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Transport in Geophysical Fluids -- overview

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The Lagrangian approach has yielded many useful insights to transport over the last 25 years or so, both in idealised flows and in realistic atmospheric and oceanic flows. One flavour of the Lagrangian approach is essentially diagnostic -- a flow field is described in an Eulerian framework either by solution of dynamical evolution equations or by processing observational data using an analysis or assimilation procedure based on an underlying predictive numerical model. The properties of this (Eulerian) flow may then be explored by calculating Lagrangian quantities. These might involve following ensembles of advected particles in forward or backward time or by calculating stretching rates following such particles. Dynamical systems theory provides more sophisticated diagnostics that may, for example, identify contours or surfaces across which there is minimal transport. A second flavour is focused on the prediction of the transport and stirring of particular advected species, e.g. chemical species or biological species, where a Lagrangian formulation may have advantages over an Eulerian formulation that solves the advection equation as a partial differential equation. A third flavour is using Lagrangian methods to solve the dynamical equations. This is relatively undeveloped in its application to atmospheric and oceanic flows, except for highly idealised systems, not least because the complexity and computational demands of the dynamical equations. The papers in this Chapter cover these three different flavours of the Lagrangian approach.

Key aspects of the development of the Lagrangian approach to the study of transport and stirring in geophysical flows have been, on the one hand, the recognition of the phenomenon of chaotic advection and the development of mathematical tools to quantify transport in dynamical systems and, on the other, the increasing availability of comprehensive data on velocity fields in the atmosphere and, to a lesser extent, the ocean. ('Stirring', sometimes called 'mixing', is the process of deformation through differential advection that leads to complex geometric structure in advected, e.g. chemical, fields and eventually leads to real physical mixing under the effect of molecular diffusion.) The paper by Aref (1984) is regarded as a key publication in the recognition of chaotic advection as a phenomenon and in the development of relevant quantitative technique. Aref (2002) provides a nice commentary on this paper and how it was received. For example, from the point of view of one referee, Aref (1984) was simply repeating work (albeit important work), that had recently been carried out in dynamical systems. But the value of Aref (1984) and related work was the recognition that the mathematical machinery then being developed in dynamical systems theory was directly relevant to interesting problems in fluid dynamics. Much of the detailed theoretical work in fluid dynamics that followed Aref's paper was focused on time-periodic flows, since time periodicity adds the essential ingredient that allows non-trivial transport and stirring behaviour. (In steady flows transport is along streamlines, so streamlines are perfect barriers to transport, and there are strong resulting constraints on stirring.) But the notion of chaotic advection extends beyond

time-periodic flows and indeed it must do so to have any relevance to the real atmosphere or ocean. From one viewpoint, the important insight of chaotic advection is that complex structures in advected tracer fields and complex patterns of tracer transport can arise in flows with relative simple space-time structure (e.g. Pierrehumbert and Yang 1993); these patterns do not depend on a highly complex space-time structure of the underlying (Eulerian) flow, e.g. that envisaged in three-dimensional turbulence. The Pierrehumbert and Yang (1993) study of the upper troposphere and the Bowman (1993) study of the Antarctic winter stratosphere are early examples of studies that apply some of the ideas and tools of chaotic advection to realistic atmospheric velocity fields. The many other studies over the last 20 years that have used relatively coarse resolution data from atmospheric models or from analysed datasets to calculate patterns of transport and stirring have relied on the notion of chaotic advection, whether or not those carrying out these studies have realised this.

A continuing challenge for dynamical systems techniques applied to chaotic advection has been to find robust methods of identifying 'coherent structures', i.e. bodies of fluid that remain to some approximation isolated or, correspondingly to identify the partial transport barriers that bound such structures. It is helpful to contrast two extreme cases, e.g. in consideration of a relatively isolated atmospheric or oceanic vortex. The vortex is a quasi-Lagrangian entity and an Eulerian view, defining the boundary of the vortex as a surface that is fixed in space, will measure large instantaneous fluxes across the boundary. Those instantaneous fluxes may largely cancel in a time-average, but a reliable estimate of the time-average flux will rely on accurate calculation of those cancellations. The opposite extreme is a purely Lagrangian view, where the boundary is defined by a material contour that is initially co-located with the dynamical boundary of the vortex. Since the boundary is a material contour then there will be no transport across it, but if the vortex is not a 'perfect' Lagrangian entity then the contour will typically be strongly deformed by deformation events and will be drawn into the region outside (or inside) the vortex and become highly distorted. The 'best' definition of the vortex boundary, e.g. in providing the most useful estimate of exchange between the vortex and its exterior, is some kind of compromise between the pure Eulerian view and the pure Lagrangian view. Dynamical systems theory has offered several such definitions, e.g. see the review by Wiggins (2005), and others have been motivated more by fluid-dynamical considerations, e.g. the hybrid Eulerian-Lagrangian effective diffusivity diagnostic (Nakamura 1996, Shuckburgh and Haynes 2003).

One approach to the objective identification of Lagrangian coherent structures has been proposed by Haller (2001 and references therein) and is based on calculation of Lagrangian stretching rates ('finite-time Lyapunov exponents') with the spatial variation of these stretching rates being used to characterise the boundaries of the coherent structures. This approach has been further developed and analysed, with comparison against other approaches, by Shadden et al (2005) and Branicki and Wiggins (2010). The paper by Sulman et al in this volume analyses the calculation of Lagrangian stretching rates in geophysical flows, considering in particular the implications of three-dimensionality in the flow. (Most calculations of stretching rates in geophysical flows assume the flow is quasi-horizontal and consider only the stretching due to the two-dimensional flow on each quasi-horizontal, e.g. density, surface.) Sulman et al emphasise the role of vertical shear in stretching. This has

previously been investigated by Haynes and Anglade (1997), Haynes and Vanneste (2004) and Smith and Ferrari (2009), amongst others, in considering the interaction between large-scale stirring and small-scale mixing processes in the atmosphere and ocean, but the implications for identification of coherent structures has not previously been considered. Another distinct aspect of three-dimensionality not considered by Sulman et al but of potential interest in certain geophysical flows is the 'resonance-induced dispersion discussed by Cartwright et al (1996).

Two further papers in this Chapter, Liberato et al and Orza et al use Lagrangian descriptions in meteorological studies. The first, Liberato et al, considers the transport of water vapour as a precursor to an extreme precipitation event in Portugal. This study, and several like it that have appeared recently, reflect a significant change over recent years in the description of water vapour transport from tropics and subtropics to the extratropics. A traditional approach might have been to express this transport in terms of Eulerian eddy fluxes. But, beginning in the early 1990s improvements in global atmospheric datasets began to show that the transport was dominated by thin longitudinally confined regions, indicating the extension of thin filaments of high water vapour concentrations into the extratropics. Newell and colleagues, e.g. Newell et al (1992), termed these features 'atmospheric rivers', though this terminology has been criticized as implying transport along a fixed horizontal path (e.g. Bao et al, 2006). Synoptic meteorologists tend to describe these features in terms of the 'warm conveyor belts' of extratropical cyclones, but it seems fair to say that the importance of these features in large-scale, as distinct from synoptic-scale, water vapour transport was recognised only relatively recently. It is natural to use Lagrangian techniques to quantify and analyse this sort of transport-- a recent comprehensive climatological description is given by Knippertz and Wernli (2010). The Liberato et al study (this volume) shows how long-range transport of water vapour from the subtropics was important in setting up the large-scale conditions required for the extreme precipitation event. One difference between the transport of water vapour identified in this and similar studies and the transport of chemical pollutants (see Brunner, this volume) is that the 'source region' for the water vapour is typically large-scale, a large section of the subtropical North Atlantic in the Liberato et al case. Thus the (relative) localisation of large concentrations of water vapour in a certain region of the extratropics results from the nature of the transport rather than any localisation of the source region. The Orza et al study uses a Lagrangian description as a basis for characterising the interannual variability of transport pathways and relation of this variability to the North Atlantic Oscillation, thereby combining the Lagrangian approach with a more traditional description of long-term atmospheric variability.

The second theme addressed in this Chapter, by the article of Konopka et al, is the use of Lagrangian methods in a systematic way to predict chemical fields over some finite region (possible global). Development of this approach was stimulated by the success in the 1990s of Lagrangian methods in providing, given estimates of large-scale wind fields, high-resolution reconstructions of chemical fields, e.g. in the stratosphere (Sutton et al 1994) and in the upper troposphere lower stratosphere region (Newman and Schoeberl 1995). The practical advantage of the Lagrangian approach over the Eulerian approach is that it straightforwardly offers solution of the advection equations, without any artificial mixing effect associated with representing advection on a Eulerian grid. However, eventually this practical advantage becomes a problem. Firstly in the real atmosphere (or ocean) there is mixing associated with molecular

diffusion, perhaps augmented by the small-scale stirring effects of three dimensional turbulence. So there is a significant problem in that a real physical effect is missing in a purely Lagrangian model. Additionally, there is a practical problem that solution of the forward-in-time advection equation by Lagrangian methods, i.e. by carrying along a number of advected particles, almost inevitably requires generation of new particles (and correspondingly removal of existing particles) to maintain a roughly uniform spatial coverage. In fact, any attempt to resolve one of these problems naturally provides a potential resolution of the other, for example the process of removing nearby particles implies a sort of mixing, since it limits the minimum spatial scales in chemical concentration fields. However, the implied mixing does not necessarily correspond to any physically realistic mixing process. The CLAMS model developed at Germany's Forschungszentrum Jülich (e.g. Konopka et al 2004) has been formulated with the aim that the implied mixing does have a physical basis, by basing the criterion for particle removal on local stretching rates. The Konopka et al paper in this volume considers this physical representation further in a three-dimensional context, emphasising that formation of small vertical scales in chemical fields (with implications for vertical mixing) is expected to depend strongly on static stability and arguing that a vertical distribution of particles based on consideration of an entropy coordinate results in mixing that is a good match to physical reality.

The final topic touched by this Chapter is the Lagrangian approach to the solution of the dynamical equations. It is certainly the case that in some areas of computational fluid dynamics Lagrangian methods are highly developed, see e.g. the review by Koumoutsakos (2005). These methods have yet to be extensively used in atmospheric modelling, though see Alam and Lin (2008) for implementation of one possible scheme and analysis of some test cases. There has been some use of Lagrangian methods for certain restricted classes of flows that arise in geophysical fluid dynamics, where the structure of the equations naturally leads itself to a Lagrangian description. One example is contour dynamics for two-dimensional and quasi-geostrophic vortex dynamics (e.g. Dritschel and Ambaum 1997) and extensions to more general sets of equations such as the primitive equations (Mohebalhojeh and Dritschel 2004) where the Lagrangian description for certain dynamical variables can be combined with an Eulerian description for others. There has also been use of what are essentially hybrid Lagrangian-Eulerian methods, where, for example, the vertical coordinate is quasi-Lagrangian and the horizontal coordinate is Eulerian (Lin 2004, Shin et al 2012). The Haertel paper in this issue presents a fully Lagrangian model whose formulation is motivated by the particular structure of stably stratified flows in hydrostatic balance. The model demonstrates some significant successes, e.g. in the simulation of Madden-Julian oscillations, perhaps as a result of beneficial effects of a Lagrangian representation of convective overturning. It might be that the next key stage in the development of this model (or other models with a new and 'unconventional' structure) is to encourage use by a wider community so that the advantages and disadvantages can be thoroughly explored.

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