Wave Activity I

Divide all quantities into mean (*x*-average) and disturbance parts

$$\chi = \overline{\chi} + \chi'$$

Apply to quasigeostrophic potential vorticity equation and linearize

$$\frac{\partial q'}{\partial t} + \overline{u}\frac{\partial q'}{\partial x} + v'\overline{q}_y = \mathcal{D}'$$

$$\overline{\frac{q'}{\overline{q}_y} \times \{ \qquad \}}$$

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\frac{\overline{q'^2}}{\overline{q}_y}\right\} + \overline{v'q'} = \frac{\overline{q'\mathcal{D}'}}{\overline{q}_y}$$

Wave Activity II

Useful identity

$$v'q' = \psi'_x\left(\psi'_{xx} + \psi'_{yy} + (\psi'_z \frac{f_0^2}{N^2})_z\right)$$

$$= \left(\tfrac{1}{2}\psi'^2_x\right)_x + (\psi'_x\psi'_y)_y - \left(\tfrac{1}{2}(\psi'_y)^2\right)_x + \left(\psi'_x\psi'_z\frac{f_0^2}{N^2}\right)_z - \left(\tfrac{1}{2}(\psi'_z)^2\frac{f_0^2}{N^2}\right)_x$$

$(\psi'_x\psi'_y) \uparrow -u'v'$; $\psi'_x\psi'_z \uparrow -gf_0v'\tilde{\rho}'/N^2\rho_0$

Wave Activity Conservation Relation

$$\frac{\partial}{\partial t}\left\{\frac{1}{2}\frac{\overline{q'^2}}{\bar{q}_y}\right\} + \frac{\partial}{\partial y}\left\{-\overline{u'v'}\right\} + \frac{\partial}{\partial z}\left\{-\frac{gf_0}{\rho_0 N^2}\overline{v'\tilde{\rho}'}\right\} = \frac{\overline{q'\mathcal{D}'}}{\bar{q}_y}$$

wave activity = ‘wave stuff’

$$\frac{\partial \mathcal{A}}{\partial t} + \frac{\partial F^{(y)}}{\partial y} + \frac{\partial F^{(z)}}{\partial z} = \mathcal{D}_{\mathcal{A}}$$

density ($\mathcal{A}$), *y*-flux ($F^{(y)}$), *z*-flux ($F^{(z)}$), dissipation ($\mathcal{D}_{\mathcal{A}}$)

‘Eliassen-Palm flux’

$$\mathbf{F} = (F^{(y)}, F^{(z)})$$

Wave Activity III

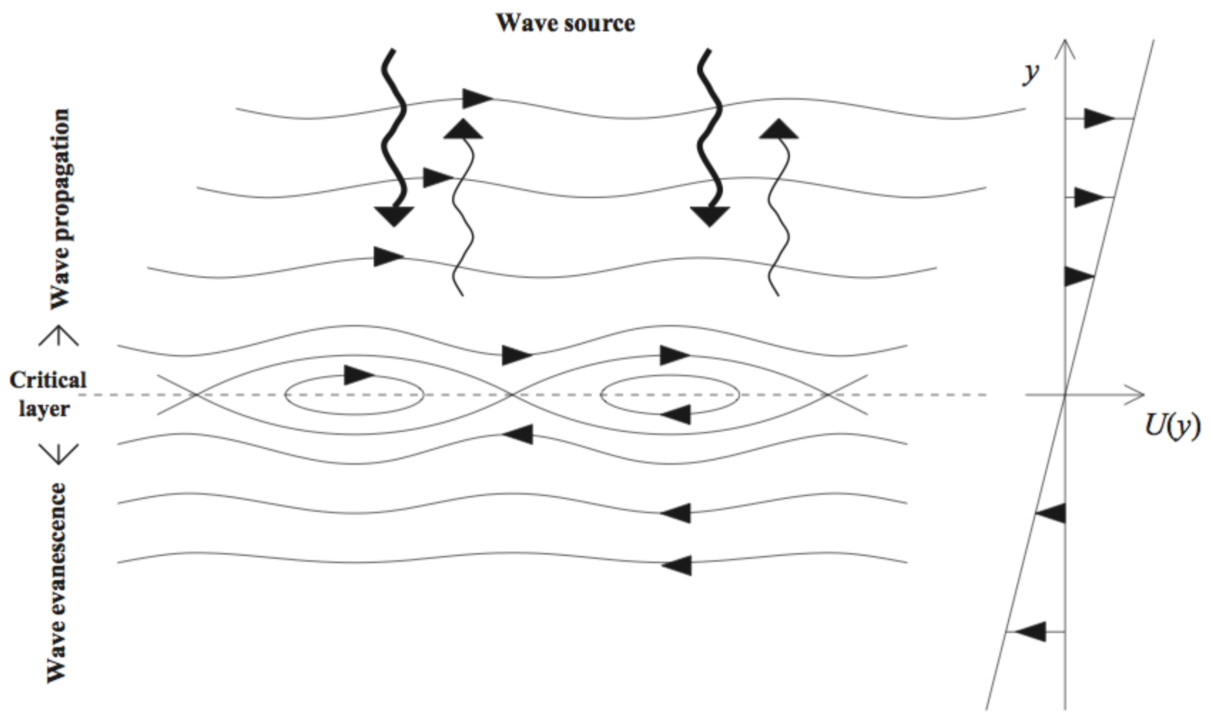
Lower boundary condition with topography h' and vertical shear $\overline{u}_z = 0$

$$F^{(z)} \left(= \overline{\psi'_x \psi'_z} \frac{f_0^2}{N^2} = -\frac{g f_0}{\rho_0 N^2} \overline{v' \tilde{\rho}'} \right) = -f_0 \overline{v' h'}$$

Lower boundary condition with vertical shear $\overline{u}_z$ and topography $h' = 0$

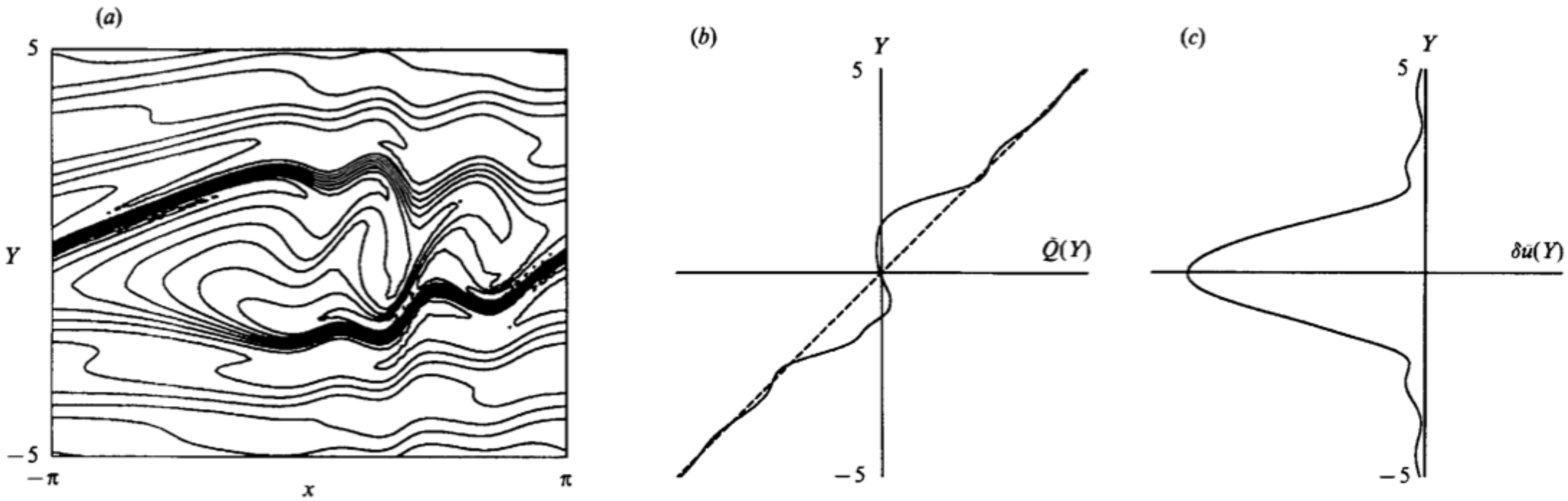
$$F^{(z)} = -\frac{\partial}{\partial t} \left(\frac{1}{2} \frac{f_0^2}{N^2} \frac{\overline{{\psi'_z}^2}}{\overline{u}_z} \right)$$

The Rossby-wave critical layer problem



See PHH 2015 Encyclopedia of Atmospheric Sciences article for further details.

McIntyre and Norton (1990)



Advective rearrangement of Q field by nonlinear effect of waves

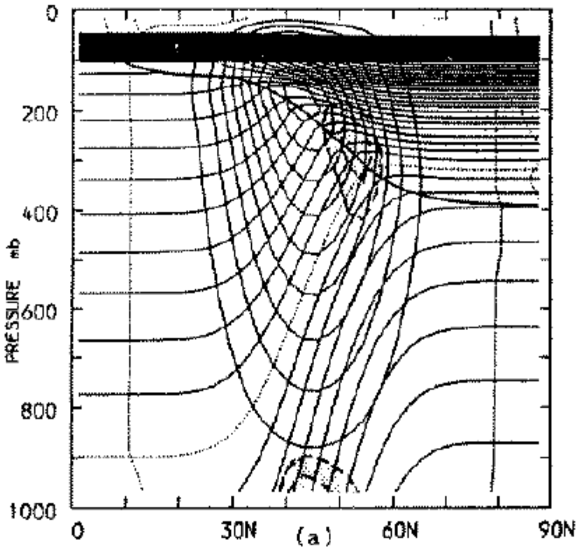
Change in $\overline{Q}$

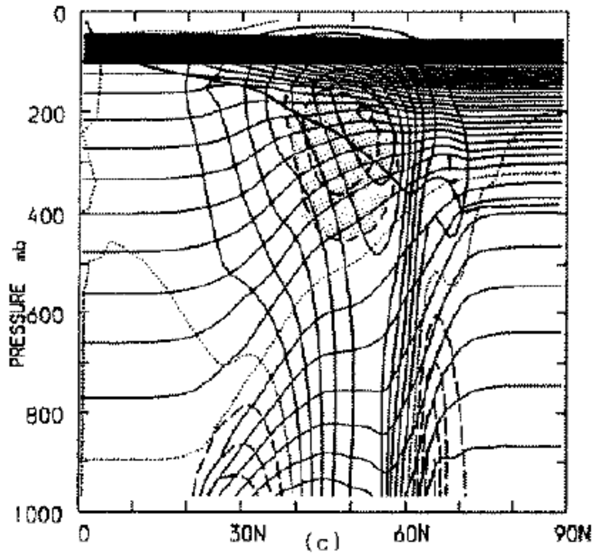
Change in $\overline{u}$

Mean flow evolution in a baroclinic lifecycle

Thorncroft et al (1993), Simmons and Hoskins (1978)

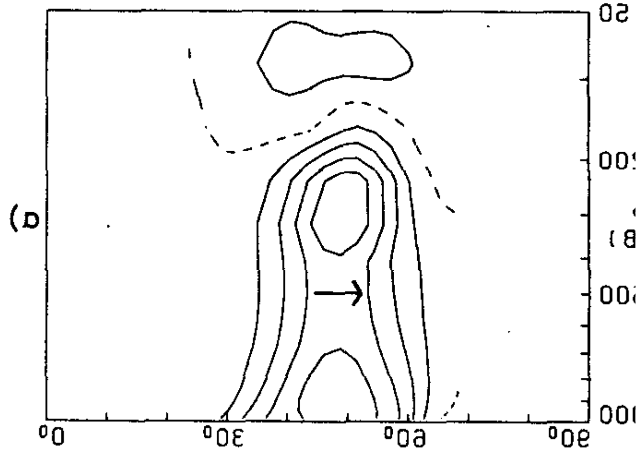
Day 0

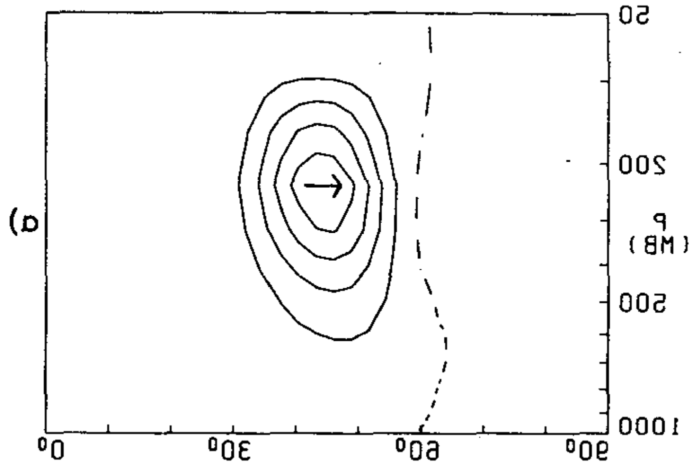




Day 7

$\overline{v'T'}$



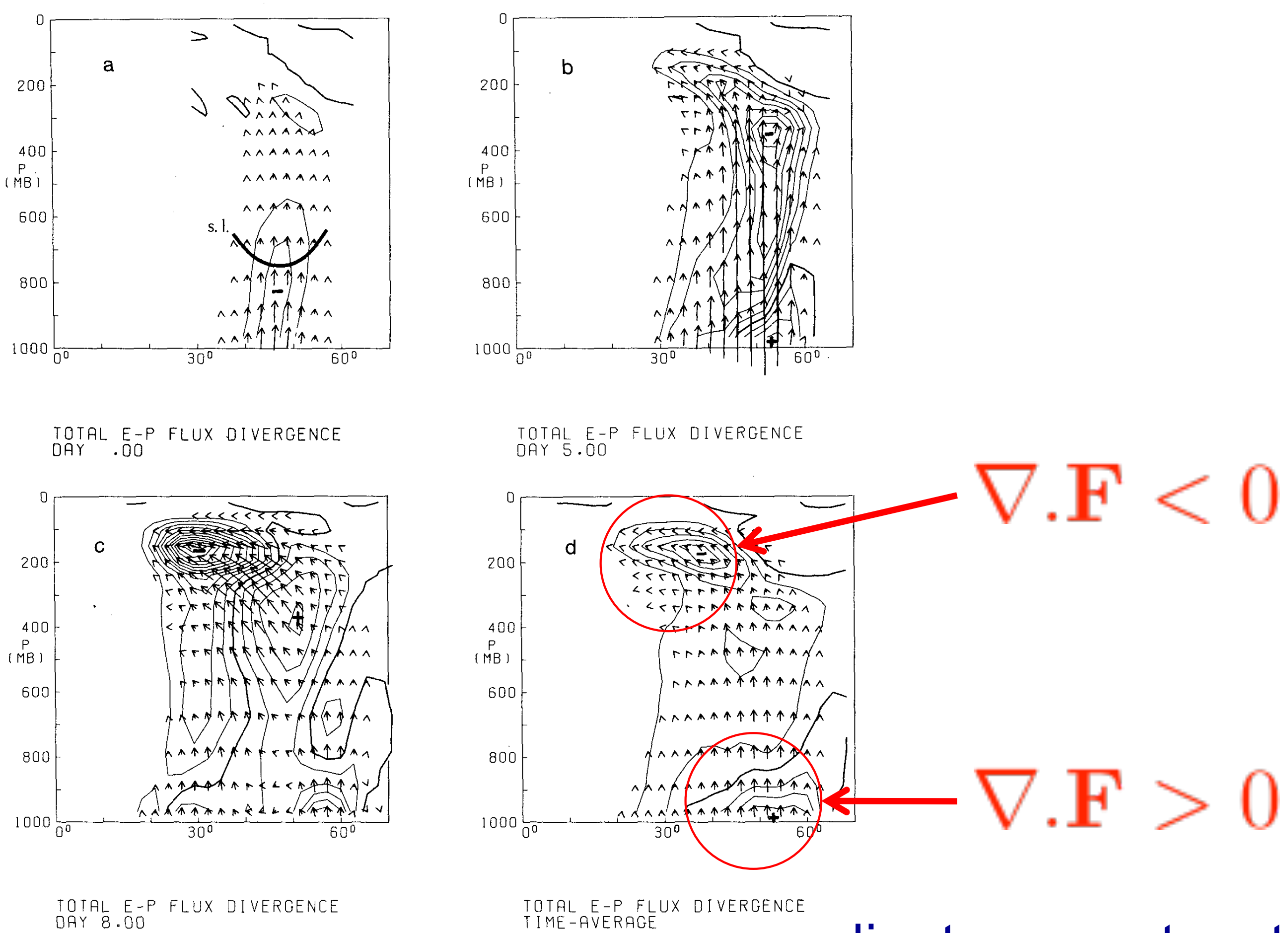


$\overline{u'v'}$

upgradient momentum transport

EP flux cross sections for the nonlinear baroclinic lifecycle -- implications for mean flow

Edmon et al (1980)



westward force

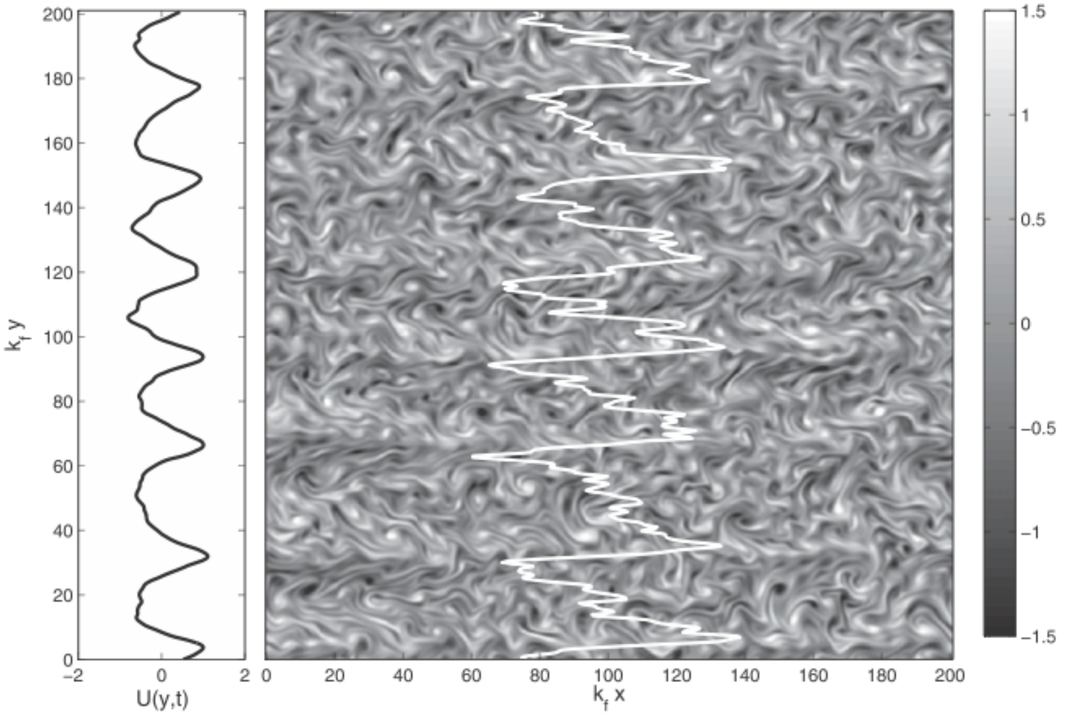
eastward force

upgradient momentum transport is a natural consequence of wave propagation

β–plane turbulence and jet formation

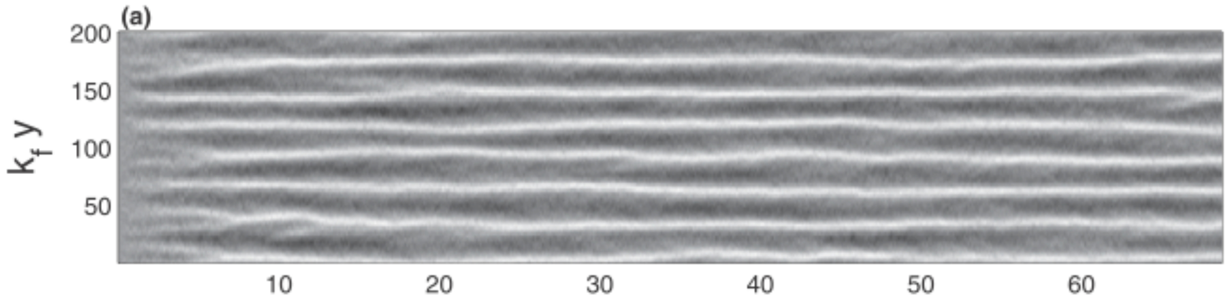
Srinivasan and Young 2012

mean flow



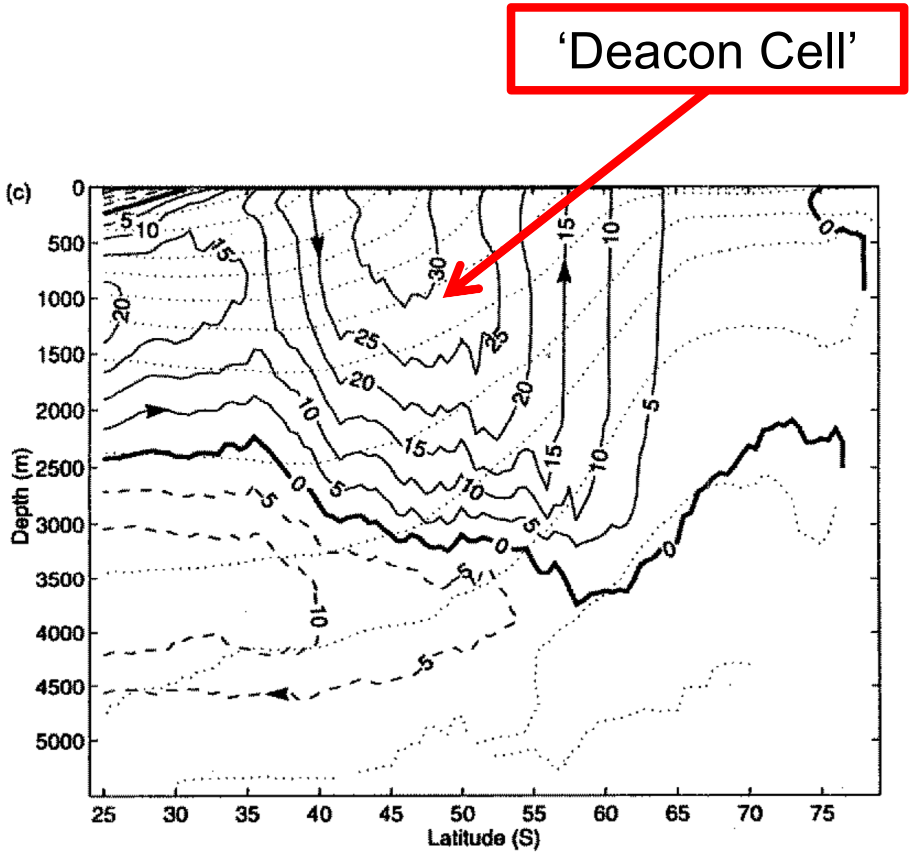
snapshot of vorticity field

Time evolution

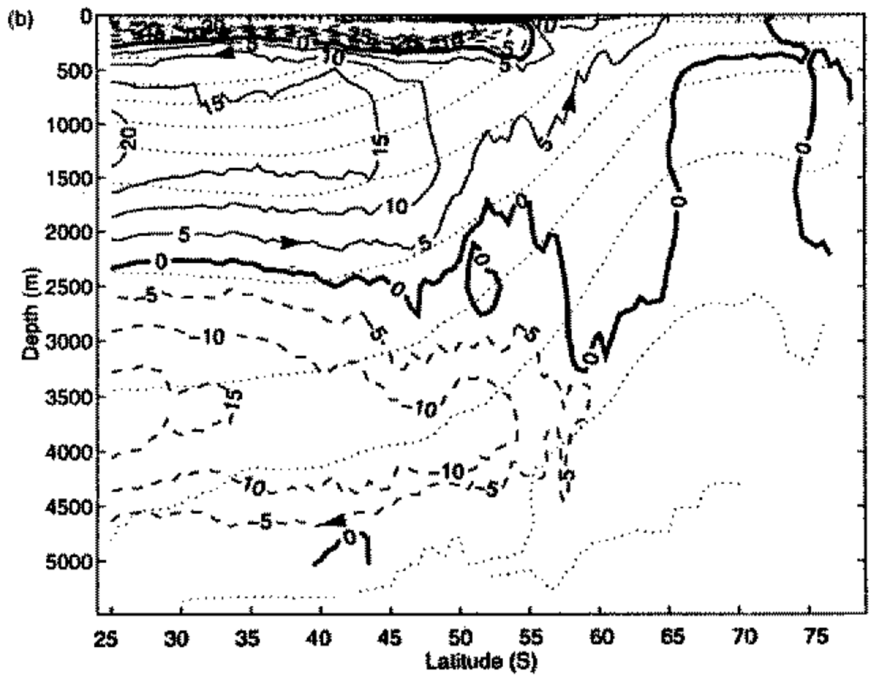


Southern Ocean circulation

Karoly et al 1997



Eulerian mean circulation



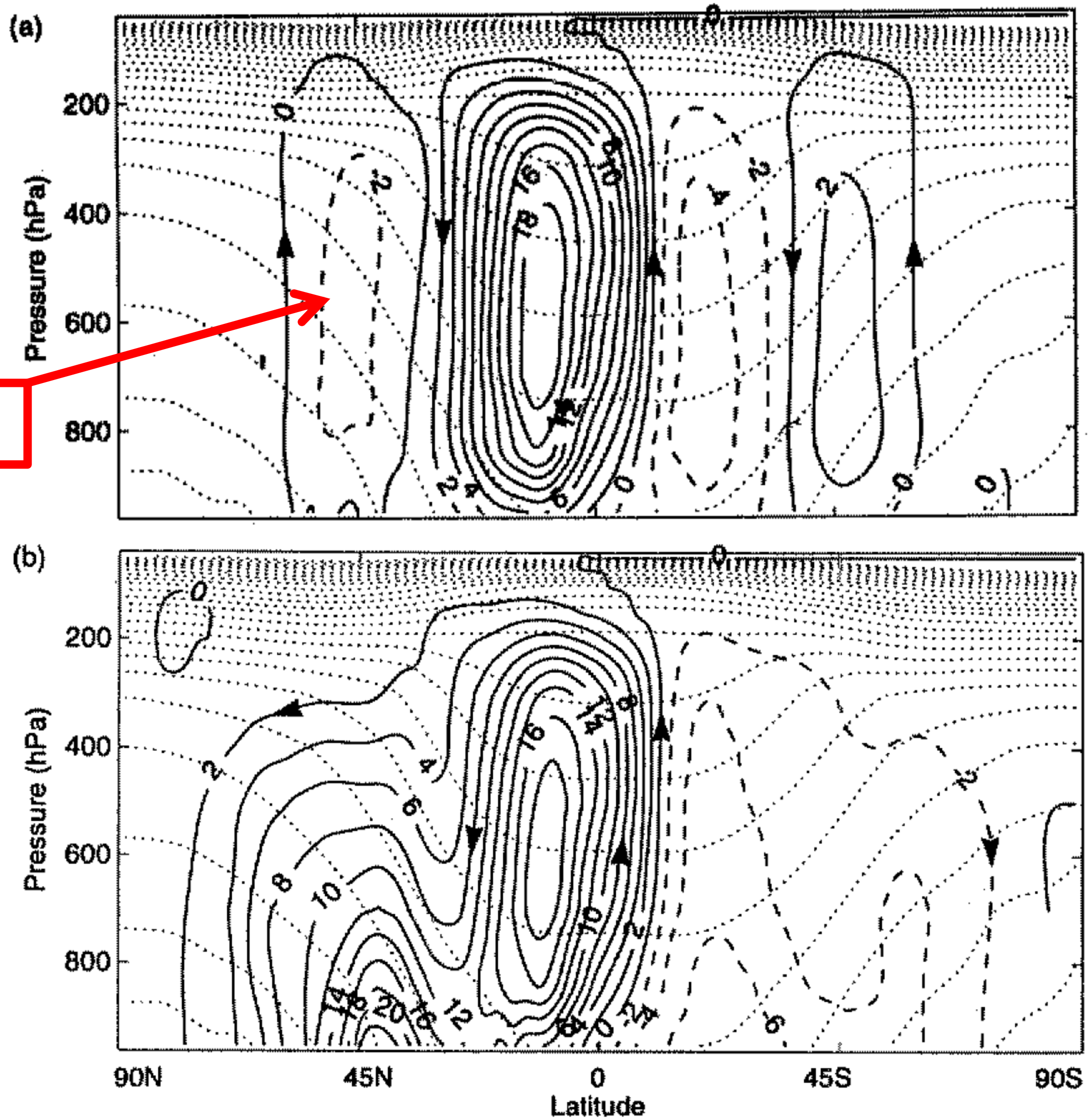
Transformed Eulerian mean circulation

Atmospheric circulation

Karoly et al 1997

Eulerian mean circulation

'Ferrel Cell'



Transformed Eulerian mean circulation