Mechanics (non-examinable) Examples sheet 1 Dr P. J. O'Donnell

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On these sheets, no attempt is made to 'model' real-life situations: no trains, cars, cyclists, lifts, etc. It is assumed that there are no 'real' forces, such as air-resistance unless they are specifically mentioned. Most questions, but not all, avoid numbers and units, preferring general algebraic formulae with consistent dimensions.

1 A particle is in equilibrium under the action of three forces of magnitudes $3P$, $5P$ and $7P$. Show that the angle between the forces with magnitudes $3P$ and $5P$ is $\cos^{-1}\frac{1}{2}$.
[If you resolve forces, you should choose two sensible directions. You could also do this geometrically, representing the three vector forces as the sides of a triangle.]

2 A particle of mass m is attached to one end of a light inextensible string, the other end of which is fixed to a point O. The particle is acted on by a horizontal force P so that the particle is in equilibrium and the string is inclined at an angle θ to the vertical. Find P in terms of m, g and θ.

3 The ends of light inextensible string are attached to two fixed points A and B at the same horizontal level. A smooth ring O of mass m, which can slide freely on the string, is acted on by a horizontal force of magnitude P. The string is taut and the two sections of the string, AO and BO, make angles of θ and ϕ respectively with the vertical through O. Assuming that the tensions in the two sections of the string are the same, show that

$$P = \frac{|\sin\phi - \sin\theta|}{\cos\phi + \cos\theta} mg.$$

4 A particle of mass m rests on a rough horizontal plane. The coefficient of friction between the particle and the plane is μ. The particle is in limiting equilibrium under the action of a force of magnitude P inclined at an angle θ to the upward normal to the plane ($0 < \theta < \pi/2$). Show that

$$P = \frac{\mu mg}{\sin\theta + \mu\cos\theta}.$$

[Recall that $F = \mu R$ for a particle in limiting equilibrium, where F is the frictional force opposing the tendency to move and R is the normal reaction.]

5 A particle of mass m rests on a rough plane inclined at an angle α to the horizontal. It is subject to a force of magnitude P acting at an angle θ to the plane, which *just* prevents the particle from slipping down the plane (so that it is in limiting equilibrium). Show that

$$P = \frac{mg\sin|\alpha - \lambda|}{\cos(\theta + \lambda)}$$

where λ is the acute angle satisfying $\tan\lambda = \mu$ (the so-called *angle of friction*) and $\alpha > \mu$. What is the corresponding result if P is such that the particle is on the point of slipping up the plane?