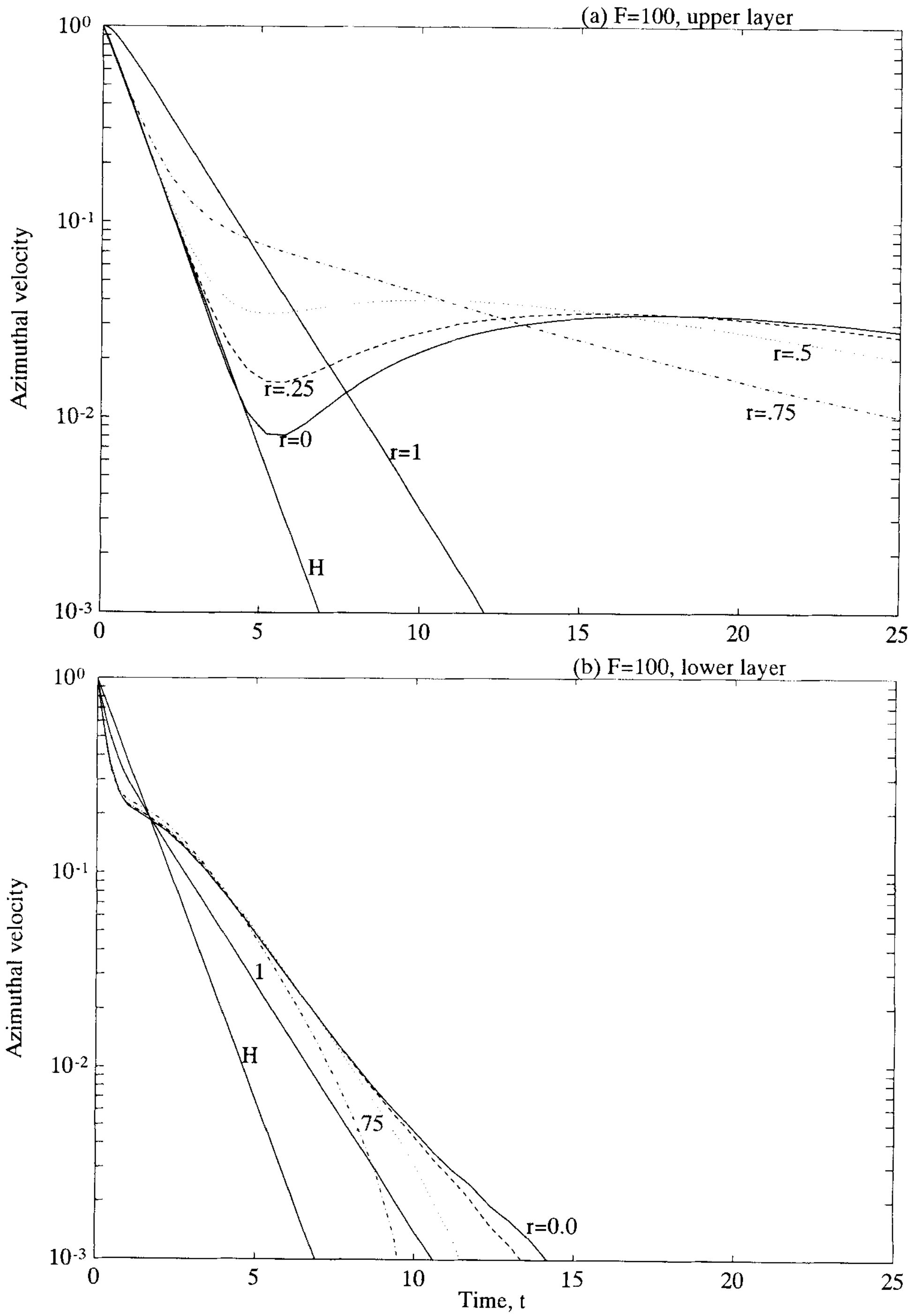
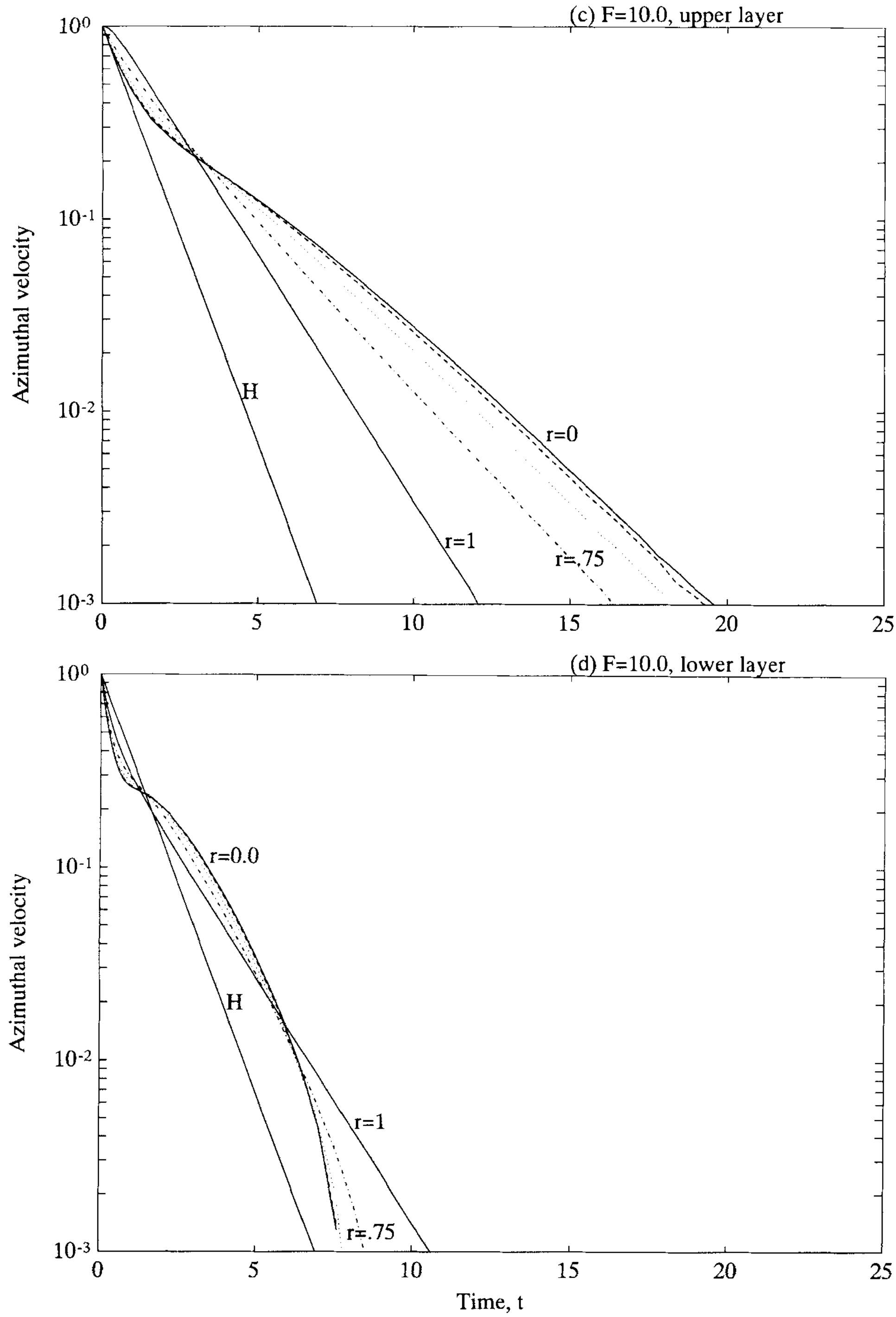


Figure 1 Pedlosky's (1967) prediction of the evolution of the velocity field in two layers of equal depth with $R/H = 2.5$. Values of $u_{\theta 1}/r$ and $u_{\theta 2}/r$ are plotted on log scales at $r = 0.0$, 0.25, 0.5, 0.75 and 1.0 for $F = 100$ [(a) and (b)], 10 [(c) and (d)], and 1.0 [(e) and (f)]. The line labeled 'H' indicates the spin-up rate of a homogeneous fluid.



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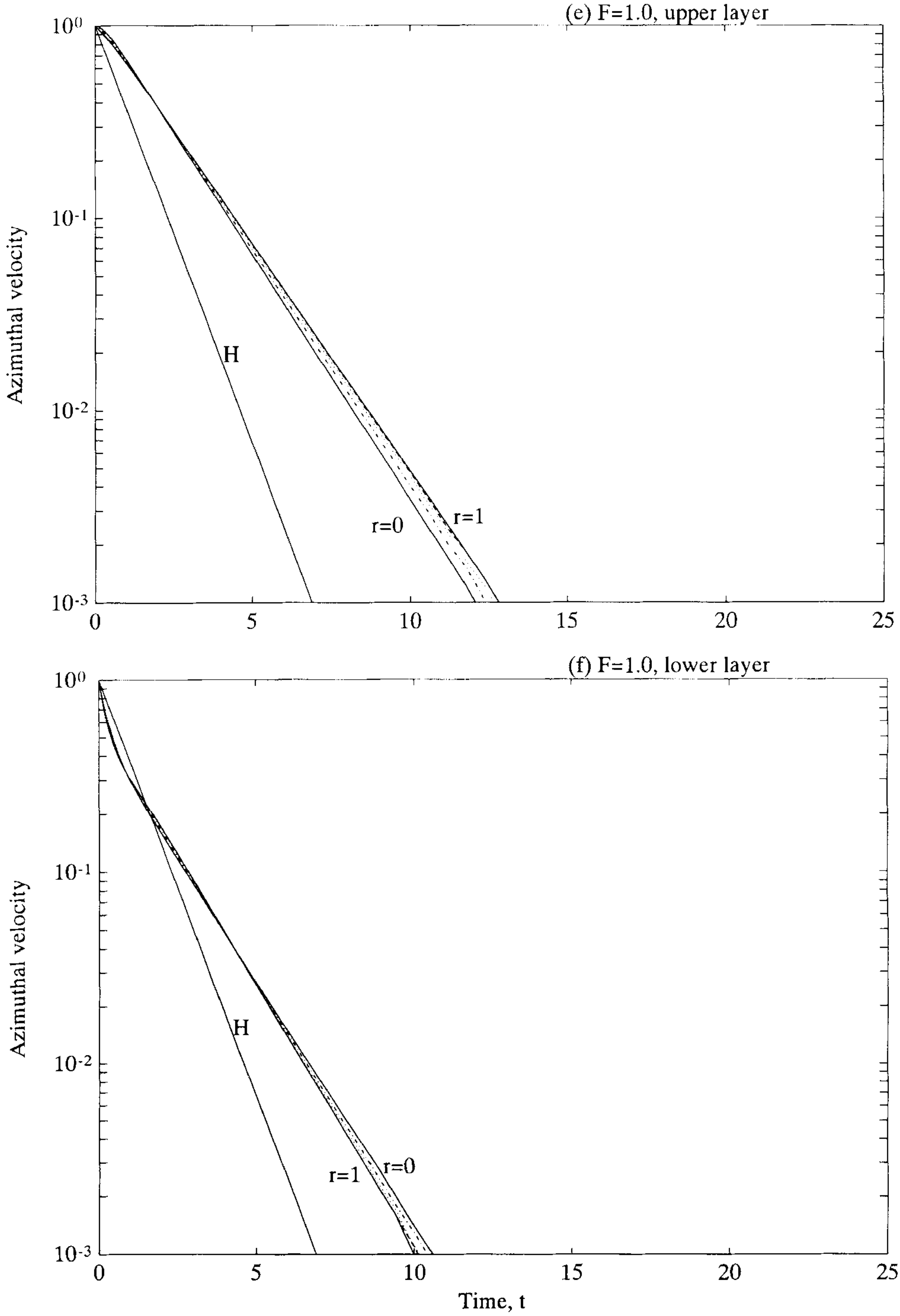


Figure 1 *Continued.*

homogeneous fluid and significant radial variations in the angular velocity develop quickly, see Figure 1(a). The third phase begins at $t \approx 5$. The spin-up rate of the upper layer becomes very small and large radial variations in the angular velocity develop. This summary is also applicable at smaller Froude numbers, see Figure 1(c)–(f), though the radial variations are smaller and the ultimate spin-up rate is faster.

Figure 2 shows the predicted evolution of the interface displacement for $F = 100$ at (a) $t = 0.5$, 1.0, 2.0 and (b) $t = 3$, 5, 10. It is evident that most of the deformation occurs in a narrow (of order $F^{-1/2}$) band around the edge of the tank during the initial phase of spin-up where the interface rises rapidly. Subsequently, this outer region descends slowly as the azimuthal velocity decays. In the interior of the cylinder, the interface remains almost level in the initial phase of spin-up, descending to a quasi-equilibrium level in $t \approx 2$. In the second phase, the interface curvature in the interior increases slightly as the interface slowly descends at the cylinder edge and rises in the center.

3. EXPERIMENTS

The experiments were performed on a precisely controlled, direct drive rotating table on which the apparatus, shown in Figure 3, was assembled. The fluids, distilled water and a Limonene-Dibromomethane mixture, were contained in an accurately machined steel cylinder ($R = 0.25$ m $\pm$ 0.001) with reinforced glass ends. The density of the lower layer was controlled by varying the relative amounts of Dibromomethane ($C_2H_4Br_2$), which is rather dense, and Limonene. These fluids were also used by Hart and Kittleman (1986) who reported the viscosity of the mixture in the density range we employed to be within 5% of that of water.[1] Density measurements were made on the day of each experiment by withdrawing several samples of both layers and using a densitometer capable of an accuracy of 5.0×10^{-5}. This allowed the density difference between the layers to be small ($4 \times 10^{-3} \leqslant \Delta\rho/\rho \leqslant 8 \times 10^{-3}$), yet carefully controlled. Though the chemicals in the lower layer are essentially immiscible with water, $C_2H_4Br_2$ did slowly diffuse into the water and decrease the density difference. However, this was a slow process and any consequences were mitigated by the frequent density measurements.

Although Dibromomethane and Limonene are hazardous and must be handled very carefully, they were used because Limonene is optically active, i.e. it rotates the plane of polarization of light by an amount that is proportional to the path length of the ray and dependent of the wavelength, or color, of the light. The polarizing filter between the white light source and the bottom of the tank ensures that only polarized light passes into the tank. The second filter in front of the camera fixes the angle of polarization of light entering the camera and so the color of the light recorded by the camera depends on the path length of the ray in the Limonene. A mirror above the table minimizes parallax to ensure that the height of the interface is accurately "mapped" over the whole flow domain. The principles of this method were

[1] Dr. John Hart now recommends the use of 1,2 Dibromotetrafluoroethane, rather than Dibromomethane since it is much less toxic and may be used with plastics.

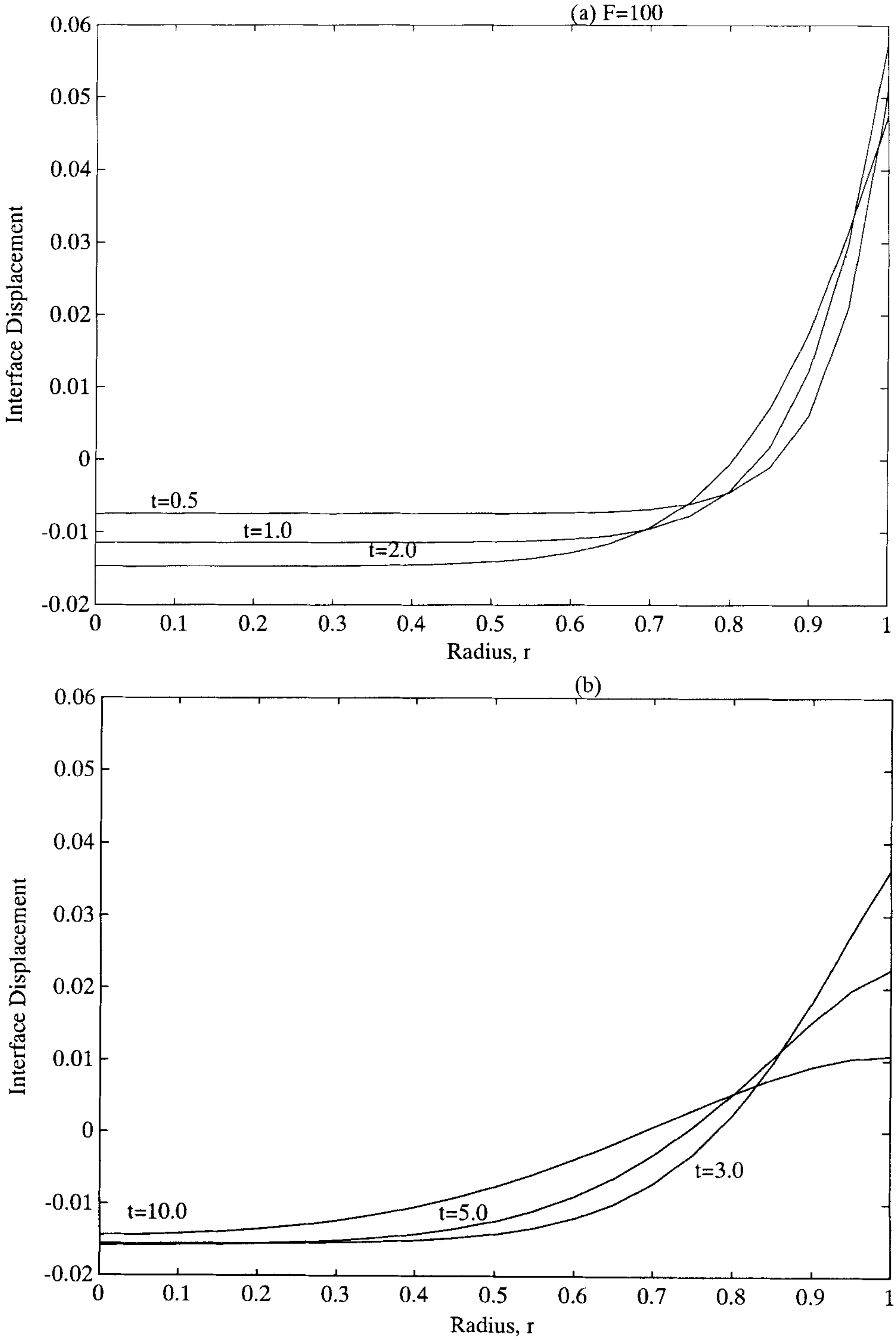


Figure 2 Pedlosky's (1967) prediction of the evolution of the interface displacement for $F = 100$ at (a) $t = 0.5$, 1.0, 2.0 and (b) $t = 3.0$, 5.0, 10.0.

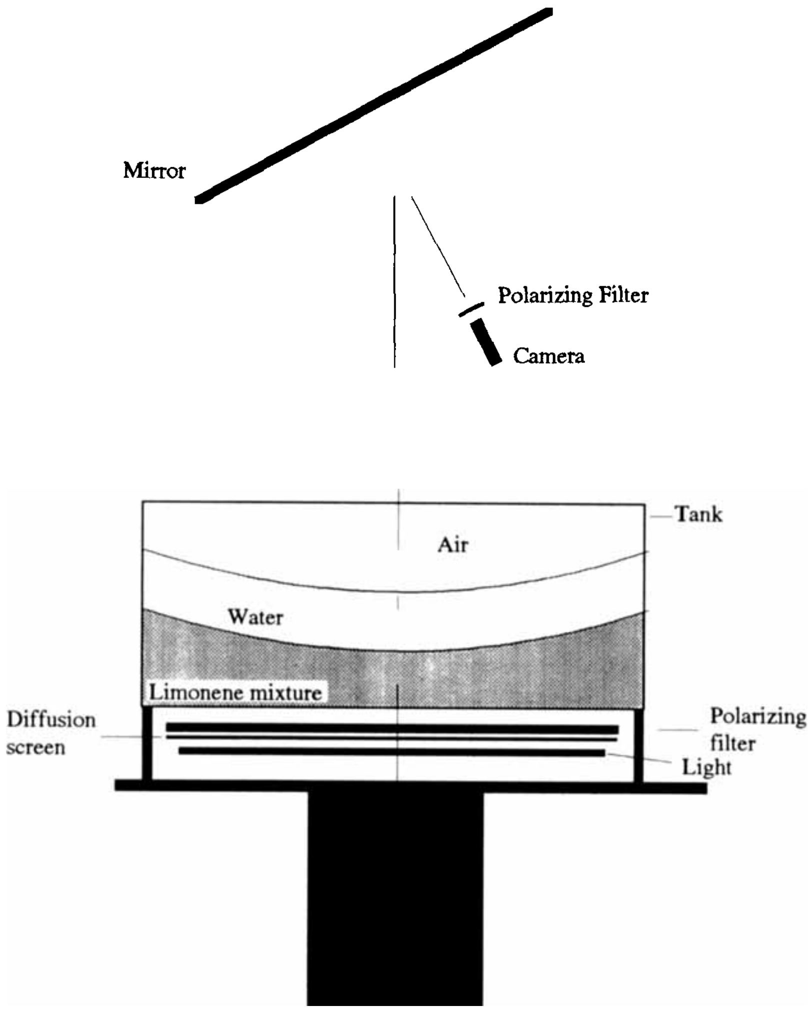


Figure 3 A schematic of experimental equipment.

further elaborated by Hart and Kittleman (1986) who also presented a technique to quantify the interface displacement. However, qualitative information on the symmetry of the color pattern were sufficient in these experiments. We found that the presence of instabilities was most easily assessed if we adjusted the angle between the two polarizing filters prior to rotation of the table so that a pale pink color appeared in the camera image. Small negative interface displacements then showed up clearly as red areas and positive interface displacements were indicated by an intense blue.

The velocity field in the upper layer was obtained in the manner discussed in O'Donnell and Linden (1991), by tracking the location of small circles of black paper (approximately 4 mm in diameter) scattered randomly on the surface of the water. To allow higher spatial and temporal resolution, this simple technique was automated by employing a video-digitizer and micro-computer to locate the positions of the particles and to compute their velocities. Since the radial component of velocity during spin-up is much smaller than the azimuthal component, and also because the uncertainty associated with estimates is limited by the ratio of the net displacement to the radius of the particles, only $u_{\theta 1}$ could be estimated with acceptable uncertainty. Near the center of the tank however, particle displacements were always small and the associated uncertainty, correspondingly large. Measurements in the interval $0 \leqslant r \leqslant 0.1$ were therefore ignored. At a radius of 0.5, relative uncertainty in the estimates of velocity was of the order of 10%.

After the tank was filled to the required depths and samples of the fluids were collected, the table was set in motion and the speed slowly increased until the desired rotation rate was reached. Direct measurements of the table speed showed variations of less than 0.001 radians/sec, and acceleration between selected speeds occurred in less than one rotation period. When all motion in the tank decayed and the interface became essentially flat (a uniform pink shade), relative motion was initiated by suddenly changing the rotation rate from Ω to $(1 + \varepsilon)\Omega$.

4. RESULTS

As the table accelerated to the new rotation rate, high frequency oscillations and Ekman layer instabilities became evident in most experiments, but disappeared after a few rotation periods. Red, and subsequently orange, colors appeared in the center area of the tank indicating a decrease in the interface level, while at the edges a corresponding thickening of the lower layer was represented by an intense blue, in qualitative agreement with Pedlosky's (1967) theory. As the azimuthal velocity decayed, a transient asymmetry in the color pattern arose in some experiments which then evolved into a higher wave number structure before disappearing. We will focus first on the evolution of the velocity field, and compare quantitatively the observations and theory and we will then return to a further discussion of the cause and consequences of the instability in Section 5.

A comparison of the measured upper layer velocity field evolution and the corresponding theoretical predictions of Pedlosky (1967) in four experiments are presented in Figure 4. The first forty terms of Pedlosky's infinite series were included in the evaluation of the solution and the small oscillations that appear in the plot are artifacts of the computation. Since it is impossible to control the positions of the particles used to measure velocity, observations are irregularly spaced with very few near the cylinder wall because of both the upwelling of the interface and the interfacial Ekman layer driven radial flow. To allow a model-data comparison we have averaged the data in radial bins and then plotted the theoretical solution at the edges of the bins. In each plot, the straight solid line indicates the decay rate for motion in a homogeneous fluid and is provided to ease comparison of plots with different scales. The "+", "*" and "○" symbols represent the average of all azimuthal velocity measurements in the intervals $0.2 \leqslant r \leqslant 0.3$, $0.45 \leqslant r \leqslant 0.55$, and $0.7 \leqslant r \leqslant 0.8$, respectively. These are to be compared with the dotted curves which show the

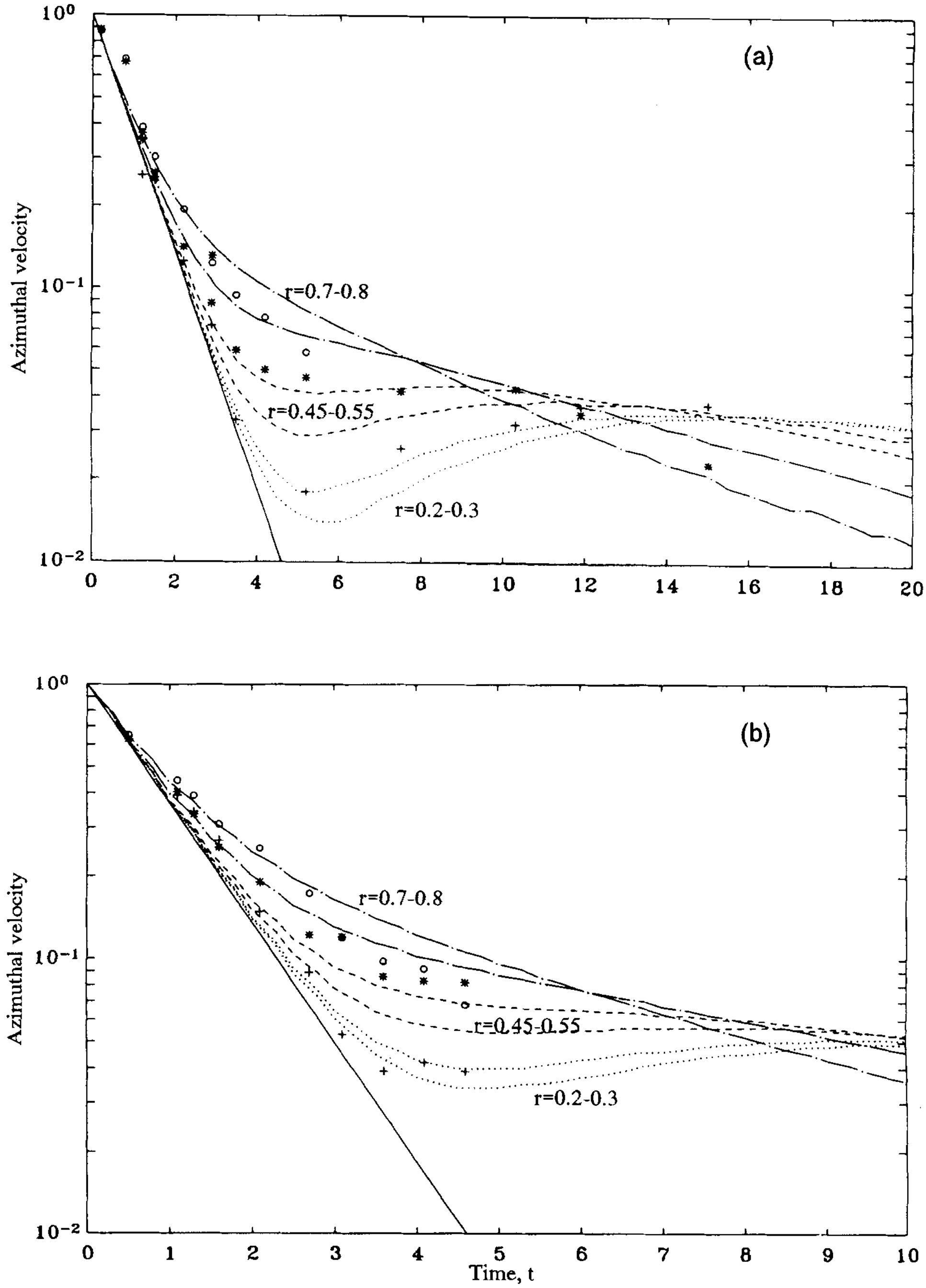
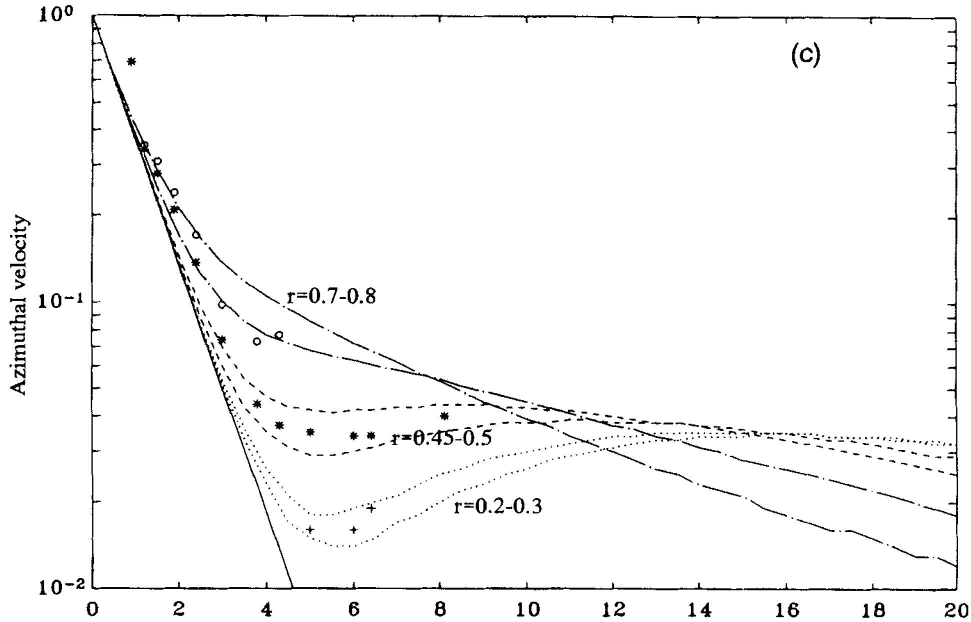


Figure 4 A comparison of the measured azimuthal velocity $u_{1,\theta}/r$ and the theoretical predictions of Pedlosky (1967). The results of experiments 9 ($F = 62.6$, $\varepsilon = 0.167$), 7 ($F = 97.9$, $\varepsilon = 0.167$), 10 ($F = 97.9$, $\varepsilon = 0.222$), and 6 ($F = 174.0$, $\varepsilon = 0.167$) are shown in (a), (b), (c) and (d) respectively.



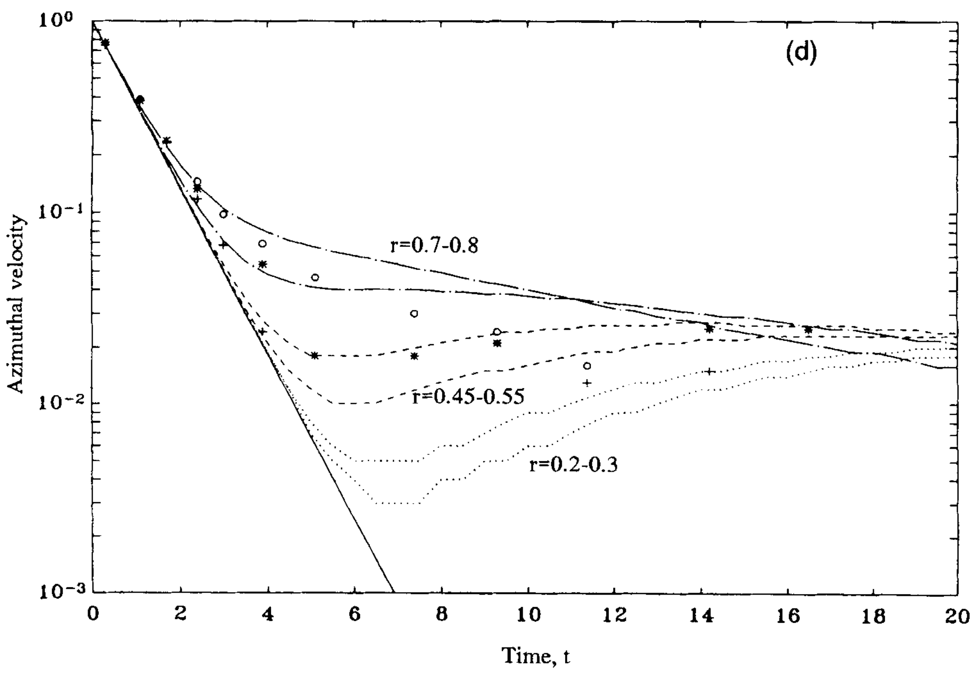


Figure 4 *Continued.*

evolution of the theoretical solution evaluated at $r = 0.2$ and 0.3; the dashed lines showing the solution at $r = 0.45$ and 0.55; and the dot-dash line indicating the solution at $r = 0.7$ and 0.8.

The observations of the upper layer velocity in all experiments at large internal Froude number exhibit the general features of the spin-up model of Pedlosky (1967). Moreover, a detailed comparison of the measurements with the theory shows good agreement around $r = 0.25$ (compare "+" symbols with the dotted curves) at all times. The measurements also support the theory in the neighborhood of $r = 0.5$, (compare "*" symbols with the dashed lines) though there is some disagreement in the early stages of experiment 9, Figure 4(a).

The disagreement between the theory and observations is largest in experiment 6, Figure 4(d), at the observation interval around $r = 0.75$, where the data indicate that spin-up occurs rather more quickly than predicted. Notice that the theory also over-predicts the velocity in the latter stages of experiments 9 and 7, see Figures 4(a) and (b). However, since the theory successfully predicts both the quantitative and qualitative behavior rather well, we conclude that it is fundamentally correct, and that the small discrepancies at large radii are the result of the omission of the $O(\varepsilon)$ nonlinearities.

5. INSTABILITIES

As noted above, the presence of transient azimuthal disturbances in the interface depth was found in approximately half of the experiments. Figure 5 presents a sequence of color photographs from experiment 7 showing the formation, evolution and decay of a typical weak instability.

Prior to the initiation of the experiment the fluid was stationary and the interface flat. This is represented by a pink color in Figure 5(a). Shortly after the initiation of the experiment an orange color appeared across much of the interior of the tank [see Figure 5(b); $t = 0.20$], indicating that the interface had descended uniformly in the center, and a pale blue appeared around the edge where it had risen. The narrow circle of pink indicates the region where the interface displacement is approximately zero. These observations are in qualitative agreement with Pedlosky's predictions. At $t = 0.47$ a weak low mode asymmetry in the color pattern became evident, see Figure 5(c), which subsequently evolved to include higher modes; see Figure 5(d) at $t = 0.67$, and Figure 5(e) at $t = 1.23$. By $t = 1.87$ [see Figure 5(f)], however, the pattern became symmetric again and the amplitude of the interface displacement was smaller. This state was persistent, compare Figures 5(f) and 5(g) ($t = 3.07$), and the remaining disturbance decayed very slowly. At $t = 7.70$, Figure 5(h), the orange shade was still visible in the center of the tank but the wide band of pink indicated the radial gradient was small. Comparison of the interface behavior with the evolution of the velocity field shown in Figure 4(b) shows that the energetic stage of the instability occurs during the initial phase of spin-up.

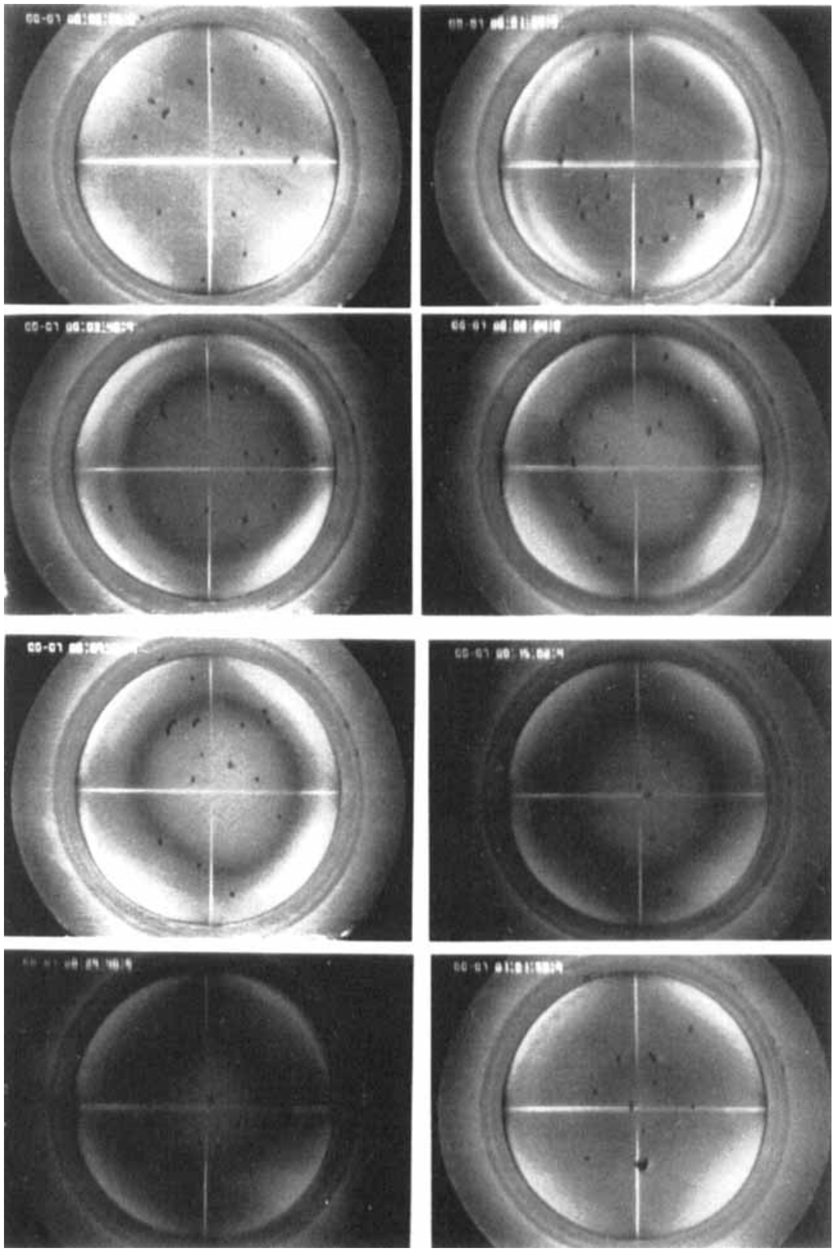


Figure 5 Photographs showing the evolution of the colour patterns in experiment 7 in which $F = 98$ and $\varepsilon = 0.167$. Photographs were taken at (a) $t = 0.0$, (b) $t = 0.20$, (c) $t = 0.47$, (d) $t = 0.67$, (e) $t = 1.23$, (f) $t = 1.87$, (g) $t = 3.07$, and (h) $t = 7.70$.

This weak instability was obvious in about half of the experiments we performed. The criterion we used was necessarily subjective however, and experiments we describe as "stable" may have been weakly unstable with slow instability growth rates. Table 1 shows the parameter range explored and indicates the existence of instabilities in the experiments.

Though there appears to have been no previous theoretical work on the stability of the spin-up process in a two-layer fluid, Hart's (1972) study of the instability of

Table 1 A summary of the experiments

Experiment	$H(m)$	k_1	k_2	ε	F	$E^{1/2}$	*Stability*
	$H_1 + H_2$	$\frac{H_1}{R}$	$\frac{H_1}{R}$	$\frac{\Delta\Omega}{\Omega}$	$\frac{4\Omega^2 R^2}{gH_1}$	$\left(\frac{\nu}{\Omega H^2}\right)^{1/2} \times 10^3$	
3	0.200	0.400	0.400	0.167	43.525	4.56	unstable
4	0.200	0.400	0.400	−0.200	30.225	5.00	unstable
5	0.200	0.400	0.400	0.286	59.242	4.23	unstable
6	0.200	0.400	0.400	0.167	174.098	3.23	stable
7	0.200	0.400	0.400	0.167	97.930	3.73	unstable
8	0.200	0.400	0.400	−0.286	59.242	4.23	unstable
9	0.200	0.400	0.400	0.167	62.675	4.17	—
10	0.200	0.400	0.400	0.222	97.930	3.73	—
17	0.100	0.200	0.200	0.091	3.540	20.3	stable
18	0.100	0.200	0.200	0.333	7.966	16.6	stable
19	0.100	0.200	0.200	0.167	41.461	9.13	stable
20	0.100	0.200	0.200	−0.500	18.427	11.2	unstable
21	0.100	0.200	0.200	0.167	26.535	10.2	stable
22	0.100	0.200	0.200	0.167	165.845	6.45	stable
23	0.100	0.200	0.200	0.167	93.288	7.45	—
24	0.100	0.200	0.200	−0.250	115.170	7.07	unstable
25	0.100	0.200	0.200	−0.500	28.793	10.0	—
26	0.100	0.200	0.200	0.333	6.378	14.9	unstable
27	0.100	0.200	0.200	0.333	14.349	12.2	unstable
28	0.100	0.200	0.200	0.327	31.180	10.0	unstable
29	0.100	0.200	0.200	0.200	31.494	10.0	—
30	0.100	0.200	0.200	0.167	45.351	9.13	unstable
31	0.100	0.200	0.200	0.167	102.041	7.45	unstable
32	0.100	0.200	0.200	0.333	11.338	12.9	unstable
33	0.100	0.200	0.200	0.250	20.156	11.2	stable
34	0.100	0.200	0.200	0.167	45.351	9.13	unstable
35	0.100	0.200	0.200	−0.200	31.494	10.0	unstable
42	0.114	0.148	0.308	0.091	3.750	15.9	stable
43	0.114	0.148	0.308	0.091	18.741	10.6	stable
44	0.114	0.148	0.308	0.082	76.654	7.49	stable
45	0.114	0.148	0.308	0.091	120.970	7.28	stable
46	0.114	0.148	0.308	0.091	364.083	5.53	stable
47	0.111	0.140	0.304	0.231	5.201	17.5	—
48	0.111	0.140	0.304	0.231	25.974	11.7	stable
49	0.111	0.140	0.304	0.231	86.712	8.68	stable
50	0.111	0.140	0.304	0.231	173.305	7.30	stable

a steady, two layer flow in a rotating cylindrical tank (angular velocity Ω) driven by a differentially rotating rigid lid (angular velocity $\Omega + \omega$) provides some guidance for the analysis of our observations. Hart (1972) showed theoretically and experimentally that this steady flow had frictional boundary layers at the top, bottom and edges of the cylinder, and that the interior fluid was in rigid rotation with differing angular velocities in each of the layers. Additionally, an analysis of the stability of the interior solutions showed that the existence of stable axisymmetric flow was determined by the internal Froude numbers, $F_{1,2}$, and Hart's parameter $Q_H = E^{1/2}/\varepsilon_H$, where $\varepsilon_H = \omega/2\Omega$ is the appropriate Rossby number for the driven instability problem. For fluids of equal viscosity and layer depth in particular, the interior velocity was shown to be $u_{\theta,1}/2\omega R = 0.75r/R$ in the upper layer, and $u_{\theta,2}/2\omega R = 0.25r/R$ in the lower layer. Further, this flow was shown to be stable in the domain of (F, Q_H) parameter space indicated in Figure 6. Note that the neutral curve labeled (m, n) refers to the m-th azimuthal and n-th radial mode.

In Hart's problem ε_H is a simple and direct measure of the difference in the angular velocities of the layers and the stability diagram, Figure 6, shows that for a particular value of F and E, the number of unstable modes increases as the shear between the layers $(u_{\theta,1} - u_{\theta,2} = \varepsilon_G \omega r)$ increases. Equivalently, as the shear is increased at constant F and E, the mode numbers of the first mode to become unstable depends strongly on the Froude number F.

As Pedlosky (1967) predicted, and as we have verified, the evolution of the azimuthal velocities during two-layer spin-up is complicated and the radial structure departs significantly from solid body rotation. The vertical shear is also complicated, as illustrated in Figure 7 which shows contours of the difference in the angular velocities

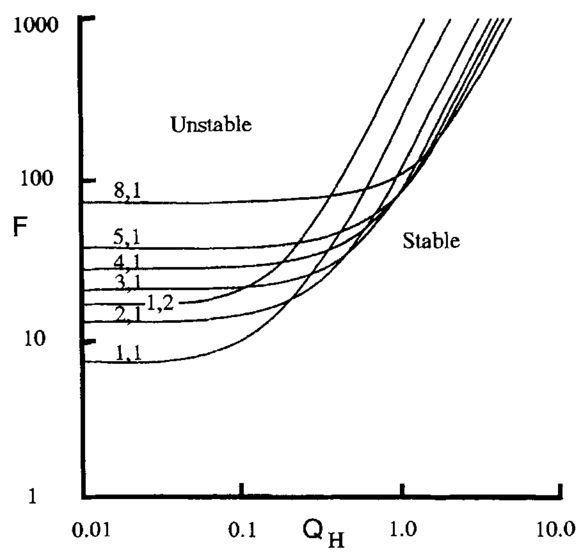


Figure 6 The stability diagram for the steady, surface stress driven, two layer baroclinic instability experiment of Hart (1972).

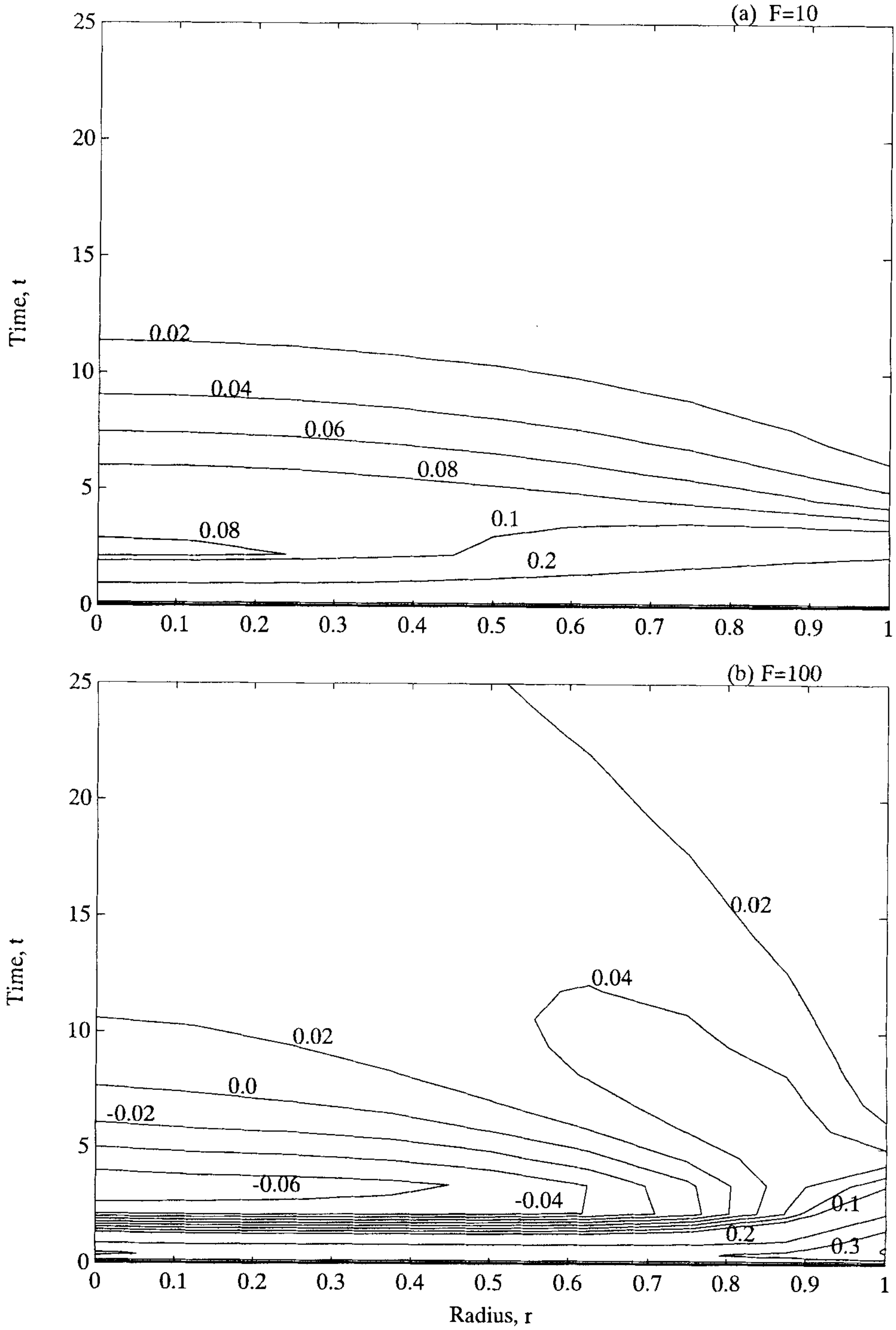


Figure 7 Difference in the angular velocities between the two layers as a function of radius and time in the spin-up theory of Pedlosky (1967) for $F = 10$ (a), and $F = 100$ (b).

between the layers, $\delta = (u_{\theta,1} - u_{\theta,2})/r$, as functions of radius and time for two values of the Froude number $F = 10$ and 100. The spin-up process begins with $\delta = 0$. It then rapidly increases as the lower layer accelerates and reaches a maximum at $t \approx 1$. Subsequently, radial variations develop and δ evolves through a secondary minimum at $t \approx 2$ before decaying slowly. Comparison of Figures 7(a) and (b) shows that the magnitude of the radial variations and the depth of the minimum are intensified at large Froude number.

Hart's stability analysis is not directly applicable to the spin-up problem for two important reasons: the flow field does not simply evolve slowly through a sequence of uniform angular velocity states, and the deformation of the free surface due to the centripetal acceleration of the fluid, results in non-uniform potential vorticity in the lower layer. However, it seems reasonable to expect that the Froude number, F, and the Hart parameter, Q, using the spin-up Rossby number ε as an appropriate measure of the interfacial shear, would be useful in the interpretation of the stability/instability information presented in Table 1. We therefore define $Q = E^{1/2}/\varepsilon$, and present the results of the experiments in (F, Q) parameter space in Figure 8. The open squares are used to indicate the stable experiments and the solid triangles represent the unstable experiments. It appears that all experiments with $Q < 0.03$ are unstable and those with $Q > 0.06$ are stable. Notice that the transition range, $0.03 < Q < 0.06$, is

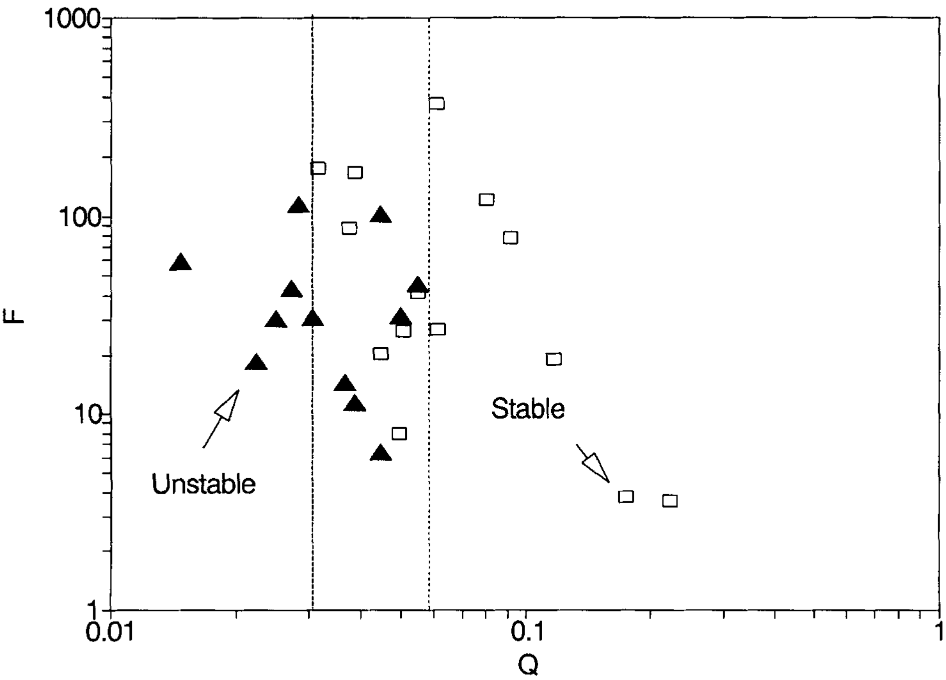


Figure 8 Stability dependence on the Froude number, F, and the Hart parameter, $Q = E^{1/2}/\varepsilon$. Experiments that were visibly unstable are denoted by the solid triangle and those that were stable are indicated by open squares. The transition region is indicated by the dashed lines.

significantly lower than the $0.25 < Q_H < 1.0$ range in Hart's stability diagram (Figure 6) for the Froude numbers used in the experiments ($F < 400$). This is simply because ε does not uniquely represent the shear during most of the spin-up process and, as can be seen by referring to Figure 7, actually overestimates it by a factor O(10).

Since Pedlosky's (1967) theory indicates that the radial gradient of the potential vorticity in the layers are of opposite sign during spin-up, and Figure 8 suggests that the instabilities in the experiments can be classified in the same manner as Hart's (1972) baroclinic instability experiments, the source of the flow asymmetries appears to be the baroclinic instability mechanism. The reason for the observed decay of the instabilities is uncertain at the moment. It is possible that as the shear between the layers evolves, the flow moves into and then out of the unstable regime. If this occurs quickly relative to the growth rate of the unstable modes, then the instabilities may never be observable. Though this interpretation is consistent with the observations, we must equivocate since interfacial tension, which has been ignored here, would also damp the high wave number modes.

6. CONCLUSIONS

The comparison of the theoretical predictions of Pedlosky (1967) for the spin-up of the azimuthal velocity in the upper of two layers of immiscible fluids in a cylindrical tank to the results of the experiments described in Section 3 show good agreement and, therefore, confirms the important role of the interfacial Ekman layer in the spin-up dynamics of immiscible layers. The experiments also demonstrate that a weak instability for $Q = E^{1/2}/\varepsilon < 0.06$ often arises in the outer area of the flow during the initial phase of the spin-up process. The instability subsequently evolves to higher wave numbers and then decays. Examination of velocity field evolution in the outer region of the tank in some experiments shows that spin-up is faster than predicted. This acceleration of spin-up may be an influence of the instability on the "mean" flow, however, there is no clear correlation between this discrepancy and the presence of interfacial instabilities. This problem deserves further study as the transport of angular momentum by eddies produced by baroclinic instability is expected to change the spin-up process.

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